E C O N O M I C S   B U L L E T I N

Inter−Generational Redistribution in an Endogenous Growth Model

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Abstract

This paper applies the Blanchard overlapping generations model to examine the effects of inter−generational redistribution on aggregate growth. The results reveal that whether or not a change in policy causes growth rates to rise or fall depends in part on whether the government has chosen to distribute in favor of the young or old, and in part on household preferences.

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1. Introduction

In previous work fiscal policy has typically been linked to economic growth either through a distortionary tax system or by government expenditures being modeled as productivity enhancing.[1] This paper adds to the literature by introducing an alternative channel through which fiscal policy affects economic growth. In the model, the government distributes utility enhancing goods/services to a group of heterogeneous agents. The model considers overlapping generations of finite lived agents of the type developed by Blanchard (1985). The government's distribution policy has an effect on the long-run growth rate of the aggregate economy because it affects the optimal consumption and savings decisions of the private agents. Whether or not a change in policy causes growth rates to rise or fall depends in part on whether the government has chosen to distribute in favor of the young or old, and in part on household preferences.

In that this paper incorporates inter-generational transfers in a model with Blanchard type agents, it is similar to work by Calvo and Obstfeld (1988) and Kosempel (2004). In those papers the authors attempted to assess the optimality of fiscal policy, whereas in this paper fiscal policy is exogenous. Our objective is to identify the effects of exogenous changes in distribution policy on growth. The Calvo and Obstfeld paper does not discuss the effects of government transfers on growth, perhaps because the production side of their model is not setup to generate long-run growth. Furthermore, although long-run growth is endogenous in Kosempel's paper, distribution policy does not affect it because of the restrictive form of the utility function that he adopted. This paper shows that distribution policy affects growth, at least when the utility function takes the general constant elasticity of substitution form.

2. The Model

The artificial economy consists of three types of agents: households, firms and a government.

2.1 The Households

At every instant of time, a large cohort is born whose size is normalized to λ. Each agent faces a constant instantaneous probability of death, which is also equal to λ. These assumptions imply a population size equal to one. It is assumed that new generations are not connected to old generations, and therefore there is no bequest motive. In the absence of bequests, households contract actuarially fair annuities with life-insurance companies. A household born at time i has a financial wealth at date t of $a(i,t)$. If the household survives it will receive $\lambda a(i,t)$ from an insurance company, but it will pay $a(i,t)$ if it dies.

The representative household of the cohort born at time i receives utility at time t by consuming a privately produced good, $c(i,t)$, and a public service, $x(i,t)$. The latter is transferred exogenously by the government. The expected lifetime utility of the representative household of cohort i at date t is given by:

$$U = \int_t^\infty u[c(i,v), x(i,v)]e^{-(\rho+\lambda)(v-t)}dv \,, \qquad (1)$$

[1] Models with public finance and economic growth have been reviewed by Barro and Sala-I-Martin (1992) and Glomm and Ravikumar (1997).

where ρ is the subjective rate of time preference, and $\rho + \lambda$ is the effective discount rate.[2] The period utility function is given by:[3]

$$u[c(i,v), x(i,v)] = \begin{cases} \dfrac{\left[c(i,v)^{1-\beta} x(i,v)^{\beta}\right]^{1-\sigma} - 1}{1-\sigma} & for \quad 0 \le \sigma < 1, \;\; \sigma > 1 \\ (1-\beta)\ln c(i,v) + \beta \ln x(i,v) & for \quad \sigma = 1 \end{cases}. \tag{2}$$

Each household is endowed with one unit of labor time, which it supplies inelastically. Let *w(t)* denote the age-independent real wage rate and *r(t)* the real interest rate. The dynamic budget constraint is given by:

$$da(i,t)/dt = [r(t) + \lambda]a(i,t) + w(t) - c(i,t), \tag{3}$$

where the first term on the right hand side gives the total return to a household's assets.

Each household chooses a consumption and savings profile to maximize its expected lifetime utility. The optimization provides us with an Euler equation for private consumption that depends on the dynamic behavior of *x(i,t)*:

$$[1-(1-\beta)(1-\sigma)]\frac{dc(i,t)/dt}{c(i,t)} - \beta(1-\sigma)\frac{dx(i,t)/dt}{x(i,t)} = r(t) - \rho\,. \tag{4}$$

2.2 The Firms

Economic models that make the growth rate of output endogenous rely on some mechanism that allows constant, or increasing returns in a factor that can be accumulated. This paper follows Romer (1986) by assuming that individual firms face diminishing returns to labor and their own capital stocks. However, due to a positive externality from aggregate capital accumulation (which Romer refers to as "learning-by-doing"), the aggregate production function for output, *Y(t),* is linear in capital, *K(t)*:[4]

$$Y(t) = AK(t). \tag{5}$$

In Romer's model, labor, *L(t),* and capital are paid according to their marginal products:

$$w(t) = (1-\tau)(1-\alpha)Y(t)/L(t)\text{, and} \tag{6}$$

$$r(t) = (1-\tau)\alpha Y(t)/K(t) - \delta = (1-\tau)\alpha A - \delta\,; \tag{7}$$

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[2] The effect of having a positive probability of death is to increase the household's effective discount rate. This result was obtained by Yaari (1965).

[3] This specification of the period utility function has also been used by Barro (1990). However, in his model, agents are infinitely lived.

[4] Upper case papers are used throughout this paper to denote aggregate variables.

where α denotes capital's share of output, δ is the rate of capital depreciation, and τ is a proportional tax applied to output.

2.3 The Government

In this paper government behavior is assumed to be exogenous. Tax revenue is used to finance a good/service that enters as an argument in the household's utility function. The government's expenditures must satisfy a budget constraint,

$$X(t) = \tau Y(t)\,; \tag{8}$$

and its distribution policy must satisfy a feasibility constraint,

$$\int_{-\infty}^{t} x(i,t)\lambda e^{-\lambda(t-i)} di = X(t)\,. \tag{9}$$

A well known feature of the AK model is that there are no transitional dynamics.[5] The aggregate variables will all experience the same growth rate at every point in time. However, despite the fact that consumption of the private good and government service grow at the same rate at the aggregate level, they do not have to grow at the same rate at the individual level. In fact, we assume that an individual's allocation of the government service follows a law of motion given by:

$$\frac{dx(i,t)/dt}{x(i,t)} = \frac{dc(i,t)/dt}{c(i,t)} - \gamma\,, \tag{10}$$

where γ is some constant. We will not make any assumption about the sign of γ. If γ is positive then the government is distributing in favor of the young, since the individual's share of the publicly provided good falls over time.[6] Conversely, if γ is negative then the government is distributing in favor of the old.

2.4 Equilibrium and Aggregation

In each time period there is a competitive equilibrium, which consists of an allocation $\{c\,(i,t),\, x\,(i,t),\, a\,(i,t)\}$ for all living households of each cohort $i \le t$; an allocation $\{Y(t),\, L(t),\, K(t)\}$ for each firm; and a set of prices $\{r\,(t),\, w\,(t)\}$; such that:

(i) the allocations received by the households solve their optimization problems, given prices;
(ii) the allocations received by the firms satisfy (6) and (7);
(iii) the government satisfies its budget constraint (8) and the feasibility constraint (9); and

[5] In the next section we prove that this holds for the current model as well.
[6] If the government wishes to devote more services to the young, then in order to satisfy (8) and (9), it must reduce transfers to the elderly. We can model this as a rise in γ, which measures the reduction in the growth rate of the transfer due to aging.

(iv) all markets clear: $L(t)=1$, $K(t)=A(t)$, $C(t)+\dot{K}(t)+X(t)=Y(t)-\delta K(t)$.

The household's Euler equation (4) can be simplified using the law of motion for *x(i,t)* from (10), and can then be integrated across all living households to produce an aggregate law of motion for private consumption,[7]

$$\frac{\dot{C}}{C}=\frac{1}{\sigma}[r-\rho-(1-\sigma)\beta\gamma]+\frac{\lambda}{\sigma}[(1-\sigma)(r+\lambda-\beta\gamma)-\lambda-\rho]\frac{K}{C}. \qquad (11)$$

Here time indicators have been dropped and a dot over a variable indicates a time derivative.[8]

Using (8) to eliminate *X(t)* from the aggregate resource constraint gives

$$\frac{\dot{K}}{K}=(1-\tau)A-\delta-\frac{C}{K}. \qquad (12)$$

Equations (11) and (12) determine the evolution of the growth rates of aggregate consumption and the capital stock. These equations are solved graphically in Figure 1. However, before the two lines could be plotted it was necessary to impose two additional assumptions on the model. First, we assumed that the lifetime utilities of the households are finite. This assumption guarantees that the second term on the right hand side of (11) is negative, regardless of the magnitude of σ. Therefore, the $\dot{C}/C$ line will always slope up. Second, we assumed that the two lines cross in the positive orthant,

$$(1-\tau)A-\delta>-\frac{[(1-\sigma)(r+\lambda-\beta\gamma)-\lambda-\rho]\lambda}{r-\rho-(1-\sigma)\beta\gamma}. \qquad (13)$$

If these two conditions are met, then there is a unique value of the consumption-capital ratio that is feasible given current production and satisfies the law of motion for aggregate consumption, and this occurs at the intersection of the two lines. As a result, there are no transitional dynamics in this model. The consumption and savings decisions of the agents put the economy immediately in the steady-state.[9]

[7] If β=0 then this equation is identical to the law of motion for consumption derived by Blanchard (1985).

[8] In deriving (11) use was made of the equilibrium condition that the aggregate financial wealth of the households equals the aggregate capital stock.

[9] Rearranging the inequality in (13) will give strict positive and negative bounds for γ in terms of the constant parameters of the model. Thus only certain levels of redistribution are consistent with a positive steady-state growth rate.

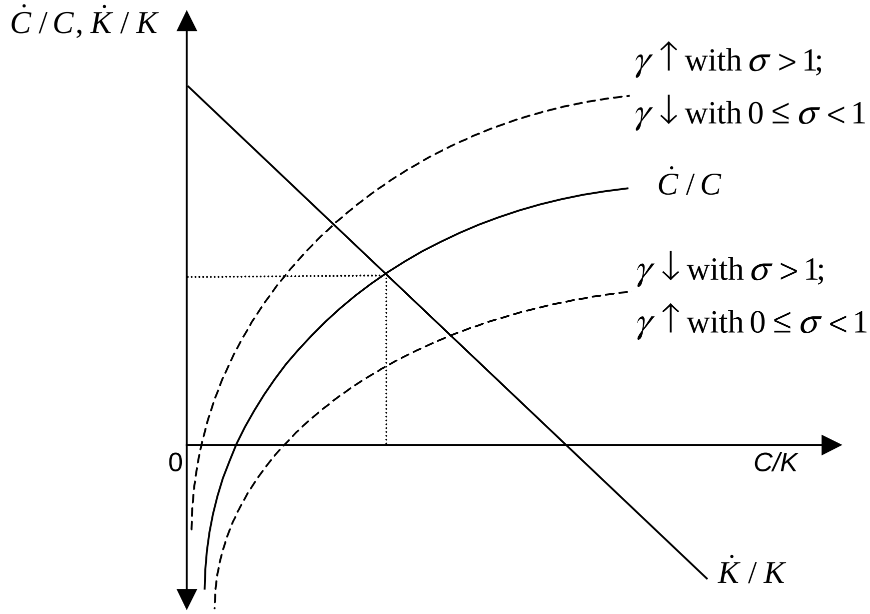

Figure 1. The Equilibrium

3. Distribution and Growth

The dotted lines in Figure 1 reveal the effects of increasing or decreasing the distribution parameter γ. Notice that the direction of movement in the $\dot{C}/C$ line in response to a change in γ depends on the utility function parameter σ. This parameter determines the sign of the cross-partial derivatives of the period utility function. The signs of these derivates are important for understanding the inter-temporal consumption smoothing behavior of the agents, and therefore their propensity to save.

Suppose that the private consumption good and the government service are substitutes in the sense that the cross-partial derivates of the period utility functions are negative. This condition requires $\sigma > 1$. Furthermore, suppose initially that $\gamma > 0$ and the government changes its distribution policy to favor the young even more, that is γ increases. This change in policy creates a larger wedge between the growth rates of the government service and the private consumption good at the individual level. The individuals in the model realize that over time their consumption of the government service will be declining relative to their consumption of the private good. Since the two goods are substitutes, in the interest of inter-temporal consumption smoothing they will save more in their youth. Furthermore, since all savings are productively invested, an increase in the savings rate will increase growth.

Now suppose that the two goods are complementary, that is, the cross-partial derivates of the period utility function are positive (which requires $0 \le \sigma < 1$). In this case, agents will respond to an increase in γ by altering their consumption profile in such a way that they will consume more in the periods in which they receive a lot of the government service. In other words, the individuals will reduce their savings and consume more when young, and this fall in savings reduces growth.

Finally, we will now consider the special case where $\sigma = 1$. Here the private good and the government service enter the period utility function separably, implying that the cross-partial derivates are zero. In this case, the household's Euler equation (4) is invariant to the policy parameter γ, and therefore a change in distribution policy will not affect savings or growth.

4. Conclusion

We have shown that the distribution of government goods and services has an effect on the long-run growth rate of the aggregate economy. Whether or not a change in policy causes growth rates to rise or fall depends in part on whether the government service raises or lowers the marginal utility of the private good, and in part on whether the government has chosen to distribute in favor of the young or the old. This occurs because the optimal consumption-savings decisions of individuals requires that the marginal utility received from the private consumption good follow a relatively smooth profile over time. If the two types of goods are substitutable, then the households will set their consumption profiles so that they have more private consumption when the provision of the government service is relatively light. In this case, a change in distribution policy that favors the young will result in a higher savings rate. This change in savings behavior drives the changes in the aggregate growth rate through changes in capital accumulation.

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