E C O N O M I C S   B U L L E T I N

The effect of differentiated emission taxes: does an emission tax favor industry?

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Abstract

Extending a standard 2x2 Heckscher−Ohlin model to incorporate emissions, this paper investigates the effect of differentiated emission taxes on output and emissions in a small open economy. The following results are derived. First, raising the emission tax imposed on one industry may increase the output of that industry. This result is quite surprising in the sense that such a paradoxical result can occur in a simple and standard model under fairly plausible values of parameters. By numerical examples and using a graphical method, it is also shown that the mechanism behind the result is the factor market adjustment effects which work through two different channels. Second, while strengthening emission taxes uniformly across industries always reduces the volume of emissions, strengthening emission tax unevenly may increase it.

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1 Introduction

Emissions from production activities are one of the major causes of various environmental problems and regulations on emissions are regarded as an important policy subject (see, for example, UNFCCC, 1997). As a policy instrument for regulating emissions, emission tax has been attracting much attention and introduced in many countries. What should be noted is that in actual policies, such emission taxes are often implemented in a differentiated way, i.e. some industries are usually imposed lower tax than other industries or there are industries that are exempted from taxes (see OECD, 1994). Thus, it is of great importance to analyze what effects such differentiated regulations have on economies.

However, the previous theoretical studies on environmental regulations usually consider uniform emission taxes and the differentiated emission taxes have not been investigated adequately. To the author's knowledge, only exception is Hoel (1996). He considers the situation where there are both participants and non-participants to an international environmental agreement and shows that the optimal emission taxes for participants may be differentiated across industries. Although he investigates an interesting aspect in environmental regulation, he does not analyze in detail how emission tax affects output and emissions.

In this paper, we intend to analyze the effects of differentiated emission taxes in a general equilibrium setting. Extending a standard 2×2 Heckscher-Ohlin (HO) model to incorporate emissions, we focus on how the differentiated emission taxes affect output and emissions. By the term "differentiated emission tax" here, we mean a policy that changes the level of emission tax on one industry. Since we assume good prices as a given constant, our model represents a small open economy.[1] Although our model is a highly simplified one, we can show clear mechanism of how emission tax affects output and emissions.

2 The Model

We employ the standard 2×2 model and, as in previous literature on the subject, incorporate emissions as the third production factor.[2] Thus, the model has a structure similar to the standard 2×3 HO model employed in Batra and Casas (1976) and Jones and Easton (1983). However, there is one important difference from their models: while

[1]We can also regard the model as a production side of an economy.

[2]This approach is commonly used in general equilibrium models for environmental analyses, for example, Yohe (1979), Copeland and Taylor (1994, 1995), and Ishikawa and Kiyono (2000). The alternative approach is to assume that emissions are a function of output (and, in some cases, abatement activity) (see, for example, Markusen 1975 and Barrett 1997). Although this emission function approach may be more straight forward, it is employed mainly for partial equilibrium analyses and rarely employed in general equilibrium models because of its lack of tractability in general equilibrium setting. Moreover, the emission function approach, if it does not consider abatement activity, means that emissions and output have a one-to-one relationship. This does not seem realistic in many situations because it is often possible to decrease emissions — for example, by introducing new equipment or by hiring more labor while keeping output constant.

all factor prices are endogenously determined in the standard 2×3 model, the factor prices corresponding to emissions in our model are policy instruments (i.e. emission tax) determined exogenously.[3]

Let v_j^i denote the amount of factor $j = K, L$ employed in sector $i = 1, 2$ and v_Z^i denote the level of emission from sector i. The production function of sector i is given by $Q_i = f^i(v^i)$ where Q_i is the output of sector i and $v^i = (v_K^i, v_L^i, v_Z^i)$. We assume that $f^i(\cdot)$ is concave and homogeneous of degree one in v^i. The unit cost function of sector i is

$$c^i(w_K, w_L, w_Z^i) \equiv \min_{\{a_{Ki}, a_{Li}, a_{Zi}\}} \left[w_K a_{Ki} + w_L a_{Li} + w_Z^i a_{Zi} \mid f^i(a_{Ki}, a_{Li}, a_{Zi}) \geq 1 \right]$$

where a_{ji} $(j = K, L, Z)$ is unit factor demand, w_j $(j = K, L)$ is the price of factor j, and w_Z^i is the specific emission tax imposed on sector i. From Shephard's lemma, $a_{ji} = \partial c^i / \partial w_j$ $(j = K, L, Z)$.

At a competitive equilibrium, unit cost must be equal to price if the commodity is actually produced and capital and labor must be fully employed. Thus, for $i = 1, 2$, and $j = K, L$:

$$c^i(w_K, w_L, w_Z^i) = p_i \qquad \sum_{i=1,2} a_{ji}(w_K, w_L, w_Z^i) Q_i = v_j \tag{1}$$

where p_i is the price of good i and v_j is the endowment of factor j. Given commodity prices, factor endowments, and emission taxes, equilibrium factor prices and outputs are determined by (1). The level of emissions from sector i is given by $v_Z^i = a_{Zi} Q_i$. Let θ_{ji} denote the cost share of factor j in sector i $(\theta_{ji} \equiv w_j a_{ji} / c_i)$ and λ_{ji} denote the fraction of factor j employed in sector i $(\lambda_{ji} \equiv Q_i a_{ji} / v_j)$. Then, equations of change are given by

$$\sum_{j=K,L} \theta_{ji} \hat{w}_j = \hat{p}_i - \theta_{Zi} \hat{w}_Z^i \qquad \sum_{i=1,2} \lambda_{ji} \hat{Q}_i = \hat{v}_j - \sum_{i=1,2} \lambda_{ji} \hat{a}_{ji} \tag{2}$$

where a hat over a variable denotes the rate of change (e.g. $\hat{w}_j \equiv dw_j / w_j$). In addition, we define $|\theta_{KL}^{ih}| \equiv \theta_{Ki}\theta_{Lh} - \theta_{Li}\theta_{Kh}$, $|\lambda_{KL}^{ih}| \equiv \lambda_{Ki}\lambda_{Lh} - \lambda_{Li}\lambda_{Kh}$ $(i, h = 1, 2, i \neq h)$, and define $Y \equiv \sum_{i=1,2} p_i Q_i$, $\alpha_j \equiv w_j v_j / Y$, and $\gamma_i \equiv p_i Q_i / Y$. α_j and γ_i represent the factor and sector shares in GDP, respectively. By definition, we have $\lambda_{ji} = \gamma_i \theta_{ji} / \alpha_j$. In the remainder of the paper, we will focus on the effects of the change in the emission taxes and set $\hat{p}_i = \hat{v}_j = 0$. From (2) and the above notations, the following relations are derived.

$$\hat{w}_K = |\theta_{KL}^{12}|^{-1}(-\theta_{L2}\theta_{Z1}\hat{w}_Z^1 + \theta_{L1}\theta_{Z2}\hat{w}_Z^2) \qquad \hat{w}_L = |\theta_{KL}^{12}|^{-1}(\theta_{K2}\theta_{Z1}\hat{w}_Z^1 - \theta_{K1}\theta_{Z2}\hat{w}_Z^2) \tag{3}$$

$$\hat{Q}_1 = |\lambda_{KL}^{12}|^{-1}(\lambda_{L2}\hat{\beta}_K - \lambda_{K2}\hat{\beta}_L) \qquad \hat{Q}_2 = |\lambda_{KL}^{12}|^{-1}(-\lambda_{L1}\hat{\beta}_K + \lambda_{K1}\hat{\beta}_L) \tag{4}$$

where $\hat{\beta}_j \equiv -\sum_{i=1,2} \lambda_{ji} \hat{a}_{ji}$.

[3] The model more similar to ours is the 2×3 model with capital mobility like Wong (1995), chapter 4 because one of the factor prices in his model (the rental rate) is also constant. See section 4 for details.

To derive the expression of $\hat{Q}_i$, we define further notations.

$$\varepsilon^i_{jl} \equiv \frac{w_l}{a_{ji}}\frac{\partial a_{ji}}{\partial w_l} \quad i = 1, 2 \;\; j, l = K, L, Z$$

ε^i_{jl} is the price elasticity of unit factor demand in sector i. If $\varepsilon^i_{jl} > (<)\ 0$, factor j and l are called *substitutes* (*complements*) in sector i.[4] ε^i_{jl} has the following three properties: (i) $\varepsilon^i_{lj} = \theta_{ji}\varepsilon^i_{jl}/\theta_{li}$, (ii) because of the zero homogeneity of a_{ji} with respect to (w_K, w_L, w^i_Z), $\varepsilon^i_{jK} + \varepsilon^i_{jL} + \varepsilon^i_{jZ} = 0$ for $j = K, L, Z$, (iii) because of the concavity of the cost function, $\varepsilon^i_{jj} \leq 0$ and $\varepsilon^i_{jj}\varepsilon^i_{ll} - \varepsilon^i_{jl}\varepsilon^i_{lj} \geq 0$. From these properties, if ε^i_{jl} is negative, both ε^i_{jk} and ε^i_{lk} must be positive, that is, there is at most one pair of complementary factors. Moreover, property (ii) and (iii) imply that the following inequality holds for $i = 1, 2$, $j, l, k = K, L, Z$, $j \neq l$, $l \neq k$, $k \neq j$:

$$\varepsilon^i_{jl} \geq -\frac{\theta_{li}\varepsilon^i_{jk}\varepsilon^i_{lk}}{\theta_{ji}\varepsilon^i_{jk} + \theta_{li}\varepsilon^i_{lk}} \tag{5}$$

This means that even if $\varepsilon^i_{jl} < 0$ (i.e. factor j and l are complements), the degree of complementarity is limited by some bound.

In addition, we define $\varepsilon_{jl} \equiv \lambda_{j1}\varepsilon^1_{jl} + \lambda_{j2}\varepsilon^2_{jl}$. ε_{jl} expresses the price elasticity of *total* factor demand and has the properties similar to ε^i_{jl}: (i) $\varepsilon_{lj} = \alpha_j\varepsilon_{jl}/\alpha_l$, (ii) $\varepsilon_{jK} + \varepsilon_{jL} + \varepsilon_{jZ} = 0$, (iii) $\varepsilon_{jj} \leq 0$. Using these notations, we can rewrite $\hat{\beta}_j$ as follows

$$-\hat{\beta}_j = \varepsilon_{jK}\hat{w}_K + \varepsilon_{jL}\hat{w}_L + \lambda_{j1}\varepsilon^1_{jZ}\hat{w}^1_Z + \lambda_{j2}\varepsilon^2_{jZ}\hat{w}^2_Z \tag{6}$$

Below, we basically consider output of sector 1 without loss of generality. The same arguments can be applied also to $\hat{Q}_2$. Combining (3), (4), and (6), we can derive the expression of $\hat{Q}_1$:

$$\hat{Q}_1 = \frac{1}{|\theta^{12}_{KL}||\lambda^{12}_{KL}|}\left(A^1_1\hat{w}^1_Z + A^1_2\hat{w}^2_Z\right) \tag{7}$$

where

$$\begin{aligned}
A^1_1 &= -B^1_1\theta_{Z1} + |\theta^{12}_{KL}|(\lambda_{K2}\lambda_{L1}\varepsilon^1_{LZ} - \lambda_{L2}\lambda_{K1}\varepsilon^1_{KZ}) \\
A^1_2 &= -B^1_2\theta_{Z2} + |\theta^{12}_{KL}|\lambda_{K2}\lambda_{L2}(\varepsilon^2_{LZ} - \varepsilon^2_{KZ}) \\
B^1_1 &= \frac{\gamma_2}{\alpha_L}(\theta_{K2} + \theta_{L2})^2\varepsilon_{KL} + \frac{\gamma_2}{\alpha_L}\theta^2_{L2}\varepsilon_{KZ} + \frac{\gamma_2}{\alpha_K}\theta^2_{K2}\varepsilon_{LZ} \\
B^1_2 &= -\frac{\gamma_2}{\alpha_L}(\theta_{K2} + \theta_{L2})(\theta_{K1} + \theta_{L1})\varepsilon_{KL} - \frac{\gamma_2}{\alpha_L}\theta_{L2}\theta_{L1}\varepsilon_{KZ} - \frac{\gamma_2}{\alpha_K}\theta_{K2}\theta_{K1}\varepsilon_{LZ}
\end{aligned}$$

Since both $|\theta^{12}_{KL}|$ and $|\lambda^{12}_{KL}|$ have the same signs, the fraction $1/|\theta^{12}_{KL}||\lambda^{12}_{KL}|$ is always positive. Note that the signs and size of all terms depend not only on factor intensities (that is, θ_{ji} and λ_{ji}), but also on the elasticities of substitution between factors (that is, ε^i_{jl}).

[4] $\varepsilon^i_{jl}/\theta_{li}$ is the well-known Allen's partial elasticity of substitution (see Chambers, 1988, p. 95). While most papers including Batra and Casas (1976), Yohe (1979), and Siebert, Eichverger, Gronych and Pethig (1980) use this Allen's measure of elasticity, we use ε^i_{jl} as Jones and Easton (1983) do.

Table 1: Three numerical examples illustrating proposition 1.

	θ_{K1}	θ_{L1}	θ_{K2}	θ_{L2}	γ_1	ε^1_{KL}	ε^1_{LZ}	ε^1_{KZ}	ε^2_{KL}	ε^2_{LZ}	ε^2_{KZ}
Case 1	0.5	0.45	0.3	0.65	0.6	0.5	0.5	-0.118	0.5	0.5	-0.171
Case 2	0.7	0.25	0.3	0.65		2.5	1.5	-0.309			
Case 3	0.3	0.65	0.5	0.45	0.7	0.5	0.5	2.5	1	0.5	1

$\theta_{Z1} = \theta_{Z2} = 0.05$ in all cases.

3 The Effects of Emission Taxes on Outputs

First, let us examine w^1_Z on Q_1, that is, the effect of the rise in emission tax on sector 1 on its output. The sign of this effect is determined by the sign of A^1_1 in (7). Since A^1_1 includes a lot of parameters, we cannot derive analytical propositions from it except for extreme cases.[5] However, we can show the following paradoxical proposition by numerical examples.

Proposition 1 *The sign of $\hat{Q}_1/\hat{w}^1_Z$ may be positive, that is, raising emission tax imposed on an industry may increase its output.*

We show and explain this proposition by giving three numerical examples.[6] The values of the parameters in the three cases are shown in Table 1 and we have $\hat{Q}_1/\hat{w}^1_Z > 0$ in all three cases (see Appendix 1).[7] In Case 1, we assume that K and Z are complements in both sectors, which is consistent with the empirical result in Chambers (1988, p. 98).

The proposition is quite counter-intuitive because the rise in emission tax should have the cost-push effect and thus lead to the downward pressure on the output of the industry. However, a close look at A^1_1 in (7) reveals that, in addition to the cost-push effect, the rise in the emission tax has another effect. Two effects can be explained as follows. First, the sector specific rise in emission tax alters factor prices in the same way as the fall in the commodity price (see the RHS of (2)). These changes in factor prices lead to the changes in factor demand and the output is adjusted so as to clear the factor markets. This effect, which we call *the cost-push* effect, is represented by the first term in A^1_1. In addition, the change in emission tax directly affects factor demand through substitution (or complementarity) between factors. This *substitution* effect is represented by the second term.[8]

For example, suppose that w^1_Z rises by one percent. This has the same impact on factor prices as a θ_{Z1} percent fall of p_1, and its impact on Q_1 is represented by $-B^1_1\theta_{Z1}$. We can show that this cost-push effect of the rise in w^1_Z through factor price

[5] For example, we can show that when K and L are perfect complements in both sectors, $\hat{Q}_1/\hat{w}^1_Z < 0$ always holds.

[6] Of course, one can easily find other various cases in which the paradoxical result happens.

[7] The constraint (5) is satisfied in all cases.

[8] Although we use the term *substitution*, it does not mean that complementarity is excluded. We use the term to represent the effects which work through substitution and complementarity between factors.

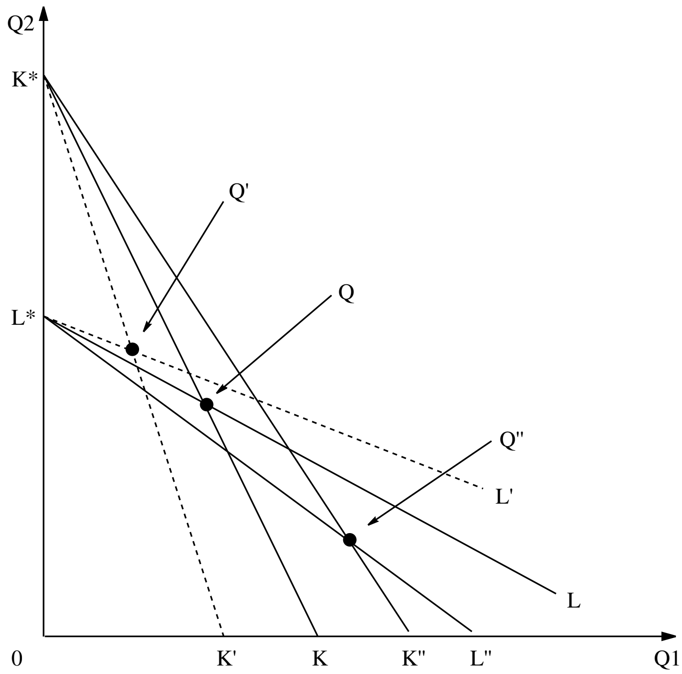

Figure 1:

adjustment always decreases the output, i.e. $B_1^1 \geq 0$ (see Appendix 2 for the proof). On the other hand, one percent rise in w_Z^1 raises the demands for capital and labor by $\lambda_{K1}\varepsilon_{KZ}^1$ and $\lambda_{L1}\varepsilon_{LZ}^1$ respectively (or reduces them if they are complements). If, for example, sector 1 has a higher capital-labor ratio than sector 2 (i.e. $|\theta_{KL}^{12}| > 0$), the increased demand for capital gives rise to a downward pressure on the output of sector 1 and the increased demand for labor gives rise to a upward pressure. This effect is represented by the second term. The proposition says that if the substitution effect works strongly in the opposite direction to the cost-push effect, the rise in the emission tax on an industry may raise the output of the industry.

Using Case 1, let us explain the intuition of two effects above. In Case 1, it is assumed that emissions are complement with capital but substitute with labor ($\varepsilon_{KZ}^1 < 0$, $\varepsilon_{LZ}^1 > 0$), and that sector 1 is more capital intensive than sector 2 ($|\theta_{KL}^{12}| > 0$). Suppose that the emission tax on sector 1 is raised. First, this raises the cost of sector 1 and generates the downward pressure on the output of that sector. On the other hand, the rise in emission tax on sector 1 decreases capital demand and increases labor demand through substitution effect and this leads to the fall in the rental rate and the rise in the wage. As a result of this, more resource is allocated to the capital intensive industry (sector 1) and the output of that sector tends to increase. What we have showed that under the numerical values of Case 1, the latter effect indeed dominates the former effect and the output of that industry increases.

Next, using Case 2 and the figure 1, we explain two effects above in detail. In Case 2, we assume, for graphical exposition, that capital and labor are perfect complements in sector 2 (i.e. $\varepsilon^2_{Kj} = \varepsilon^2_{Lj}$ for $j = K, L, Z$).[9] Figure 1 depicts the equilibrium in the output space. The horizontal and vertical axes represent the outputs of sector 1 and 2 respectively. Let the full employment lines for capital and labor at the initial equilibrium be denoted by line K*K and L*L whose slopes are given by a_{K1}/a_{K2} and a_{L1}/a_{L2}. Since in Case 2, the capital-labor ratio in sector 1 is higher than that in sector 2, line K*K is steeper than line L*L. The outputs at the initial equilibrium are given by the point Q where both factor markets are cleared.

Now suppose that the emission tax on sector 1 is raised by 1%. First, let us consider the cost-push effect. From (3), 1% rise in w^1_Z leads to (0.0235/0.38)% fall in w_K and (0.015/0.38)% rise in w_L because the capital-labor ratio in sector 1 is higher than that in sector 2 (the Stolper–Samuelson effect). Since capital and labor are substitutes in sector 1, these changes in factor prices lead to the rise in a_{K1} and the fall in a_{L1}: $\hat{a}^{\mathrm{CP}}_{K1} = 0.28609$ and $\hat{a}^{\mathrm{CP}}_{L1} = -0.93421$ (the superscript CP means cost-push effect).[10] On the other hand, from the perfect complementarity between K and L in sector 2, both $\hat{a}^{\mathrm{CP}}_{K2}$ and $\hat{a}^{\mathrm{CP}}_{L2}$ are zero (see Appendix 1). Therefore, by the cost-push effect, the full employment lines shift to K*K' and L*L', and outputs shift to Q' where the output of sector 1 decreases. As has already been pointed out, the cost-push effect always works in this direction.

Next, consider the substitution effect. The substitution effect of a tax on input coefficients in sector 1 is given by $\hat{a}^{\mathrm{ST}}_{K1} = \varepsilon^1_{KZ} \times 1 = -0.309$ and $\hat{a}^{\mathrm{ST}}_{L1} = \varepsilon^1_{LZ} \times 1 = 1.5$. Thus, the substitution effect works in the opposite direction to the cost-push effect. Moreover, since the size of the substitution effects is larger than that of the cost-push effects (i.e. $|\hat{a}^{\mathrm{ST}}_{K1}| > |\hat{a}^{\mathrm{CP}}_{K1}|$ and $|\hat{a}^{\mathrm{ST}}_{L1}| > |\hat{a}^{\mathrm{CP}}_{L1}|$), the substitution effect dominates the cost-push effect. Taking account of two effects, the full employment lines shift to K*K" and L*L" and the new equilibrium output shifts to Q". Therefore, in the example above, the rise in the emission tax on sector 1 increases the output of sector 1.

Both Case 1 and 2 include complementary factors. However, it does not mean that complementary factors are necessary for the paradoxical result to occur. This is shown by Case 3 in which all factors are substitutes in both sectors.

Other cases

In the previous paragraphs, we have seen $\hat{Q}_1/\hat{w}^1_Z$. Here, for comparison, let us see the effect of the uniform rise in emission taxes on the output of sector 1. Since the uniform rise in emission taxes means $\hat{w}^1_Z = \hat{w}^2_Z > 0$, the effect is represented by $A^1_1 + A^1_2$. From

[9] Note that perfect complementarity does not mean Leontief technology (i.e., no substitution). Leontief technology is represented by $\varepsilon^i_{jl} = 0$.

[10] From the definition of $a_{ji} = a_{ji}(w_K, w_L, w^1_Z)$,

$$\hat{a}_{ji} = \varepsilon^i_{jK}\hat{w}_K + \varepsilon^i_{jL}\hat{w}_L + \varepsilon^i_{jZ}\hat{w}^i_Z$$

From this, $\hat{a}_{ji}$ is decomposed as follows: $\hat{a}_{ji} = \hat{a}^{\mathrm{CP}}_{ji} + \hat{a}^{\mathrm{ST}}_{ji}$ where $\hat{a}^{\mathrm{CP}}_{ji} = \varepsilon^i_{jK}\hat{w}_K + \varepsilon^i_{jL}\hat{w}_L$ and $\hat{a}^{\mathrm{ST}}_{ji} = \varepsilon^i_{jZ}\hat{w}^1_Z$. $\hat{a}^{\mathrm{ST}}_{ji}$ represents the substitution effect of the change in emission tax on input coefficients and $\hat{a}^{\mathrm{CP}}_{ji}$ represents the cost-push effect through factor price adjustment.

(7), we have

$$A_1^1 + A_2^1 = -\frac{\gamma_2}{\alpha_K \alpha_L}\left[(\theta_{K2}+\theta_{L2})(\theta_{Z1}-\theta_{Z2})(\gamma_1\theta_{K1}\varepsilon_{KL}^1+\gamma_2\theta_{K2}\varepsilon_{KL}^2)\right.$$
$$+\,\theta_{L2}(\theta_{L2}-\theta_{L1})(\gamma_1\theta_{K1}\varepsilon_{KZ}^1+\gamma_2\theta_{K2}\varepsilon_{KZ}^2)$$
$$\left.+\theta_{K2}(\theta_{K2}-\theta_{K1})(\gamma_1\theta_{L1}\varepsilon_{LZ}^1+\gamma_2\theta_{L2}\varepsilon_{LZ}^2)\right]$$

We can show that output of sector 1 can increase when emission taxes are uniformly raised. For example, consider the case: $\varepsilon_{LZ}^i = \varepsilon_{KZ}^i = 0$ for $i = 1, 2$, and $\theta_{Z1} < \theta_{Z2}$. In this case, we have $A_1^1 + A_2^1 > 0$. The reason why output of sector 1 increases in the above example is very simple. By $\varepsilon_{LZ}^i = \varepsilon_{KZ}^i = 0$, the substitution effect disappears and only the cost-push effect works and since $\theta_{Z1} < \theta_{Z2}$, sector 1 has a lower share of emissions and the cost-push effect works more adversely on sector 2. Thus, in the above example, the uniform rise in emission taxes increases output of sector 1.

The above example shows that the uniform rise in emission taxes can increase output of sector 1 as the rise in w_Z^1 can. However, there is a large difference between two cases. As the above example shows, in the case of uniform rise, the substitition effect is not necessary for Q_1 to rise. On the other hand, as the arguments in the previous sections show, in the case of differentiated rise, the substitition effect needs to exist and moreover its size must outweight that of the cost-push effect. So, the causes for decrease in output are completely different in two cases.

Finally, let us see $\hat{Q}_2/\hat{w}_Z^1$, that is, the effect of the rize in emission tax to sector 1 on output of sector 2. The sign of $\hat{Q}_2/\hat{w}_Z^1$ is determined by the sign of A_1^2, which is derived by exchanging 1 and 2 of A_2^1 in (7). Since the cost-push effect in this case is likely to work favorably to sector 2 (that is, B_1^2 is likely to be positive), the rise in w_Z^1 is likely to increase Q_2.[11] We can easily create an exmple where this indeed holds. For example, set $\varepsilon_{LZ}^1 = \varepsilon_{LZ}^2 = 0$, $\varepsilon_{KZ}^1 = \varepsilon_{KZ}^2 = 0$. Then, we have $A_1^2 > 0$.

4 The Effects of Emission Taxes on Emissions

In this section, we consider the effects of emission regulations on the level of emissions. As to emissions, it is the total emissions, rather than those of individual sectors, that matter. Thus, we focus on the total volume of emissions. The total volume of emissions is determined by $v_Z^i = a_{Zi}Q_i$, and $\hat{v}_Z = \sum_i \lambda_{Zi}\hat{v}_{Zi}$. Thus, the rate of change in total emissions is given by

$$\hat{v}_Z = \sum_{i=1,2}\lambda_{Zi}\hat{Q}_i + \sum_{i=1,2}\lambda_{Zi}\hat{a}_{Zi}$$
$$= \sum_{i=1,2}\lambda_{Zi}\hat{Q}_i + \sum_{j=K,L}\varepsilon_{Zj}\hat{w}_j + \sum_{i=1,2}\lambda_{Zi}\varepsilon_{ZZ}^i\hat{w}_Z^i$$

By $\hat{w}^K$ and $\hat{w}^L$ in (3), and $\hat{Q}_i$ in (7), the above equation is reduced to

$$\hat{v}_Z = \frac{\beta}{|\theta_{KL}^{12}||\lambda_{KL}^{12}|}\sum_{i=1,2}C^i\hat{w}_Z^i \tag{8}$$

[11] Although we can show $B_1^1 > 0$, the sign of B_1^2 cannot be determined. Thus, B_1^2 may be negative.

where

$$
\begin{aligned}
C^i = {} & \theta_{Zi}\left[-(\theta_{Kh}+\theta_{Lh})\alpha_K(\theta_{Zi}-\theta_{Zh})\varepsilon_{KL}\right. \\
& \qquad \left. +\theta_{Lh}\alpha_K(\theta_{Li}-\theta_{Lh})\varepsilon_{KZ}+\theta_{Kh}\alpha_L(\theta_{Ki}-\theta_{Kh})\varepsilon_{LZ}\right] \\
& + \gamma_i|\theta_{KL}^{ih}|\left[\theta_{Ki}(\theta_{Li}-\theta_{Lh})\varepsilon_{KZ}^i-\theta_{Li}(\theta_{Ki}-\theta_{Kh})\varepsilon_{LZ}^i\right] \qquad i \neq h \\
\beta \equiv {} & \gamma_1\gamma_2/\alpha_K\alpha_L\alpha_Z
\end{aligned}
$$

First, we consider the effects of the uniform change in emission taxes, therefore, we set $\hat{w}_Z^1 = \hat{w}_Z^2 = \hat{w}_Z$. Then, we can show that $\hat{v}_Z/\hat{w}_Z < 0$, that is, uniformly strengthening emission taxes on both industries always reduces the total level of emissions (see Appendix 3).

Next, let us examine the sector specific change in emission tax. The sign of this effect is represented by C^i. Note that since C^i depends on both factor intensities and elasticities of substitution, its sign cannot be easily determined by some simple conditions. Therefore, we consider a special case in which $\varepsilon_{LZ}^i = 0$ for $i = 1, 2$. This means that labor and emission are neither substitutes nor complements in both sectors. This case seems plausible in reality because in most realistic situations emission is likely to be more closely related to capital than labor. For example, some types of air pollutants can often be removed by adopting special equipments, but they are hardly removed by employing more labor.

In this case, C^i reduces to

$$
\begin{aligned}
C^i = {} & \theta_{Zi}\left[-(\theta_{Kh}+\theta_{Lh})\alpha_K(\theta_{Zi}-\theta_{Zh})\varepsilon_{KL}+\theta_{Lh}\alpha_K(\theta_{Li}-\theta_{Lh})\varepsilon_{KZ}\right] \\
& \qquad + \gamma_i|\theta_{KL}^{ih}|\theta_{Ki}(\theta_{Li}-\theta_{Lh})\varepsilon_{KZ}^i
\end{aligned}
$$

This yields the following proposition.

Proposition 2 *If $\varepsilon_{LZ}^i = \varepsilon_{LZ}^h = 0$, $a_{Ki}/a_{Kh} > a_{Li}/a_{Lh} > a_{Zi}/a_{Zh}$ and $\theta_{Li} > \theta_{Lh}$, $\hat{v}_Z/\hat{w}_Z^i$ is positive.*

Proof *Since $\varepsilon_{LZ}^i = \varepsilon_{LZ}^h = 0$, we have $\varepsilon_{KL} > 0$, $\varepsilon_{KZ} > 0$, and $\varepsilon_{KZ}^i > 0$. And since $a_{Ki}/a_{Kh} > a_{Li}/a_{Lh} > a_{Zi}/a_{Zh}$, we have $|\theta_{KL}^{ih}| > 0$ and $\theta_{Zi} - \theta_{Zh} < 0$. Thus, the proposition immediately follows. Q.E.D.*

This means that, for example, if sector i is relatively capital intensive and sector h is relatively emission intensive, and if the cost share of labor is larger in sector i than in sector h, a rise in emission tax on sector i (holding emission tax on sector h constant) increases the total level of emission. The intuitive reasoning is simple. Under the conditions in the proposition, the rise in emission tax on sector i decreases the output of sector i and increases that of sector h. Since the expanded sector is relatively intensive in emissions and the other sector is relatively non-intensive, the rise in emissions from the expanded sector dominates the fall in emissions from the contracted sector, and thus, total emissions increase. The same kind of arguments is valid in the case of $\varepsilon_{KZ}^i = 0$. Therefore, we can conclude that strengthening emission tax may increase the total volume of emissions according to the way in which taxes are imposed.

5 Further Discussions

In this section, we provide further discussions on our model and its results so as to make our contribution clear. The first point is the relation between our model and the standard 2×3 model. Since our model includes two primary factors and incorporates emission as the third production factor, it has the structure similar to the standard 2×3 model employed in Batra and Casas (1976) and Jones and Easton (1983). But there is one important difference between the standard 2×3 model and ours. In the standard 2×3 model, all primary factors are treated symmetrically: endowments of three factors are given exogenously and all factor prices are determined endogenously. On the other hand, the price of emission in our model (emission tax) is given exogenously and the volume of emission is determined endogenously. Due to this difference, the results from the standard 2×3 model are not applicable to our analysis.

The model more similar to ours is the 2×3 model with international capital movements employed in Wong (1995, Chap. 4). In his model, the country is assumed to be a small open economy and the rental price for capital is exogenously given. This means that Wong's model has one exogenously given factor price like ours. However, he assumes uniform rental prices among industries and therefore our result of differentiated emission taxes cannot be derived from his analysis.

The second point is the relation between our model and a model with 2×2 structure. A lot of theoretical analyses employ a 2×2 model with one primary factor and emissions (e.g. Rauscher, 1994; Ishikawa and Kiyono, 2000). Due to the simplicity of the model, they often analyze more complicated policy issues than ours such as optimal emission tax and international trade etc. But as long as such a structure is employed, the result derived in this paper are excluded because when emission tax is imposed in a small open economy with such a structure, the production is always specialized to one sector and one cannot analyze the interaction of two production sectors.[12] In this sense, the model of 2×2 structure is not suited to our purpose and our results of differentiated emission taxes cannot be derived in such a model.

As these arguments show, our result is a new insight and not what has been showed in the previous studies.

6 Concluding remarks

In this paper, we have considered the two sector economy with two primary factors and emissions and have explored the effects of differentiated emission taxes on output and emissions. Our findings are summarized as follows. First, increasing the emission taxes imposed on an industry may increase its output. It is surprising that such a paradoxical result arises in a simple standard model with plausible parameter values. I have also shown that the mechanism behind this result is the general equilibrium effect that operates through factor-market adjustment and that the emission taxes affect factor demand through two different effects (the cost-push and substitution effects).

[12]Proposition 7 in Ishikawa and Kiyono (2000) shows this.

Second, while strengthening emission taxes uniformly across industries always reduces emissions, strengthening emission taxes unevenly may increase them.

As a policy instrument for regulating emissions, emission tax has been attracting much attention and introduced in many countries. However, our analysis indicates that according to the way in which emission taxes are introduced, they may have unintended and detrimental effects on an economy. Therefore, it seems that greater attention should be paid to how emission taxes are introduced.

Appendix 1

A^1_1 can be rewritten as $A^1_1 = -\gamma_2(\gamma_1 X_1 + \gamma_2\theta_{Z1}\theta_{K2}X_2)/\alpha_K\alpha_L$ where $X_1 = (\theta_{K2} + \theta_{L2})^2\theta_{Z1}\theta_{K1}\varepsilon^1_{KL} + \theta_{L2}\theta_{K1}[\theta_{L2} - \theta_{L1}(\theta_{K2}+\theta_{L2})]\varepsilon^1_{KZ} + \theta_{K2}\theta_{L1}[\theta_{K2} - \theta_{K1}(\theta_{K2}+\theta_{L2})]\varepsilon^1_{LZ}$, and $X_2 = (\theta_{K2}+\theta_{L2})^h\varepsilon^2_{KL} + (\theta_{L2})^2\varepsilon^2_{KZ} + \theta_{K2}\theta_{L2}\varepsilon^2_{LZ}$. Inserting numerical values of Table 1 into this equation leads to $\hat{Q}_1/\hat{w}^1_Z > 0$. In Case 2, we assume the perfect complementarity between K and L in sector 2 (i.e. $\varepsilon^2_{Kj} = \varepsilon^2_{Lj}$ for $j = K, L, Z$). In this case, from the property (ii) of ε^i_{jl}, we have $\varepsilon^2_{KZ} = \varepsilon^2_{LZ} = -(1 + \theta_{K2}/\theta_{L2})\varepsilon^2_{KL}$, thus $X_2 = 0$. When there is a pair of complementary factors, the constraint (5) must be satisfied. In both Case 1 and 2, this constraint is indeed satisfied.

Appendix 2

The proof of $B^i_i \geq 0$. If all factors are substitutes, $B^i_i \geq 0$ is clear. Thus, we have to prove $B^i_i \geq 0$ when there is a pair of factors which are complements. We provide the proof in the case of $\varepsilon^i_{KL} < 0$ for $i = 1, 2$, that is, the case where capital and labor are complements in both sectors. Similar arguments can be applied to the other cases.

We can rewrite B^i_i as

$$B^i_i = \frac{\gamma_h}{\alpha_K\alpha_L}\Big\{\gamma_i\Big[(\theta_{Kh}+\theta_{Lh})^2\theta_{Ki}\varepsilon^i_{KL} + \theta^2_{Lh}\theta_{Ki}\varepsilon^i_{KZ} + \theta^2_{Kh}\theta_{Li}\varepsilon^i_{LZ}\Big] + \gamma_h\Big[(\theta_{Kh}+\theta_{Lh})^2\theta_{Kh}\varepsilon^h_{KL} + \theta^2_{Lh}\theta_{Kh}\varepsilon^h_{KZ} + \theta^2_{Kh}\theta_{Lh}\varepsilon^h_{LZ}\Big]\Big\}$$

Then, from (5), we have

$$\begin{aligned}
B^i_i \geq{}& \frac{\gamma_h}{\alpha_K\alpha_L}\Big\{\gamma_i\Big[-(\theta_{Kh}+\theta_{Lh})^2\theta_{Ki}\frac{\theta_{Li}\varepsilon^i_{KZ}\varepsilon^i_{LZ}}{\theta_{Ki}\varepsilon^i_{KZ}+\theta_{Li}\varepsilon^i_{LZ}} \\
&+ \theta^2_{Lh}\theta_{Ki}\varepsilon^i_{KZ} + \theta^2_{Kh}\theta_{Li}\varepsilon^i_{LZ}\Big] \\
&+ \gamma_h\Big[-(\theta_{Kh}+\theta_{Lh})^2\theta_{Kh}\frac{\theta_{Lh}\varepsilon^h_{KZ}\varepsilon^h_{LZ}}{\theta_{Kh}\varepsilon^h_{KZ}+\theta_{Lh}\varepsilon^h_{LZ}} \\
&+ \theta^2_{Lh}\theta_{Kh}\varepsilon^h_{KZ} + \theta^2_{Kh}\theta_{Lh}\varepsilon^h_{LZ}\Big]\Big\} \\
={}& \frac{\gamma_h}{\alpha_K\alpha_L}\Big[\frac{\gamma_i}{\theta_{Ki}\varepsilon^i_{KZ}+\theta_{Li}\varepsilon^i_{LZ}}(\theta_{Ki}\theta_{Lh}\varepsilon^i_{KZ} - \theta_{Li}\theta_{Kh}\varepsilon^i_{LZ})^2 \\
&+ \frac{\gamma_h}{\theta_{Kh}\varepsilon^h_{KZ}+\theta_{Lh}\varepsilon^h_{LZ}}(\theta_{Kh}\theta_{Lh})^2(\varepsilon^h_{KZ} - \varepsilon^h_{LZ})^2\Big] \geq 0
\end{aligned}$$

Appendix 3

The sign of $\hat{v}_Z/\hat{w}_Z$ depends on the sign of $C^1 + C^2$. It is given by

$$\begin{aligned}
C^1 + C^2 ={}& -\beta[(\theta_{Z1}-\theta_{Z2})^2\alpha_K\varepsilon_{KL} + (\theta_{L1}-\theta_{L2})^2\alpha_K\varepsilon_{KZ} \\
&+ (\theta_{K1}-\theta_{K2})^2\alpha_L\varepsilon_{LZ}]
\end{aligned}$$

Following the same procedure as when we prove $B_i^i \geq 0$, we can show $C^1 + C^2 \leq 0$. However there is an easier way to prove it.

Since the equivalence between tax and quota holds in the model, the effects of the uniform rise in emission tax can be derived from the effects of the reduction in country-wide emission quota. If we interpret emission quota as factor endowment, the latter effect has already been derived in Batra and Casas (1976), theorem 1: the decrease in the supply of a factor (emission quota) always raises the reward of that factor (emission permit price). This result and the equivalence between both policies imply that the uniform increase in emission tax always reduces the volume of emission.

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