E C O N O M I C S B U L L E T I N

Spectral Density Bandwidth Choice and Prewhitening in the Generalized Method of Moments Estimators for the Asset Pricing Model

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Abstract

This paper investigates the performances of GMM estimates using kernel methods with and without prewhitening and the VARHAC method in a representative agent exchange economy. A Monte Carlo study is conducted to evaluate the issues of estimating the spectral density functions, e.g., parametric vs. nonparametric, data−based bandwidth selection, and prewhitening procedures. The Monte Carlo results show that kernel methods with prewhitening procedure outperform others in terms of statistical inferences. The deviations from true parameter values, however, are larger for kernel methods with prewhitening procedure. Therefore, there exists efficiency/bias trade−off when choosing HAC covariance estimation method.

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1. Introduction

In a representative agent exchange economy, a certain type of asset pricing model is proposed by Lucas (1978) with a set of Euler equations of per capital consumption in the equilibrium. Empirical works on the set of Euler equations and the budget-constraint (or market-clearing) conditions raise problems on how to calibrate the economy in a reasonable set of parameter values (e.g., Mehra and Prescot 1985, Hall 1988, Tauchen 1986, and Kocherlakota 1990). Fundamentally, there are two approaches to find the estimates of parameters. The first approach is to estimate the reduced-form model with restrictions using the maximum likelihood estimation (MLE), and the second approach is to estimate the model using the generalized method of moments (GMM). The former approach needs researchers to assume or plug in a specific distribution and economic dynamics, while the latter approach does not need to do so. Ever since the seminal paper of Hansen (1982), the GMM estimation method has been commonly used in many applied research works. The merit of the GMM is that it requires certain specific moment conditions instead of assumed distributions. Thus, the linear and non-linear models are easier to be taken care of within the GMM framework. Given the GMM estimation, the non-linearity of a model is captured through the weighting matrix (i.e., variance-covariance matrix of orthogonality conditions). Consequently, an optimal weighting matrix results in efficient GMM estimates in terms of statistical inferences and model specification.

The generalized method of moments is widely used in the asset pricing models (e.g., Hansen and Singleton 1982, Kocherlakota 1990, Epstein and Zin 1991, MacKinlay and Richardson 1991, Hodrick 1992, and Cochrane 2001). The essence of the estimation of the weighting matrix in the GMM is the estimation of the spectral density function (at the frequency zero). The issue is important, because the performance of the GMM estimators or test statistics (e.g., test for over-identification) can be sensitive to the quality of the spectral density estimates. It is known in the literature (e.g., Ferson and Foerster 1994, Hansen, Heaton, and Yaron 1996, and Kan and Zhou 1999) that the GMM may perform poorly if one uses a poor spectral density function to estimate the weighting matrix. Cochrane (2000, 2001), in fact, points out that the only important comparison is not between the MLE and GMM, but between methods for estimating spectral density functions (at the frequency zero).

The purpose of this paper is to examine the performances of parameter estimates of the asset pricing model under different spectral density functions using the GMM procedure. At the same time, the issue of how to choose a consistent heteroskedasticity and autocorrelation covariance (HAC) matrix in the asset pricing model is considered herein. Andrews (1991) considers the kernel estimator of the HAC with an automatic optimal data-dependent bandwidth selection in an asymptotic truncated mean squared error criterion. Newey and West (1994) suggest an alternative way to search for the optimal bandwidth automatically. In addition, Andrews and Monahan (1992) consider the pre-whitening kernel-based HAC estimator which can reduce the bias of the kernel HAC estimator, as the pre-whitening procedure may flatten the uneven part of the spectral density function and obtain an unbiased kernel estimator. Therefore, this paper also investigates the effects of pre-whitening on the kernel-based HAC estimators.

In contrast to the usual HAC estimators that focus on kernel methods, Den Haan and Levin (1998) propose a method, called VARHAC, to construct a HAC estimator using the vector autoregressive spectral estimation. The basic idea behind the VARHAC method is to filter errors using VAR models first so as to obtain pre-whitened errors and then calculate the HAC covariance matrix through the covariance matrix of those pre-whitened errors. Their simulation results show that the VARHAC covariance estimator outperforms kernel-based HAC estimators given a high-order autoregressive error structure. Therefore, the VARHAC method is of interest in this paper to serve as a comparison with the kernel methods. The study starts with simulated economic data under different experimental settings in a Lucas-typed asset pricing model. The GMM procedure is then applied to the simulated data. For each experiment, 1000 replicates are performed randomly for sample sizes of 50 and 100, respectively, in order to provide reliable inferences.

The remainder of this paper is organized as follows: Section 2 describes the HAC estimation methods. Section 3 discusses the model economy and the experimental design. Section 4 compares the performances among distinct HAC estimators through different aspects. Finally, section 5 concludes.

2. The GMM Procedure and Estimation of Variance-Covariance Matrix

The GMM estimates the unknown parameter, e.g., $\theta = [\beta, \gamma]'$, by minimizing a quadratic norm of the orthogonality condition, $g_T(\theta)$, i.e., a weighted sum of squared pricing errors:

$$\hat{\theta} = \arg\min_{\theta} Q_T(\theta) \tag{1}$$

$$Q_T(\theta) = g_T(\theta)' W_T g_T(\theta),$$

with a weighting matrix, W_T, and $g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} g_t(\theta)$ which is the sample mean of the pricing errors. The orthogonality condition for the asset pricing model can also be derived from the pricing errors, $e_t(\theta)$, using the vector of instrumental variables, z_t. Constructing orthogonality conditions using instrumental variables is the same as adding the returns of managed portfolios to the model and hoping to capture all of the model's predictions (Cochrane 2001, p. 198). We then write the orthogonality conditions as:

$$E[e_t(\theta) \otimes z_t] = E[g_t(\theta)] = 0, \tag{2}$$

where $\otimes$ is the Kronecker product.

The instrumental variables employed in the paper consist of a conforming vector of ones and lag one values of consumption growth, w_t, and asset returns, R_t. That is, the vector of instrumental variables, z_t, is defined as $[1, w_{t-1}, R_{t-1}]'$. Hansen's *J*-statistic is used as a specification test to examine whether the model fits well for the data. The *J*-statistic represents whether the pricing errors are too big or not if the model is true, i.e.:

$$J = Tg_t(\theta^*)'W_T g_t(\theta^*) \tag{3}$$

The *J*-statistic converges to a χ^2 distribution with degrees of freedom equal to the difference between the number of moments and the number of parameters.

The optimal weighting matrix can be shown (e.g., Hansen 1982) to be S^{-1}, where $S = \sum_{j=-\infty}^{\infty} g_t(\theta)g_{t-j}(\theta)'$ is the spectral density matrix at the frequency zero for $g_t(\theta)$. Given the parameter estimates, $\hat{\theta}=(\hat{\beta},\hat{\gamma})$, and $\left\{g_t(\hat{\theta})\right\}_{t=-\infty}^{\infty}$, one could estimate the optimal weighting matrix by (e.g., Tauchen, 1986) $\tilde{W}_T = \tilde{S}_T^{-1}$, where $\tilde{S}_T = \frac{1}{T}\sum_{t=1}^{T} g_t(\hat{\theta})g_t(\hat{\theta})'$ if $\left\{g_t(\hat{\theta})\right\}_{t=-\infty}^{\infty}$ are serially uncorrelated. However, in practice, $\left\{g_t(\hat{\theta})\right\}_{t=-\infty}^{\infty}$ are usually serially correlated and then the weighting matrix is misleading and inconsistent. Therefore, as in Tauchen (1986), we assume that consumption growth and dividend growth are negatively auto-correlated in our Monte Carlo experiments below.

Andrews (1991) considers a consistent HAC estimator of the spectral density function at the frequency zero using kernel methods with an automatic bandwidth selection. The HAC estimator in Andrews (1991) is calculated as follows:

$$\tilde{S}_T = \sum_{j=-T+1}^{T-1} k(j/\tilde{m}(T))\tilde{\Omega}_j \tag{4}$$

$$\tilde{\Omega}_j = \begin{cases} \frac{1}{T}\sum_{t=j+1}^{T} V_t V_{t-j}' & \text{for} \quad j \ge 0 \\ \frac{1}{T}\sum_{t=-j+1}^{T} V_{t+j} V_j' & \text{for} \quad j < 0 \end{cases}$$

where $\tilde{S}_T$ is the estimated variance-covariance matrix, V_t is the interested vector or the error vector which equals $g_t(\theta)$ as of equation (2) in the GMM estimation in this paper, *k* is a kernel function, and $\tilde{m}$ is the bandwidth. The optimal bandwidth is shown to be:

$$m^*(T) = c_q(\alpha^*(q)T)^{1/(2q+1)} \tag{5}$$

$$c_q = \left(qk_q^2 \Big/ \int k^2(x)dx\right)^{1/(2q+1)}$$

$$\alpha^*(q) = \frac{2(vecf^{(q)}\omega vecf^{(q)})}{tr\omega(I+K\ \ )f \otimes f}$$

$$f^{(q)} = (2\pi T)^{-1}\sum_{j=-\infty}^{\infty} |j|^q EV_t V_{t-j}'$$

where *K* is a commutation matrix transforming *vec(A)* to $vec(A)'$, and ω is a weight matrix. The term c_q is a constant depending on the chosen kernel function. For the quadratic spectral (QS) kernel, c_q is equal to 1.3221.

On the other hand, Newey and West (1994) propose an alternative HAC estimator for estimating the optimal bandwidth from truncated sample autocovariances:

$$m^*(q) = \hat{\tau}T^{1/(2q+1)} \tag{6}$$

$$\hat{\tau} = c_\tau \{\hat{s}^{(q)} / \hat{s}^{(0)}\}^{2/(2q+1)}$$

$$\hat{s}^{(q)} = \sum_{j=-n}^{n} |j|^q \tilde{\Omega}_j$$

where n is the lag selection parameter depending on the kernel function. Their Monte Carlo simulation results suggest that their procedure performs well although there is size distortion and the selection of bandwidth may be more important than the choice of kernel functions. Andrews and Monahan (1992) conversely show that the pre-whitening procedure is able to effectively reduce the bias of the kernel estimator and improve the confidence interval's coverage probabilities.

The estimate of HAC with the pre-whitening procedure can be calculated as follows:

$$\tilde{S}_T = [I_T - \sum_{p=1}^{b} \hat{A}_p]^{-1} \hat{\Sigma}_T [I_T - \sum_{p=1}^{b} \hat{A}'_p]^{-1} \tag{7}$$

$$\hat{\Sigma}_T = \sum_{j=1-T}^{T-1} k(j/\hat{m}_T) \hat{\Omega}_j$$

$$\hat{\Omega}_j = \begin{cases} \frac{1}{T} \sum_{t=j+1}^{T} \hat{e}_t \hat{e}'_{t-j} & \text{for} \quad j \geq 0 \\ \frac{1}{T} \sum_{t=1}^{T-j} \hat{e}_{t+j} \hat{e}'_j & \text{for} \quad j < 0 \end{cases}$$

$$\hat{e}_t = V_t - \sum_{p=1}^{b} \hat{A}_p V_{t-p}$$

where $\hat{A}_p$ is usually obtained using the ordinary least squares estimation by assuming that all elements in V_t have the same number of AR lags. Moreover, Den Haan and Levin (1998) offer a VAR (Vector Autoregressive) procedure to calculate the HAC covariance matrix, called VARHAC. Their procedure pre-whitens error vectors (or interested vectors) by first using the VAR filter and then calculating the desired variance-covariance matrix through the HAC covariance estimate of pre-whitened errors.

In the first step, a VAR model is adopted to filter an error series using either AIC or BIC criterion:

$$\hat{e}_t = \sum_{j=0}^{\bar{p}} \hat{A}_j V_{t-j} = V_t + \sum_{j=1}^{\bar{p}} \hat{A}_j V_{t-j} \ , \tag{8}$$

where $\hat{e}_t$ is a filtered error vector, $\hat{A}_j$ is the coefficient matrix with $\hat{A}_0$ equal to the identity matrix, and $\bar{p}$ is the maximum lag order. The coefficient $\hat{A}_j$ is obtained using the ordinary least squares estimation by allowing that each element in V_t may have a different number of AR lags in its own past and the other elements. That is, equation (8) can be a restricted VAR representation. Given equation (8), the covariance of the filtered errors is estimated as: $\hat{\Sigma}_T = \frac{\sum_{t=\bar{p}+1}^{T} \hat{e}_t \hat{e}'_t}{T - \bar{p}}$. The VARHAC spectral estimator of a weighting matrix is thus calculated as: $\tilde{S}_T = \left[\sum_{j=0}^{\bar{p}} \hat{A}_j\right]^{-1} \hat{\Sigma}_T \left[\sum_{j=0}^{\bar{p}} \hat{A}'_j\right]^{-1}$.

The major difference between the VARHAC and the pre-whitening kernel-based HAC is that the VARHAC obtains the covariance of filtered errors using the standard sample variance method, while the pre-whitening kernel-based HAC obtains the covariance of pre-whitened residuals using the kernel-based procedure. It is noted that the objective of using the pre-whitening procedure in the pre-whitening kernel-based HAC is to flatten the relevant portion of the spectral density function, but not to actually pre-whiten the original series, while the VAR filter in the VARHAC is employed to obtain whitened original series.

3. The Simulation Economy

Following suit with Lucas (1978), Breeden (1979), and Tauchen (1986), a representative agent has a time and state separable utility function of the constant relative risk aversion family:

$$E_t\left[\sum_{k=0}^{\infty}\beta^k u(c_{t+k})\right], \qquad (9)$$

$$u(c) = [\tfrac{1}{1-\gamma}]c^{1-\gamma}$$

where β is the discount rate, *c* is the consumption, and γ is the coefficient of relative risk aversion (CRRA). Therefore, under a complete and frictionless market, the asset returns can be expressed as follows in terms of the price-dividend ratio:

$$R_{it} = \left[\frac{1+v_{it}}{v_{it-1}}\right]x_{it} \quad i = 1,2,\ldots\ldots, M\ , \qquad (10)$$

where v_{it} is the price-dividend ratio for asset *i* at time *t* and x_{it} is the dividend growth variable for asset *i* at time *t*. Given the price-dividend ratio, the consumption growth, and the dividend growth, the equilibrium in the economy is found to be $v_{it} = \beta E[(1+v_{it+1})w_{t+1}^{-\gamma}x_{it+1}]$. The pricing error function can now be defined as $e_t(\theta) = \beta R_{t+1}w_{t+1}^{-\gamma} - 1$.

The experiments conducted here follow the setup of Tauchen (1986) in which the stochastic processes of consumption growth and dividend growth are assumed to be the following autoregressive processes of order 1, respectively. That is:

$$\ln x_t = a_1 + b_{11}\ln x_{t-1} + b_{12}\ln w_{t-1} + \varepsilon_{1t} \qquad (11)$$
$$\ln w_t = a_2 + b_{21}\ln x_{t-1} + b_{22}\ln w_{t-1} + \varepsilon_{2t}$$

where ε_{1t} and ε_{2t} are random variables of the jointly normally-distributed white noises.

We approximate this system using the N-state Markov chain for N=16 as in Kocherlakota (1990), since Kocherlakota shows that a 16-state Markov chain is good enough to capture the real economic system. Table 1 illustrates the parameter values employed in each experiment. The experiments 1, 2, 3, and 4 follow the same fashion as experiments 9, 10, 11, and 12 in Tauchen (1986), but we adopt the parameter values in Kocherlakota (1990). There are 1,000 replications generated for each experiment with sample sizes of 50 and 100. The initial state is also drawn from the stationary distribution of the Markov chain.

4. Monte Carlo Results

Table 2 reports the mean values and standard deviations of estimated discount rates and CRRA in panels A and B, respectively. The point estimates of the discount rate are biased downward for Models 1–4. In general, the standard errors of estimates using the kernel methods with pre-whitening are uniformly larger than using the kernel methods without pre-whitening and the VARHAC. This indicates that estimators generated using kernel methods with pre-whitening are spread wider than other methods. At the same time, the standard errors become larger when the parameter values are bigger. Furthermore, the standard errors of estimates are reduced when the sample size increases from 50 to 100.

Table 3 shows that the mean biases of the estimated discount rate and CRRA are biased downward for most of the experiments. The mean biases of the estimated CRRA, however, are biased upward in experiment 4 of size 50 and experiments 2 and 3 for the kernel methods without pre-whitening and the VARHAC. On the other hand, the estimates of kernel methods with pre-whitening diverge more than the other methods. The root mean squared errors (RMSE) of the discount rate and CRRA estimates are larger for kernel methods with pre-whitening than for kernel methods without pre-whitening and the VARHAC. In addition, the results of the mean absolute deviations of estimated betas indicate that kernel methods with pre-whitening produce a larger deviation from the true value than kernel methods without pre-whitening and the VARHAC. Consequently, the kernel methods with pre-whitening produce more biases and tend to generate downward estimates.

Table 4 reports the coverage rates of estimated discount rates and CRRA at a 95% confidence interval. In general, kernel methods with pre-whitening have larger coverage rates than kernel methods without pre-whitening. Within pre-whitening methods, NW (Newey and West 1994) with pre-whitening and AM (Andrews and Monahan 1992) perform better than the VARHAC in all cases, especially in experiments 2, 3, and 4 of the actual economic parameter settings. For example, in experiment 4 which does not allow AR(1) inter-dependence between consumption and dividend growths, the differences in coverage rates between kernel methods with pre-whitening and the VARHAC method are much larger, while the magnitude of differences is not so much in experiment 3 which allows inter-dependence between consumption and dividend growths. This indicates that the VARHAC method works better when data are actually dependent in the true data generating process, but the method turns worse when data turn out to be less correlated. The coverage rates also rise when the sample size increases from 50 to 100, which upholds the belief that the larger sample size introduces more confidence in statistical inferences. For the rejection rates of the *J*-statistic, it is found that kernel methods with pre-whitening generally have values under 5% while the kernel methods without pre-whitening and the VARHAC have values larger than 5% in some experiments. Consequently, using pre-whitening HAC estimators in GMM models can deduce much more reliable model specification inferences.

As for the bandwidth estimates, Table 5 presents the mean values of them. In general, the Andrews method without pre-whitening has much smaller bandwidth estimates than the rest of the kernel methods. Within the pre-whitening kernel methods, the NW (Newey and West 1994) method tends to need larger bandwidth than the AM (Andrews and Monahan 1992) method. For the VARHAC, the AIC lag selection rule generates larger lag orders than the BIC lag selection rule. This corresponds with conclusions from Shibata (1976) and Den Haan and Levin (2000) that AIC tends to exaggerate the lag order penalty function and suggests more lags than BIC.

5. Conclusion

Based on simulated economic data in the asset pricing model of a representative agent exchange economy, we examine GMM performances of different HAC estimators by employing kernel methods with an automatic bandwidth selection and the VARHAC method. The findings indicate that kernel methods with a pre-whitening procedure (Newey and West 1994, and Andrews and Monahan 1992) possess larger coverage rates of parameter estimates than kernel methods without pre-whitening (Newey and West 1994, and Andrews 1991) and the VARHAC (Den Haan and Levin 1998) method. This suggests that there are efficiency gains if the research objective focuses on the statistical inferences when using kernel methods with pre-whitening. On the other hand, kernel methods without pre-whitening and the VARHAC method produce parameter estimates with smaller biases than kernel methods with pre-whitening. Therefore, there exists an efficiency/bias trade-off on the choice of the HAC estimation methods.

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Table I. Parameter Settings of the Model Economy

Experiment	Gamma	Beta	AR Coefficient		Error Covariance		Intercepts
1	13.7	1.139	0.117	0.414	0.0140	0.00177	0.004
			0.017	-0.161	0.00177	0.0012	0.021
2	13.7	1.139	0.117	0.000	0.0140	0.000	0.000
			0.000	-0.161	0.000	0.0012	0.000
3	13.7	1.139	0.117	0.414	0.01	0.000	0.000
			0.000	-0.161	0.000	0.0012	0.000
4	13.7	1.139	0.117	0.000	0.0140	0.00177	0.000
			0.000	-0.161	0.00177	0.0012	0.000

Table II. Means and Standard Deviations of Estimated Discount Rate and CRRA

Panel A: Estimated Discount Rate

Exp	Size	Pre		Non-Pre		VARHAC	
		NW	AM	NW	AW	AIC	BIC
1	50	0.9616	0.9587	1.0958	1.1023	1.0993	1.1023
		(0.2357)	(0.2338)	(0.1401)	(0.1204)	(0.1421)	(0.1333)
	100	0.9744	0.9847	1.1229	1.1204	1.1196	1.1208
		(0.2327)	(0.2226)	(0.1120)	(0.1028)	(0.1134)	(0.1197)
2	50	1.0787	1.0663	1.1836	1.1902	1.1802	1.1905
		(0.2248)	(0.2419)	(0.1416)	(0.1296)	(0.1461)	(0.1289)
	100	1.0152	1.0163	1.1813	1.1816	1.1750	1.1772
		(0.2589)	(0.2719)	(0.1143)	(0.1285)	(0.1331)	(0.1279)
3	50	1.0866	1.0924	1.1643	1.1716	1.1711	1.1699
		(0.2072)	(0.2662)	(0.1211)	(0.1038)	(0.1062)	(0.0982)
	100	1.0757	1.0406	1.1542	1.1535	1.1525	1.1535
		(0.1602)	(0.2077)	(0.1011)	(0.1054)	(0.1061)	(0.1046)
4	50	1.0693	1.0481	1.1673	1.1724	1.1683	1.1683
		(0.2175)	(0.2495)	(0.1180)	(0.1336)	(0.0957)	(0.1103)
	100	1.0660	1.0694	1.1367	1.1379	1.1373	1.1374
		(0.1757)	(0.1702)	(0.1262)	(0.1206)	(0.1214)	(0.1187)

Panel B: Estimated CRRA

Exp	Size	Pre		Non-Pre		VARHAC	
		NW	AM	NW	AW	AIC	BIC
1	50	6.0280	6.1897	7.7029	7.9241	7.7366	7.9935
		(15.1617)	(14.4256)	(8.2181)	(6.3561)	(7.5817)	(7.3308)
	100	12.5932	12.2000	11.5109	11.5675	11.4571	11.5654
		(15.3203)	(17.0371)	(7.4408)	(7.4863)	(7.1542)	(8.7056)
2	50	6.9848	6.1252	6.3331	6.7982	6.0862	7.0293
		(15.9604)	(17.1033)	(13.0325)	(9.0847)	(11.5336)	(10.5145)
	100	9.1315	7.8248	9.8012	9.3075	9.4207	9.1509
		(26.8841)	(21.5592)	(8.0135)	(9.9573)	(9.6626)	(10.4579)
3	50	9.9657	9.5628	8.8985	8.5461	8.7504	8.7436
		(15.5605)	(14.3256)	(7.5987)	(7.1862)	(7.2657)	(7.2468)
	100	12.6391	12.9132	10.7341	10.5088	10.7899	10.5214
		(11.7498)	(14.1702)	(6.3768)	(7.0025)	(6.7981)	(7.0744)
4	50	9.7700	9.9158	9.1084	8.8252	8.8943	9.0863
		(16.0398)	(17.9521)	(8.1897)	(9.8152)	(8.0316)	(7.6322)
	100	13.8050	14.8495	12.0165	11.8530	11.9508	11.7607
		(11.8562)	(10.5753)	(7.5059)	(7.5698)	(7.7927)	(7.6819)

Based on 1000 replications. Standard deviation is in the parenthesis. Exp: experiment; Size: sample size; CRRA: Constant Relative Risk Aversion; Pre: estimators with pre-whitening; Non-Pre: estimators without pre-whitening; NW: Newey and West (1994); AM: Andrews and Monahan (1992); AW: Andrews (1991); VARHAC: vector autoregressive HAC, Den Haan and Levin (1998).

Table III. Mean Biases, RMSE, and Mean Absolute Biases of Estimated Discount Rate and CRRA

Panel A: Estimated Discount Rate

Exp	Size	Pre		Non-Pre		VARHAC	
		NW	AM	NW	AW	AIC	BIC
1	50	-0.1774	-0.1803	-0.0432	-0.0367	-0.0397	-0.0367
		(0.2096)	(0.2114)	(0.0945)	(0.0879)	(0.0947)	(0.0945)
		[0.2950]	[0.2952]	[0.1466]	[0.1259]	[0.1474]	[0.1382]
	100	-0.1646	-0.1542	-0.0161	-0.0185	-0.0194	-0.0182
		(0.1909)	(0.1852)	(0.0768)	(0.0742)	(0.0769)	(0.0758)
		[0.2850]	[0.2708]	[0.1131]	[0.1044]	[0.1150]	[0.1210]
2	50	-0.0603	-0.0726	0.0445	0.0512	0.0412	0.0515
		(0.1447)	(0.1569)	(0.1056)	(0.1019)	(0.1042)	(0.1011)
		[0.2326]	[0.2525]	[0.1483]	[0.1393]	[0.1517]	[0.1388]
	100	-0.1238	-0.1226	0.0423	0.0425	0.0360	0.0382
		(0.1808)	(0.1923)	(0.0881)	(0.0917)	(0.0924)	(0.0894)
		[0.2868]	[0.2982]	[0.1218]	[0.1353]	[0.1378]	[0.1334]
3	50	-0.0524	-0.0465	0.0253	0.0326	0.0321	0.0309
		(0.1231)	(0.1261)	(0.0886)	(0.0840)	(0.0858)	(0.0807)
		[0.2137]	[0.2701]	[0.1237]	[0.1088]	[0.1109]	[0.1029]
	100	-0.0633	-0.0984	0.0152	0.0145	0.0135	0.0145
		(0.0966)	(0.1260)	(0.0724)	(0.0734)	(0.0729)	(0.0728)
		[0.1721]	[0.2298]	[0.1021]	[0.1064]	[0.1069]	[0.1056]
4	50	-0.0697	-0.0908	0.0283	0.0334	0.0293	0.0293
		(0.1176)	(0.1348)	(0.0770)	(0.0799)	(0.0755)	(0.0761)
		[0.2283]	[0.2654]	[0.1213]	[0.1377]	[0.1000	[0.1141]
	100	-0.0730	-0.0696	-0.0023	-0.0011	-0.0017	-0.0016
		(0.1024)	(0.0954)	(0.0794)	(0.0780)	(0.0792)	(0.0769)
		[0.1902]	[0.1838]	[0.1262]	[0.1205]	[0.1213	[0.1186]

Panel B: Estimated CRRA

Exp	Size	Pre		Non-Pre		VARHAC	
		NW	AM	NW	AW	AIC	BIC
1	50	-7.6719	-7.5102	-5.9970	-5.7758	-5.9633	-5.7064
		(13.2754)	(13.1848)	(7.3391)	(6.8350)	(7.3001)	(7.1951)
		[16.9855]	[16.2571]	[10.1703]	[8.5860]	[9.6430]	[9.2871]
	100	-1.1067	-1.4999	-2.1890	-2.1324	-2.2428	-2.1345
		(11.5828)	(11.5954)	(5.1911)	(5.2300)	(5.2859)	(5.2645)
		[15.3526]	[17.0945]	[7.7526]	[7.7805]	[7.4941]	[8.9592]
2	50	-6.7151	-7.5747	-7.3668	-6.9017	-7.6137	-6.6706
		(13.1392)	(13.9108)	(8.7764)	(8.3194)	(9.3816)	(8.6586)
		[17.3082]	[18.6978]	[14.9649]	[11.4055]	[13.8152]	[12.4476]
	100	-4.5684	-5.8751	-3.8987	-4.3924	-4.2792	-4.5490
		(14.9938)	(15.7678)	(6.6757)	(7.2254)	(7.3935)	(7.3644)
		[27.2563]	[22.3350]	[8.9080]	[10.8785]	[10.5633]	[11.3996]
3	50	-3.7342	-4.1371	-4.8014	-5.1538	-4.9495	-4.9563
		(9.6922)	(9.5605)	(7.1836)	(7.2060)	(7.0519)	(7.0325)
		[15.9947]	[14.9041]	[8.9853]	[8.8404]	[8.7884]	[8.7766]
	100	-1.0608	-0.7867	-2.9658	-3.1911	-2.9100	-3.1785
		(7.1729)	(8.5677)	(5.1119)	(5.3136)	(5.2176)	(5.3322)
		[11.7917]	[14.1850]	[7.0298]	[7.6921]	[7.3916]	[7.7524]
4	50	-3.9299	-3.7841	-4.5915	-4.8747	-4.8056	-4.6136
		(10.3305)	(11.3237)	(6.8080)	(7.1047)	(6.8644)	(6.7505)
		[16.5065]	[18.3378]	[9.3854]	[10.9547]	[9.3560]	[8.9150]
	100	0.1050	1.1495	-1.6834	-1.8469	-1.7491	-1.9392
		(7.0337)	(6.4101)	(5.1994)	(5.1487)	(5.2399)	(5.1776)
		[11.8507]	[10.6323]	[7.6887]	[7.7882]	[7.9828]	[7.9191]

Based on 1000 replications. RMSE is in the parenthesis. Mean Absolute Deviation is in the bracket. Deviation = (estimate - true value); Exp: experiment; Size: sample size; CRRA: Constant Relative Risk Aversion; Pre: estimators with pre-whitening; Non-Pre: estimators without pre-whitening; NW: Newey and West (1994); AM: Andrews and Monahan (1992); AW: Andrews (1991); VARHAC: vector autoregressive HAC, Den Haan and Levin (1998).

Table IV. Coverage Rates and Rejection Rates of Estimated Discount Rate and CRRA

Exp	Size	Pre		Non-Pre		VARHAC	
Panel A: Estimated Discount Rate							
		NW	AM	NW	AW	AIC	BIC
1	50	0.7400	0.7880	0.6770	0.7380	0.7420	0.7290
		(0.0000)	(0.0020)	(0.0990)	(0.1650)	(0.2310)	(0.3080)
	100	0.8700	0.7650	0.9070	0.9060	0.8880	0.8870
		(0.0060)	(0.0020)	(0.1200)	(0.1130)	(0.1900)	(0.1840)
2	50	0.7490	0.8190	0.4510	0.5020	0.5270	0.5090
		(0.0000)	(0.0000)	(0.0410)	(0.1170)	(0.2290)	(0.3030)
	100	0.8500	0.6610	0.5840	0.5610	0.5700	0.5610
		(0.0040)	(0.0040)	(0.1620)	(0.2170)	(0.3620)	(0.3840)
3	50	0.8190	0.9110	0.5340	0.5930	0.5930	0.5990
		(0.0000)	(0.0000)	(0.0300)	(0.0550)	(0.1570)	(0.2030)
	100	0.9560	0.8900	0.6130	0.6250	0.6280	0.6280
		(0.0000)	(0.0000)	(0.0540)	(0.0670)	(0.1270)	(0.1300)
4	50	0.9000	0.9300	0.5710	0.5870	0.6010	0.5980
		(0.0000)	(0.0000)	(0.0400)	(0.0770)	(0.1760)	(0.2490)
	100	0.9760	0.9280	0.6040	0.6080	0.6100	0.6080
		(0.0000)	(0.0010)	(0.0470)	(0.0710)	(0.1630)	(0.1670)
Panel B: Estimated CRRA							
Exp	Size	Pre		Non-Pre		VARHAC	
		NW	AM	NW	AW	AIC	BIC
1	50	0.5460	0.5970	0.4480	0.4700	0.4840	0.4820
		(0.0000)	(0.0020)	(0.0990)	(0.1650)	(0.2310)	(0.3080)
	100	0.7270	0.7650	0.6970	0.7080	0.6940	0.7080
		(0.0060)	(0.0020)	(0.1200)	(0.1130)	(0.1900)	(0.1840)
2	50	0.6430	0.7000	0.4610	0.4520	0.4590	0.4490
		(0.0000)	(0.0000)	(0.0410)	(0.1170)	(0.2290)	(0.3030)
	100	0.6970	0.6610	0.5440	0.5390	0.5470	0.5370
		(0.0040)	(0.0040)	(0.1620)	(0.2170)	(0.3620)	(0.3840)
3	50	0.7080	0.7570	0.4450	0.4580	0.4720	0.4800
		(0.0000)	(0.0000)	(0.0300)	(0.0550)	(0.1570)	(0.2030)
	100	0.8740	0.8900	0.6040	0.6170	0.6190	0.6100
		(0.0000)	(0.0000)	(0.0540)	(0.0670)	(0.1270)	(0.1300)
4	50	0.7530	0.7700	0.5000	0.5100	0.5270	0.5250
		(0.0000)	(0.0000)	(0.0400)	(0.0770)	(0.1760)	(0.2490)
	100	0.8970	0.9280	0.6610	0.6750	0.6610	0.6740
		(0.0000)	(0.0010)	(0.0470)	(0.0710)	(0.1630)	(0.1670)

Based on 1000 replications. Coverage Rate is of a 95% confidence interval. Rejection rate at the 5% level is in the Parenthesis. Exp: experiment; Size: sample size; CRRA: Constant Relative Risk Aversion; Pre: estimators with pre-whitening; Non-Pre: estimators without pre-whitening; NW: Newey and West (1994); AM: Andrews and Monahan (1992); AW: Andrews (1991); VARHAC: vector autoregressive HAC, Den Haan and Levin (1998).

Table V. Mean Values of Bandwidths and Lag Orders in VAR Filters

Exp	Size	Pre		Non-Pre		VARHAC	
		NW	AM	NW	AW	AIC	BIC
1	50	4.4885	3.9665	4.2796	1.7312	0.5720	0.0210
	100	4.5012	4.9627	3.8980	1.5087	0.6000	0.0100
2	50	6.6134	3.9711	5.1715	1.4726	0.4680	0.0080
	100	6.3871	5.1266	4.5786	1.5393	0.2650	0.0000
3	50	4.0218	4.4788	4.1235	1.4304	0.5060	0.0050
	100	4.3711	5.102	3.9146	1.2582	0.4080	0.0100
4	50	4.3950	4.4247	4.5540	1.4081	0.6380	0.0070
	100	4.1787	5.0351	4.0700	1.3276	0.5720	0.0030

Exp: experiment; Size: sample size; Pre: estimators with pre-whitening; Non-Pre: estimators without pre-whitening; NW: Newey and West (1994); AM: Andrews and Monahan (1992); AW: Andrews (1991); VARHAC: vector autoregressive HAC, Den Haan and Levin (1998). Lag orders of the first instrumental variable in the AIC and the BIC are reported.