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On the allocation of commodities by queuing and the prevention of violence

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Abstract

In times of acute scarcity, demand for a commodity greatly exceeds its supply. In such situations, queuing mechanisms are frequently used to allocate scarce goods to citizens. However, inordinately long queues lead to excessive wait times and this can lead to violence. As such, the general purpose of this paper is to theoretically analyze the problem of preventing violence in a queuing context. To this end, we first formulate a queuing model with a finite capacity. Next, we determine the smallest capacity that will keep the likelihood of violence below an exogenously specified value. Finally, we illustrate the working of our model with a simple numerical example.

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1. Introduction

In many countries and especially in developing countries, it is common to use *queuing* mechanisms to allocate commodities or goods to citizens. Examples of goods that are often allocated in this manner during normal economic times include rice,[1] groundwater,[2] and banking services.[3] A basic feature of these goods allocation mechanisms is that they involve waiting in line by citizens. In other words, citizens have to actually wait in queue to obtain the good that is being allocated by either a public or a private provider.

These queuing mechanisms take on particular significance during times of acute scarcity brought on by one or more governmental policies. In fact, during such difficult times, it is frequently the norm in many developing nations to allocate the available supplies of the relevant scarce goods by means of the aforementioned queuing mechanisms. What is important for our purpose is that in times of acute scarcity, demand for a good greatly exceeds its supply. As a result, if a good characterized by excess demand is to be allocated to citizens by means of a queuing mechanism then, in the time period of interest, it is important for a public or a private provider to regulate the *length* of the queue. Concrete examples from two different countries show why.

In recent times, President Hugo Chavez's desire to bring the powerful state owned oil company *Petroleos de Venezuela* (PDVSA) under his influence has caused considerable political turmoil in Venezuela. As the Economist (Anonymous, 2003a) has reported, there was an acute shortage of both cooking gas and petrol. As a result, in some provider facilities, there were queues of citizens that could be measured not in hours but in days. This situation gave rise not only to a secondary market for places in the fuel queues but it also resulted in considerable economic unrest and *violence*.

As a second example, consider the case of contemporary Zimbabwe. Because of President Robert Mugabe's misguided policies, Zimbabwe is in poor economic shape. As a result, many essential commodities are in short supply and it is routine for both public and the remaining private providers to allocate these scarce commodities by means of queuing mechanisms. The Economist (Anonymous, 2002; 2003b) describes instances where queues for essential commodities were so long that the frustrations of Zimbabwean citizens boiled over and this led, as in Venezuela, to *violence*.

The examples in the previous two paragraphs tell us why it is essential for a public or a private provider to regulate the *length* of the relevant queues. As a referee has suggested, it can be useful to think of inordinately long queues—that result in excessive wait times—as a necessary condition for

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1 See Gunawardana (2000) for a discussion of Sri Lanka.

2 See Wood (1999) for an account of the state of Bihar in India.

3 See Woldie (2003) for a discussion of Nigeria.

violence. In addition to this, a deteriorating law and order situation and/or corruption can also lead to violence.[4] Given this discussion, it is clear that if the provider of a scarce good by means of a queuing mechanism is to prevent violence then, in the germane time period, (s)he should be able to regulate the capacity of the queue. Such an action effectively caps the length of the queue and this prevents the phenomenon of overly long citizen wait times.

Despite the obvious importance of the question of optimally determining the capacity of a queue in the context of the allocation of scarce commodities, to the best of our knowledge, this question has received scant attention in the economics literature. Lui (1985) and more recently Batabyal and Nijkamp (2004) and Batabyal and Yoo (2004) have used queuing models to analyze bribery and corruption. Although all three of these cited papers refer to the optimal queue capacity determination question, they do *not* explicitly analyze this question from the standpoint of the prevention of violence. Consequently, the general purpose of our paper is to theoretically analyze the problem of preventing violence in a queuing context.

The rest of this paper is organized as follows. Section 2 delineates a Markovian queuing model[5] with finite capacity. Section 3 first determines the smallest capacity that will keep the likelihood of violence below an exogenously specified value. Next, this section illustrates the working of our model with a simple numerical example. Section 4 concludes and then discusses three ways in which the research of this paper might be extended.

2. The Queuing Theoretic Framework

Consider a particular region within a developing country in a time of acute economic scarcity. Within this nation, we focus on a public or private provider who is in charge of allocating an essential but scarce commodity to citizens. It takes this provider a *random* amount of time to allocate the good in question to citizens. We suppose that this random allocation time is exponentially distributed with parameter β.[6] To obtain the good under study, citizens arrive at our provider's facility in accordance with a Poisson process with rate α. They then join a queue and wait in this queue until it is their turn to be served by the provider. In the parlance of queuing theory, our provider is the single server and

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We are not suggesting that long queues will *always* lead to violence. In fact, in the next paragraph, we explicitly model this aspect of the problem by supposing that the occurrence of violence is a *probabilistic* event. In this regard, it is also helpful to recognize that in planned economies such as the former Soviet Union and China before 1978, there often were long queues but such queues did not always lead to violence.

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See Taylor and Karlin (1998, chapter 9) and Ross (2003, chapter 8) for textbook accounts of Markovian queuing models.

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In queuing models, it is very common to assume that the service times (in our case the good allocation times) are exponentially distributed. Indeed, this is the standard assumption in all Markovian queuing models. For more details on this point, see Taylor and Karlin (1998, pp. 547-557) and Ross (2003, pp. 480-495). In addition, economists analyzing the allocation of goods by queuing mechanisms have also frequently assumed that the service times are exponential. For a more detailed corroboration of this claim, see Lui (1985) and Batabyal (2005).

we are analyzing a variant of the prominent model known as the $M/M/1$ queuing model.[7]

To model the problem of preventing violence, suppose that whenever the total number of citizens in our provider's facility exceeds a capacity C, there is violence because this provider is unable to allocate the commodity in question in a reasonable amount of time. From the perspective of the waiting citizens, this violence occurs because the queue length is too long and hence the wait times to obtain the commodity being allocated by our provider are excessive. Now, note that the occurrence of violence is not a deterministic but a *stochastic* event. Therefore, we suppose that our provider would like to keep the likelihood of violence below some exogenously specified value. Because this exogenously stipulated value is a probability, we can always write this probability as a fraction $1/x$ where $x>1$.[8]

The reader will note that the focus of our provider is on the *length* of the queue and the probability $1/x$. As such, we have to have some criterion under which the smallest capacity C will be computed to keep the likelihood of violence below $1/x$. Given our queuing theoretic approach to the problem of preventing violence, the most reasonable such criterion is the steady state probability distribution of the queue length.[9] Given this criterion, we are now ready to state the central question of this paper and that question is this: What is the smallest capacity C that will keep the likelihood of violence, under the steady state probability distribution of the queue length, below an exogenously specified value $1/x$? We now proceed to answer this question.

3. The Prevention of Violence

3.1. Optimal capacity determination

We begin by describing the concept of a steady state probability. To this end, let $Y(t)$ be the number of citizens in our provider's facility at time t. Then, the steady state probability P_k tells us the probability, as time approaches infinity, that there are exactly k citizens in the provider's facility. Mathematically, we have

$$P_k = \lim_{t\to\infty} Prob\{Y(t)=k\},\ k=0,1,2,... \tag{1}$$

Now, from equation 2.9 in Taylor and Karlin (1998, p. 549) we conclude that when the citizen arrival rate to our provider's facility (α) is less than the parameter (β) of the exponentially distributed random commodity allocation time, steady state probabilities for the queue length exist and they are

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Lui (1985) also uses a $M/M/1$ model to analyze issues pertaining to bribery and corruption.

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As an example, if our provider would like to keep the likelihood of violence below 0.01 then $x=100$ and the relevant fraction would be 1/100.

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See Taylor and Karlin (1998, pp. 548-550) and Ross (2003, pp. 478-483) for additional details on this concept.

given by[10]

$$P_k = (1 - \frac{\alpha}{\beta})(\frac{\alpha}{\beta})^k, \;\; k = 0,1,2,... \tag{2}$$

We are interested in determining the smallest capacity C that will keep the likelihood of violence, under the steady state probability distribution of the queue length, below the exogenously specified value $1/x$. Mathematically, we seek the smallest C for which the condition

$$\sum_{k=C+1}^{\infty} P_k = (\frac{\alpha}{\beta})^{C+1} \leq \frac{1}{x} \tag{3}$$

holds. The inequality in (3) above can be expressed in a more convenient form by first taking the logarithm and then simplifying the resulting expression. After several steps of algebra, we find that the smallest C satisfies a particular inequality and that inequality is

$$C + 1 \geq \frac{\log_e(x)}{\log_e(\beta/\alpha)}. \tag{4}$$

Inspection of the inequality in (4) tells us that the smallest C we seek depends on the Poisson arrival rate (α), the parameter of the exponentially distributed commodity allocation time (β), and the denominator (x) of the exogenously specified probability $(1/x)$.

We can think of the rate (α) as a demand parameter. Looked at in this way, the magnitude of this parameter tells us the level of demand for the good being allocated by our provider. Similarly, the parameter (β) can be thought of as an efficiency parameter for the supplier, i.e., the provider. Interpreted in this way, as the magnitude of (β) increases, the mean good allocation time decreases and hence the efficiency of supply increases. The inequality in (4) and this discussion together tell us that in a time of economic scarcity for the commodity under study, a provider who wishes to keep the likelihood of violence under the steady state probability distribution of the queue length below $1/x$, will choose the capacity of the queue in such a way so that this capacity plus unity is at least as big as the ratio on the right-hand-side (RHS) of the inequality in (4). Now, it should be clear to the reader that the ratio on the RHS of (4) is, in fact, a function of the aforementioned demand and supply efficiency parameters. We now provide a numerical example to show how our basic result might be operationalized in a specific circumstance.

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In the remainder of this paper, we assume that $\alpha < \beta$ holds. Without this condition, the queue length would be unbounded and, as such, it would not be possible to meaningfully analyze the $M/M/1$ queuing model.

3.2 A numerical example

Suppose that citizens arrive at our provider's facility in accordance with a Poisson process with rate $\alpha=2$. Further, let the parameter of the exponentially distributed commodity allocation time be $\beta=3$. Finally, suppose that our provider would like to keep the likelihood of violence below 0.001. Clearly, 0.001 can be written as 1/1000 and hence $x=1000$. Now substituting $\alpha=2$, $\beta=3$, and $x=1000$ in the inequality in (4) tells us that the smallest C satisfies

$$C+1\geq\frac{\log_e(1000)}{\log_e(3/2)}=\frac{6.9078}{0.4055}=17.0353. \tag{5}$$

It should be clear to the reader that the smallest C must be a positive integer. Therefore, inspection of the expression in (5) above tells us that when $(\alpha, \beta, x)=(2, 3, 1000)$, the smallest C equals 17. In other words, if the likelihood of violence is to be kept below 0.001 then, in the time period of interest, once seventeen citizens have entered the provider's facility, (s)he should not allow any more citizens in.

4. Conclusions

In this paper, we provided a theoretical perspective on the violence prevention problem confronting a public or private provider in times of acute scarcity where queuing mechanisms are used to allocate essential commodities to citizens. In particular, we first formulated a queuing model with a finite capacity. Next, we determined the smallest capacity that will keep the likelihood of violence, under the steady state probability distribution of the queue length, below an exogenously specified value. Finally, we used a simple numerical example to show how the result of our analysis can be operationalized in a particular setting.

The analysis of this paper can be extended in a number of different directions. In what follows, we suggest three potential extensions. First, in times of economic scarcity, quotas are sometimes used to allocate essential commodities. In our queuing theoretic analysis, the idea of a quota was *implicit* but not explicit. To see this clearly, note the following. In the first paragraph of section 2, we said that it takes a *random* amount of time to allocate the commodity in question to citizens. One way to understand why it takes a random amount of time is to suppose that our provider has to first determine what a particular citizen's quota for the good in question is and only then is (s)he able to provide the good to this citizen. A more explicit way of modeling the quota would be to posit that the provider's random allocation time is exponentially distributed not with parameter (β) but with parameter $(q\beta)$, where $q\in(0,1)$ measures the magnitude of the quota. *Inter alia*, this way of modeling the quota would make the allocation time parameter a *smaller* number and hence the mean allocation time would be a *bigger* number to allow for the fact that the commodity in question can be given to a citizen only after the quota has been determined. Second, we studied one kind of violence prevention problem, i.e., the prevention of violence stemming from excessive wait times which in turn arise when queue lengths are inordinately long. A second kind of problem arises when citizens who have been shut out from the provider's facility engage in violent acts to get into the facility. Consequently, one way to extend the analysis in this paper would be to study a model in which the possibility of both kinds of violence is explicitly modeled. Finally, we studied a queuing model in which we implicitly assumed that the provider is not venal. However, as the analyses of Basu *et al*. (1992), Kulshreshtha (2003), and others have shown, in many developing countries, this assumption is untenable. Therefore, it would be useful to extend the model of this paper to include the case in which citizens pay bribes to increase the admissible length of the queue. Studies that analyze these aspects of the problem will increase our understanding of the connections between the allocation of essential commodities by means of queuing mechanisms during difficult economic times and the problem of preventing violence.

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