E C O N O M I C S   B U L L E T I N

Public debt, the terms of trade and welfare in an overlapping generations model with lifetime uncertainty

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Abstract

This article reconsiders the relationship between government debt and welfare in a two−country overlapping−generations model with lifetime uncertainty and international product differentiation. It has recently been proposed that a higher steady−state debt level may be welfare−enhancing in this setting. It is pointed out that this proposition does not adequately account for the effect of debt policy on individual agents' intertemporal consumption profiles. While a higher debt may indeed raise aggregate steady−state consumption, the lifetime utility of all steady−state cohorts will actually drop, unless the elasticity of substitution between domestic output and imports is extremely low. These particular results illustrate a more general caveat pertaining to any normative policy analysis in settings with overlapping generations of intertemporally optimizing agents: Attempts to draw welfare inferences on the basis of comparisons of aggregate consumption paths can be misleading.

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1. Introduction

In a recent contribution to the dynamic fiscal policy literature, Ghosh (1998) uses a two-country two-commodity extension of the Blanchard (1985) overlapping-generations model with lifetime uncertainty to study the relation between government debt and steady-state welfare. The analysis suggests in particular that a country may raise the steady-state welfare level of its residents through an increase in the long-run level of government debt, provided the substitutability between domestic goods and imports is sufficiently low and international capital flows are prohibited. While the increase in the public debt partially crowds out private capital and reduces domestic output, the associated terms-of-trade improvement can dominate the negative production effect on welfare.

The present paper points out that this proposition is based on an aggregate welfare function that does not adequately account for the effect of debt policy on individual agents' intertemporal consumption profiles. While a higher debt may indeed raise *aggregate* steady-state consumption, the lifetime utility of all steady-state cohorts will nevertheless drop in the absence of an intergenerational lump-sum redistribution scheme, unless extreme and empirically implausible values for the elasticity of substitution between domestic and imported goods are assumed.

The following section briefly recapitulates the analytical framework. Section 3 derives a representation of individual agents' welfare in the steady state. Section 4 demonstrates that higher long-run debt is in fact associated with lower steady-state lifetime utility under plausible parameter settings, and section 5 draws conclusions.

2. The Analytic Framework

The model distinguishes two countries (a,b) and two imperfectly substitutable tradable goods (1,2). Each country is completely specialized in its export good. Good 1 is produced by country a and serves as the numeraire. Both countries are populated by overlapping cohorts of finitely-lived agents who face a constant instantaneous probability of death λ. At date v, the representative member of a cohort born at date s in country a maximizes expected lifetime utility as of v,

$$u_a(s,v) = \int_v^\infty \ln q_a(s,t) e^{(\theta+\lambda)(v-t)} dt \,, \tag{1}$$

where

$$q_a(s,t) = [\alpha_a c_{1a}(s,t)^{-\rho} + (1-\alpha_a) c_{2a}{}^{-\rho}]^{-1/\rho} \tag{2}$$

subject to

$$\dot{w}_a(s,t) = [r_a(t) + \lambda) w_a(s,t)] + y_a(t) - P_a(t) q_a(s,t) \,. \tag{3}$$

Here $c_{ia}(s,t)$ denotes consumption of good i at date t by a member of cohort s in country a, ρ governs the elasticity of substitution $\sigma=1/(1+\rho)$ between home goods and imports, θ is the subjective discount rate, w denotes the agent's non-human wealth consisting of capital and government debt, y is labour income net of lump-sum taxes, r is the rate of return to capital, and P is the true price index associated with felicity index q, so that $P_a q_a = c_{1a} + \pi c_{2a}$, where π denotes the relative price of good 2 or the terms of trade. The term $\lambda \cdot w(s,t)$ in (3) enters due to the presence of a Blanchard-type reverse life insurance scheme: The agent receives a premium flow $\lambda \cdot w(s,t)$ while alive in return for bequeathing his terminal wealth to the insurance company.

The optimal consumption expenditure plan must obey

$$\dot{c}_a(s,t) = (r_a(t) - \theta)\, c_a(s,t)\,, \quad c_a \equiv c_{1a} + \pi\, c_{2a} = P_a q_a\,. \tag{4}$$

Integrating (3) and (4), optimal individual expenditure can be expressed as a linear function of individual wealth:

$$c_a(s,t) = (\lambda + \theta)[w_a(s,t) + h_a(t)]\,, \tag{5}$$

where h(t) denotes the present value of the individual's expected labour income net of taxes, which is by assumption age-independent:

$$h_a(t) = \int_t^{\infty} y_a(s) e^{-\int_t^s (r_a(v)+\lambda)dv} ds\,. \tag{6}$$

Cohort sizes at birth are time-invariant and normalized to be λ, so the total population size per country at any point in time is unity. Aggregating across cohorts and using capitals to express the economy-wide counterparts of individual variables, country a's aggregate consumption expenditure dynamics are described by[1]

$$\dot{C}_a(t) = (r_a(t) - \theta)\, C_a(t) - \lambda(\lambda + \theta) W_a(t)\,, \quad W_a = K_a + D_a\,, \tag{7}$$

where D is the level of government debt. The production technology in each country is linear-homogeneous in domestic capital K (accumulated through investment use of domestic output) and labour (L=1), and takes the Cobb-Douglas form $F(K_i)=K_i^{\gamma}$. The aggregate capital stock in a evolves according to

$$\dot{K}_a(t) = F(K_a(t)) - C_a(t) \tag{8}$$

and $r_a=F'(K_a)$. Ghosh assumes zero government expenditure on goods, hence

$$\dot{D}_a(t) = r_a(t)\, D_a(t) - T_a(t)\,, \tag{9}$$

where T(t) denotes tax revenue and - due to the normalization of cohort size - likewise the lump-sum tax faced by any individual agent alive in t. Since international capital flows are prohibited[2], trade is balanced at all times , i.e. $C_{1b}=\pi C_{2a}$.

Measuring W_b, C_b, and D_b in units of good 2, the equations of motion for country b take an analogous form.

3. Individual Welfare in the Steady State

In a steady state, the aggregate capital stock K, *aggregate* consumer expenditure C and the stock of debt D are stationary. Using (7) and (8), the steady-state capital stock is implicitly given by

$$F(K_a^*) = \frac{\lambda(\lambda+\theta)}{F'(K_a^*) - \theta}(K_a^* + D_a^*)\,. \tag{10}$$

[1] See Blanchard (1985) or Blanchard and Fischer (1989) for a detailed exposition.

[2] See Buiter (1987) for an analysis of fiscal policy under perfect international financial capital mobility in a similar two-country two-commodity perpetual youth model.

Hence

$$\frac{dK_a^*}{dD_a^*} = \frac{\lambda(\lambda+\theta)}{F_a F_a^{''} + F_a^{'}(F_a^{'}-\theta) - \lambda(\lambda+\theta)} \tag{11}$$

or equivalently, using $F_a^* = F'^*_a(K_a^* + D_a^*) + Y_a^*$ with (10),

$$\frac{dK_a^*}{dD_a^*} = \frac{\lambda(\lambda+\theta)}{F_a F_a^{''} - (F_a^{'}-\theta)Y_a/(K_a^* + D_a^*)} \tag{12}$$

i.e. higher debt *always* crowds out domestic productive capital in the steady state.[3]

The long-run equilibrium terms of trade are implicitly determined by

$$\frac{F(K_a^*)}{F(K_b^*)} = \frac{(\Lambda_a(\pi^*) + \pi^*)\Lambda_b(\pi^*)}{\Lambda_b(\pi^*) + \pi^*}, \tag{13}$$

where

$$\Lambda_i(\pi) \equiv \frac{C_{1i}}{C_{2i}} = \left[\frac{\alpha_i}{1-\alpha_i}\right]^{\sigma} \pi^{\sigma}, \quad i = a,b.$$

For the welfare analysis, Ghosh derives a steady state "indirect utility function" by using the steady-state solutions for the *aggregate* consumption quantities C_{1a} and C_{2a} in (2), yielding

$$U_a^* = \ln F(K_a^*) + \frac{1}{\rho}[-\ln(1-\alpha_a) + \ln\pi^* - (1+\rho)\ln(\Lambda_a^* + \pi^*)]. \tag{14}$$

U_a is in fact an index of aggregate real consumption at any point in time, yet this index is inappropriate for a normative welfare analysis since it does not take into account that individual cohorts are intertemporal utility maximizers with time-variant lifetime consumption profiles. The arguments of the direct utility function (1) are the *individual* consumption quantities c_{1a} and c_{2a}, which contrary to their macro counterparts are *not* time-invariant in an undisturbed steady state: From (4) it is evident that for an individual born at some date s in the steady state, lifetime consumption expenditure evolves according to

$$c_a^*(s,t) = c_a^*(s,s)e^{(r_a^*-\theta)(t-s)}, \tag{15}$$

where (using (5) and noting that individuals are born with zero financial wealth)

$$c_a^*(s,s) = (\lambda+\theta)h_a^* = \frac{(\lambda+\theta)y_a^*}{r_a^*+\lambda}, \quad y_a^* = F(K_a^*) - r_a^*(K_a^* + D_a^*). \tag{16}$$

The allocation of expenditure across the two goods is like at the macro level governed by $c_{1a} = \Lambda c_{2a}$, and thus the relevant consumption paths for the derivation of the steady-state indirect utility function dual to (1)-(2) are

$$c_{1a}^*(s,t) = \frac{\Lambda_a^* c_a^*(s,t)}{\Lambda_a^* + \pi^*}, \quad c_{2a}^*(s,t) = \frac{c_a^*(t)}{\Lambda_a^* + \pi^*}. \tag{17}$$

Using (17) with (15), (16), (2), (1) and integrating[4], the appropriate indirect utility function for a

[3] Ghosh (1998) suggests that a sufficient condition for crowding out is $\gamma \leq 0.5$. However, (12) shows that crowding out takes place in the model irrespective of the value of γ, provided an economically meaningful steady state so that $Y_a = y_a > 0$ exists.

steady-state welfare analysis - which is not concerned with the effects on welfare of cohorts alive during the transition process after a policy shock - can be written in the form

$$u_a^*(\theta+\lambda) = \frac{1}{\rho}[\ln\pi^* - (1+\rho)\ln(\Lambda_a^* + \pi^*)] + \ln y_a^* - \ln(r_a^* + \lambda) + \frac{r_a^* - \theta}{\theta + \lambda} + A, \qquad (18)$$

where A=ln(p+θ)-ln(1-α)/ρ is a policy-invariant constant.

A normative analysis based on the index of aggregate consumption (14) proposed by Ghosh(1998) is potentially misleading, since it is possible that $dU_a^*/dD_a^*>0$ although $du_a^*/dD_a^*<0$; i.e. this index may rise even though all cohorts born in the new steady state are worse off than they would have been in a steady state with lower government debt. The reason is that (14) does not capture the effect of changes in D^* on the non-stationary steady-state time profile of individual consumption expenditure paths: A rise in D^* lowers K^* and net-of-tax labour income y^* while r^* rises, and thus consumption expenditure of each new cohort starts from a lower level according to (16). However, the growth rate of consumption (r^*-θ) over an individual's lifetime rises. In short, individual consumption grows faster but from a lower base in a steady state with higher debt. Since new cohorts face a higher tax from birth onwards but accumulate government bonds only gradually with age, a higher debt is associated with redistribution from the young to the old.

It is therefore possible that the aggregates C_{1a} and C_{2a} both rise entailing a rise in the aggregate consumption index U_a, while each individual cohort is actually worse off, since the effect of the drop in initial consumption on lifetime utility dominates the positive consumption growth effect. The following replication of Ghosh's numerical simulation analysis illustrates the point.

4. Simulation Analysis

Following Ghosh (1998), we set γ=0.2, λ=θ=0.05, $\alpha_a=(1-\alpha_b)=0.7$, and ρ=5. Like in Ghosh, the welfare analysis is restricted to a steady-state comparison and does not take account of changes in lifetime utility of cohorts alive during the transition process between steady states.[5] In the initial steady state with $D_a^*=D_b^*=0$, the two economies are symmetric mirror images. Table 1 shows the steady-state effects of an increase in country a's long-run government debt to $D_a=1$, i.e. a rise in the debt/income ratio from zero to about 75 percent.[6]

Capital stock and output drop in country a, and the terms of trade improve. Aggregate consumption quantities of both goods rise and hence Ghosh's aggregate consumption index rises, yet due to the adverse effect on intertemporal consumption profiles discussed in the preceding section all cohorts born in the new steady state are actually worse off than they would be in a steady-state with zero debt.

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[4] See appendix for a detailed derivation of (18).

[5] A social welfare analysis that takes the transition process into account would require the introduction of an intertemporal aggregate welfare function weighing the lifetime utilities of individual cohorts. See Calvo and Obstfeld (1988) for the time-consistent specification of social welfare functions in OLG models with lifetime uncertainty.

[6] Since government spending is assumed to be zero in the model for notational convenience, the policy shock takes the form of a free uniform distribution of perpetual bonds at one point in time across living cohorts in conjunction with the introduction of a lump-sum tax to cover debt service payments as in Blanchard (1985).

It is tempting but not admissible to argue that the aggregate consumption quantities could be redistributed intergenerationally in lump-sum fashion, so that individual consumption paths become stationary and U_a becomes the correct welfare index. However, it is evident from (4) that the intertemporal consumption profile of individual agents is governed by $r_a(K_a)-\theta$ under decentralized decision-making and cannot be changed without affecting the steady-state capital stock. Thus, in the presence of a transfer scheme that redistributes income from the old to the young at each point in time, the steady-state solution derived above would no longer be valid.

Figure 1 shows the relation between u_a^* and public debt for alternative values of the substitution parameter ρ. As the graph indicates, the proposition "that for $\rho>1$ (i.e. $\sigma<1/2$) total welfare rises with rise in D_a^*" (Ghosh, 1998) is not correct, given that any meaningful index of country a's "total welfare" in the model should be an increasing function of individual lifetime utility u_a. Only for extremely large ρ values, which entail an elasticity of substitution between domestic and imported goods close to zero[7] and hence an extremely strong terms-of-trade response, du_a^*/dD_a^* is locally non-negative in the quantitative model.

Table 1: Steady-State Effects of an Increase in Country A's Government Debt

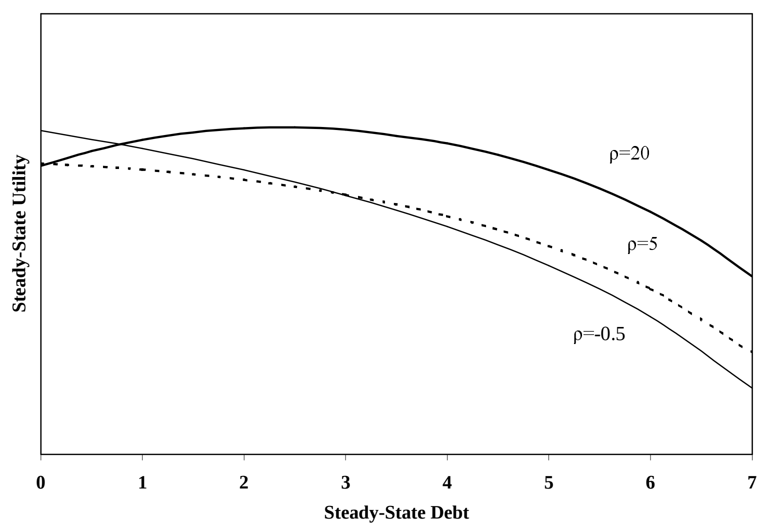
Variable	Description	$D_a^* = 0$	$D_a^* = 1$	% Change
K_a^*	Capital stock	4.050	3.821	-5.7
r_a*	Return to capital	0.065	0.068	+4.7
π^*	Inverse terms of trade	1.000	0.897	-10.3
C_{1a}^*	Aggregate consumption of good 1	0.708	0.729	+3.0
C_{2a}^*	Aggregate consumption of good 2	0.6158	0.6455	+4.8
$q_a^*(s,s)$	Initial consumption of new cohort	0.466	0.440	-5.6
$F(K_a^*)$	Aggregate output	1.323	1.308	-1.2
y_a^*	Individual labour income net of tax	1.058	0.978	-7.6
U_a	Index of aggregate consumption	-0.399	-0.362	+9.4
u_a	Individual lifetime utility	-6.114	-6.372	-4.2

One further observation on Ghosh's original numerical analysis may be noted en passant. For the case $\rho=5$ considered in Table 1 above, the author suggests that the U_a^*-maximizing debt level is around 24.33. However, no feasible steady-state exists for a debt level of this size, even though the macro equations (7)-(10) would seem to allow the computation of non-negative long-run

[7] The existing econometric evidence does not appear to support such low orders of magnitude for σ. See Willenbockel (1994:233-6) for a survey of empirical studies. Consensus estimates for σ used in applied general equilibrium trade models are typically well-above unity.

solutions for all macro aggregates. Yet for D_a=24.33, the tax $T=r_aD_a$ would exceed gross labour income F_a-r_aK_a so that net labour income y_a^* and human wealth h_a^* would be negative for all agents. Since agents are born with zero financial wealth, they would already be insolvent at birth. In order to maintain consistency with the micro structure of the model, a non-negativity constraint for y must be added to the macro equations to avoid economically meaningless simulation results.

Figure 1: Steady-State Government Debt and Individual Lifetime Utility



5. Concluding Remarks

Our analysis has re-examined the steady-state relationship between government debt and welfare in a two-country overlapping-generations model with lifetime uncertainty and international product differentiation. It has recently been proposed that a higher steady-state debt level may be welfare-enhancing in this setting. It is pointed out that this proposition does not adequately account for the adverse effect of debt policy on individual agents' intertemporal consumption profiles. While a higher debt may indeed raise aggregate real consumption at each point in time, each cohort consumes less when young and more when old, and the lifetime utility of all steady-state cohorts will actually drop, unless the elasticity of substitution between domestic output and imports is extremely low.

These particular results indicate and illustrate a more general caveat pertaining to normative policy analyses in any setting with overlapping generations of intertemporally optimizing agents: Attempts to draw welfare inferences on the basis of comparisons of aggregate consumption paths can be fundamentally misleading.

Appendix: Derivation of Equation (18)

Using (17) in the CES felicity index (2), we have

$$\ln q_a^*(s,t) = \ln c_a^*(s,t) + \psi \ , \ \psi \equiv \frac{1}{\rho}[\ln \pi^* - (1+\rho)\ln(\Lambda^* + \pi^*) - \ln(1-\alpha)] \ . \qquad \text{(A-1)}$$

Using this result in (1), the expected lifetime utility of an agent born at any date s in the steady state can be written

$$u_a^* = \frac{\psi}{\theta + \lambda} + \int_s^\infty \ln c_a^*(s,t) e^{(\theta+\lambda)(s-t)} dt \ . \qquad \text{(A-2)}$$

We know from (15) that

$$\ln c_a^*(s,t) = \ln c_a^*(s,s) + (r_a^* - \theta)(t-s) \qquad \text{(A-3)}$$

Thus the integral term in (A-2) takes the form

$$\int_s^\infty \ln c_a^*(s,t) e^{-(\theta+\lambda)(t-s)} dt = \frac{\ln c_a^*(s,s)}{\theta+\lambda} + (r_a^* - \theta)\int_s^\infty (t-s) e^{-(\theta+\lambda)(t-s)} dt \ . \qquad \text{(A-4)}$$

The integral on the RHS of (A-4) can be solved via integration by parts and is equal to $(\theta+\lambda)^{-2}$. Recalling (16), we end up with

$$u_a^* = \frac{\psi}{\theta+\lambda} + \frac{\ln y_a^* + \ln(\theta+\lambda) - \ln(r_a^* + \lambda)}{\theta+\lambda} + \frac{r_a^* - \theta}{(\theta+\lambda)^2} \ , \qquad \text{(A-5)}$$

which is equivalent to (18).

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