E C O N O M I C S   B U L L E T I N

The Ahmad−Stern approach revisited

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Abstract

This paper extends the methodology first proposed by Ahmad and Stern for the design of tax reforms that are optimal at the margin. The extension centers on a sharper approximation of welfare measures. The original approach and its variant are illustrated in the case of the current Mexican tax system.

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1. Introduction

Many theoretical papers have been written on the theory of optimal taxation for the last three decades. However, somewhat paradoxically, most of the results in that area are of little relevance for economic policy-makers. This is so because the theory of optimal taxation imposes a large set of informational requirements that are unlikely to be met in practice: For all the representative agents and firms in the economy, one needs to have reliable estimates of their behavioral responses to all possible changes in taxes and transfers.

Given that constraint, some authors have proposed instead simpler approaches that might serve as guides in the design of optimal tax systems. One of them, known as the marginal tax reform methodology, was first advanced by Ahmad and Stern (1984) [AS, from now on]. Their approach, to be reviewed below, assesses the impact of tax reforms by means of first-order approximations of the relevant variables, in such a way that policy-makers need information only about actual data (not fitted values), and aggregate rather than individual demand responses.

The attractiveness of such a simplification is attested by a number of empirical papers that have applied that methodology over the years. There is, however, a drawback: As it has been forcefully argued in a general context by Banks, Blundell and Lewbel (1996), the measurement of social welfare through the use of first-order approximations may lead to systematic biases, since substitution effects can be non-trivial.

Taking note of that admonition, this paper extends the AS marginal tax analysis by means of sharper approximations of welfare measures, in order to have a more robust approach. Toward that end, the next section reviews the key issues involved in the AS methodology, while the third section presents the extension. Finally, the fourth section illustrates the original approach and its variant using as an example the current Mexican tax system.

2. The Ahmad-Stern approach

According to AS, the optimality of an indirect tax structure may be evaluated by comparing, for all the relevant goods, the marginal social welfare cost of raising revenue via an increase in the excise tax on each of them. Clearly, optimality requires that such a marginal cost should be equal for all goods, otherwise a Pareto improvement could be implemented by lowering the tax on the good with the higher marginal cost, and by raising the tax on the good with the lower marginal cost.

To clarify matters, consider the following simple model entertained by AS: On the production side prices are fixed, and all firms exhibit constant returns to scale, so that indirect tax changes are only reflected as consumer price changes. Given the I goods, indexed by $i = 1, 2,..., I$, let $\mathbf{p}$ denote the corresponding (fixed) producer price vector. Thus, if $\mathbf{t}$ is the vector of specific taxes, then $\mathbf{q} = \mathbf{p} + \mathbf{t}$ is the final consumer price vector. There are, furthermore, H households, indexed by $h = 1, 2,..., H$. For each household h, the consumption bundle that maximizes utility $u^h(\mathbf{x}^h)$ subject to the corresponding linear budget constraint is denoted as $\mathbf{x}^h(\mathbf{q},m^h)$, while the associated indirect utility function is expressed as $v^h(\mathbf{q},m^h)$.

We also assume the existence of a social welfare function $W(u^1,...,u^H)$, which can be rewritten in terms of prices and incomes as:

$$V(\mathbf{q},m^1,...,m^H) = W(v^1(\mathbf{q},m^1),...,v^H(\mathbf{q},m^H)) \qquad (1)$$

After defining the aggregate demand vector by

$$\mathbf{X}(\mathbf{q}, m^1, \ldots, m^H) = \sum_h \mathbf{x}^h(\mathbf{q}, m^h)$$

we can calculate the government tax revenue as:

$$R = \mathbf{t}'\,\mathbf{X} = \sum_i t_i X_i \tag{2}$$

Now suppose that the excise tax on good *i* is to be increased at the margin. Given equations (1) and (2), the marginal social cost of that tax increase may be defined as the corresponding marginal decrease in social welfare relative to the corresponding marginal increase in government revenue:

$$\lambda_i = -\frac{\partial V / \partial t_i}{\partial R / \partial t_i} \tag{3}$$

where the negative sign on the right-hand side of (3) is needed to make positive the marginal social cost. This is so because $\partial V/\partial t_i$ will always be negative, and, furthermore, we would expect $\partial R/\partial t_i$ to be positive in general.

According to Roy's identity we know that

$$\frac{\partial v^h}{\partial t_i} = -\frac{\partial v^h}{\partial m^h} x_i^h$$

where the first term on the right-hand side of the equation is the private marginal utility of income. Let us now consider its social counterpart: the social marginal utility of income of household *h*, which is defined as:

$$\beta^h = \frac{\partial V}{\partial v^h} \frac{\partial v^h}{\partial m^h} \tag{4}$$

We shall have to say more about these functions soon, but at this point we may note that each β^h may be thought as a welfare weight, since, using the last two equations, the numerator in (3) can be written as the negative of the sum across households of the consumption of good *i*, each level weighted by its corresponding beta:

$$\frac{\partial V}{\partial t_i} = -\sum_h \beta^h x_i^h \tag{5}$$

In a similar fashion, taking the partial derivative with respect to t_i in (2), the impact on government revenue of a marginal increase in the excise tax is found to be

$$\frac{\partial R}{\partial t_i} = X_i + \sum_k t_k \frac{\partial X_k}{\partial t_i} = X_i + \sum_k \frac{t_k X_k}{q_i} \varepsilon_{ki} \tag{6}$$

where ε_{ki} is the uncompensated cross-price elasticity of the aggregate demand for good k with respect to price i.

Finally, after defining $\tau_k = t_k / q_k$ (the proportion of the tax relative to the price), we can then use equations (3), (5) and (6) to find the marginal social cost of taxation of good i:

$$\lambda_i = \frac{\sum_h \beta^h q_i x_i^h}{q_i X_i + \sum_k \tau_k \varepsilon_{ki} q_k X_k} \tag{7}$$

An extensive discussion of the meaning of this expression is given in Ahmad and Stern (1984, p. 265). For our purposes, it suffices to note that in order to apply the AS methodology, which requires computing and comparing each marginal social cost across the I goods, we would just need the following data: the final consumer prices, the welfare weights for all households, the consumption levels, and the aggregate cross-price elasticities.

Thus, it would not seem to be necessary to estimate a full demand system. However, this last appreciation would be correct *only if* the welfare weights defined in (4) were independent of prices. Ahmad and Stern were, of course, fully aware of that fact and so they assumed in their model, as is commonly done in most of the applied papers on the subject, that the indirect social welfare function could be locally approximated by a function independent of prices. More specifically, they made use of the following function popularized by Atkinson (1970):

$$V^A(m^1,...,m^H) = k \sum_h \frac{[m^h]^{1-e}}{1-e} \tag{8}$$

where e is a nonnegative parameter that reflects the degree of aversion to social inequality, k is a constant of normalization, and where the arguments of the function may be taken to be, say, total expenditure per household.

Using (4) and (8), each social marginal utility of income β^h may be calculated by taking the derivative of the social indirect utility function with respect to m^h. Furthermore, those authors suggested, the constant k may be chosen in such a way that the welfare weight for the poorest household is equal to one (and hence marginal social costs are always relative to the poorest household). That is to say, assuming that households are ordered according to their ascending incomes total expenditures, the welfare weight for household h would be given by $\beta^h = (m^1/m^h)^e$. Thus, for instance, when $e = 0$, the social marginal utility of income is equal to one for all households and there is no aversion to social inequality; while if, say, $e = 1$, then a household with an income twice as large as the poorest would have a social marginal utility half as large. That is, as the parameter of inequality aversion is increased, the relative weight of the poorest household is increased as well.

It is important to note, however, that the assumption of independence of prices that lies behind (8) is quite restrictive. Indeed, as shown by Banks, Blundell and Lewbel (1996, Theorem 1), the welfare weights defined in (4) are independent of prices if and only if the indirect social welfare function is of the form

$$V(\mathbf{q}, m^1,...,m^H) = \sum_h [\kappa^h \ln m^h - a^h(\mathbf{q})]$$

for some functions a^h and constants κ^h. To see how restrictive this last condition is, extend (8) to include the general class of indirect social welfare functions due to Bergson (1938):

$$W(v^1(\mathbf{q},m^1),...,v^H(\mathbf{q},m^H)) = k\sum_h \frac{v^h(\mathbf{q},m^h)^{1-e}}{1-e}$$

According to the theorem, the only members in the class that would have welfare weights independent of prices would be the ones for which each indirect utility function is multiplicatively separable in prices and income, and for which the parameter of inequality aversion is equal to one. Thus, in the particular case of (8) the local approximation argument is formally correct *only* when e is near to one.

Summing up, it is clear from the presentation above that the AS approach is neatly tied and very easy to apply (see Section 4 below), but it is also evident that a fragile aspect of that methodology is its local character. This drawback was already recognized by the own authors: "We do not argue that our methods are robust with respect to parameter estimates and model specification, and one should not expect them to be so" (Ahmad and Stern, 1984, p. 295).

3. An algebraic extension

Given that all tax reforms are far from being marginal, it would be interesting to extend the AS methodology using at least second-order approximations as recommended by Banks, Blundell and Lewbel (1996). In our context, such an extension requires that, both, the numerator and the denominator in (3) be replaced by sharper approximations.

More formally, we would like to compute for each good the approximate impact on welfare that would have a tax increase that is small, but not marginal. That is, in principle, we would like to estimate for each good the following expression

$$\Lambda_i = -\frac{\Delta V / \Delta t_i}{\Delta R / \Delta t_i} \tag{9}$$

with

$$\frac{\Delta V}{\Delta t_i} = \frac{W(v^1(q_i + \Delta t_i, \mathbf{q_{-i}}, m^1),...,v^H(q_i + \Delta t_i, \mathbf{q_{-i}}, m^H)) - W(v^1(\mathbf{q},m^1),...,v^H(\mathbf{q},m^H))}{\Delta t_i} \tag{10}$$

and

$$\frac{\Delta R}{\Delta t_i} = \frac{R(q_i + \Delta t_i, \mathbf{q_{-i}}, m^1,...,m^H) - R(\mathbf{q}, m^1,...,m^H)}{\Delta t_i} \tag{11}$$

and where, as usual, by $\mathbf{q_{-i}}$ is meant the vector that includes all the elements of $\mathbf{q}$ except for the *i-th* component.

The second-order Taylor expansion of (10) is given by

$$\frac{\Delta V}{\Delta t_i} \approx \frac{\partial V}{\partial t_i} + \frac{\Delta t_i}{2}\frac{\partial^2 V}{\partial t_i^2}$$

so that

$$\frac{\Delta V}{\Delta t_i} \approx -\sum_h \beta^h \left[1 + \frac{\Delta t_i}{2q_i}\left(\varepsilon_\beta^h + \varepsilon_{ii}^h\right)\right] x_i^h \tag{12}$$

where, for each household h, the first elasticity inside the parentheses refers to the price elasticity of the welfare weight, as defined in (4), while the second one refers to the own-price elasticity of individual demand. Likewise, the second-order Taylor expansion of (11) gives

$$\frac{\Delta R}{\Delta t_i} \approx \left[1 + \frac{\Delta t_i}{q_i}\varepsilon_{ii}\right] X_i + \sum_k \frac{\tau_k q_k}{q_i}\left[\varepsilon_{ki} + \frac{\Delta t_i}{2q_i}\left(\varepsilon_{ki}(\varepsilon_{ki} - 1) + q_i \frac{\partial \varepsilon_{ki}}{\partial t_i}\right)\right] X_k \tag{13}$$

Finally, after simplifying (12) and (13), the following variant to the Ahmad-Stern approach is suggested: To analyze the approximate impact on social welfare that would have a tax increase that is small, but not necessarily marginal, instead of the first-order approximation given in equation (7) above, use for each good the following second-order approximation:

$$\Lambda_i = \frac{\sum_h \beta^h \left[1 + \frac{\Delta t_i}{2q_i}\left(\varepsilon_\beta^h + \varepsilon_{ii}^h\right)\right] q_i x_i^h}{\left[1 + \frac{\Delta t_i}{q_i}\varepsilon_{ii}\right] q_i X_i + \sum_k \tau_k \left[1 + \frac{\Delta t_i}{2q_i}(\varepsilon_{ki} - 1)\right] \varepsilon_{ki} q_k X_k} \tag{14}$$

Since the numerator on the right-hand side of equation (14) contains both each welfare weight and its price elasticity, the measure suggested in this paper requires, of course, the estimation of the full demand system [and the use of (4)]. But the extra effort seems worthwhile: Equation (14) not only recognizes the fact that welfare weights do depend on prices, but also that tax changes involve more than marginal variations, and that a tax reform typically includes a differential treatment across goods.

4. An application to Mexico

Regarding the Mexican tax system, Urzúa (1994 and 2001) and Campos (2002) provided the first studies that estimated the welfare consequences of several tax reforms. Their work, however, did not follow the AS methodology, since they estimated full demand systems to make global welfare comparisons. Nevertheless, for the purpose of our example we can use here the demand system recently estimated by Campos (2002), which is based on an official income and expenditure survey taken in the year 2000 (INEGI, 2001).

That author aggregated the consumption goods into five categories, using as the composition criterion the differential treatment accorded by Mexican tax laws to the value added tax (VAT): A 0% VAT rate for the case of non-processed food, medicines and education (which is actually exempted), and a 15% VAT rate for processed food, clothing, appliances, and alcoholic beverages (including tobacco).[1] A brief description of those five composite goods is given in the

[1] Note that in our model t_k is a quantity tax, not an *ad-valorem* tax. Thus, given a 15% VAT rate, the corresponding τ_k (the proportion of the tax relative to the consumer price) is 0.15/1.15 ≈ 13%.

first column of Table 1 below, while the other five columns present the corresponding uncompensated cross-price elasticities.

Table 1
Composite Goods and Uncompensated Cross-Price Elasticities

	1	2	3	4	5
1. Non-processed food and dairy products	-0.636	0.033	0.013	0.260	0.152
2. Processed food, clothing and appliances	-0.026	-0.672	0.012	-0.027	-0.041
3. Alcoholic beverages and tobacco	-0.062	0.038	-0.816	0.295	2.140
4. Medicines	-0.255	-0.608	0.022	-1.032	-0.185
5. Education	-0.106	-0.289	-0.031	-0.218	-0.955

Source: Campos (2002, tables 3 and 5).

Using Table 1, we are now ready to illustrate the AS approach and the variant suggested in this paper. First note that in order to allow for a direct comparison between both approaches we have to make use of Atkinson's function, given in equation (8) above, since the AS approach assumes that the indirect social welfare function is independent of prices. Thus, at the risk of losing theoretical soundness and numerical precision, we have to approximate (14) as:

$$\Lambda_i \approx \frac{\sum_h \beta^h \left[1+\frac{\Delta t_i}{2q_i}\varepsilon_{ii}^h\right] q_i x_i^h}{\left[1+\frac{\Delta t_i}{q_i}\varepsilon_{ii}\right] q_i X_i + \sum_k \tau_k \left[1+\frac{\Delta t_i}{2q_i}(\varepsilon_{ki}-1)\right]\varepsilon_{ki} q_k X_k} \tag{15}$$

Aside from the fact that (15) is more realistic than (7), insofar as it allows for non-marginal tax changes, both equations are similar in terms of their informational requirements. The only difference is that (15) requires estimates of the own-price elasticities of household demands. In our case, given that the demand system was estimated using a single cross-section of 10,108 different households, we estimate those demand responses by the average of the corresponding elasticities in each income decile.

Making now use of Table 1, as well as of raw data on prices, consumption levels and aggregate demand responses, all of them classified by deciles, Table 2 presents for each good an estimate of the marginal social welfare cost λ_i, as given in (7). Those results are shown for four different levels of inequality aversion: From e=0, when there is no aversion whatsoever, to e=3, when the welfare of the poorest has a substantial relative weight in the social welfare function. Table 2 also presents the results obtained when we use second-order approximations to compute the variant Λ_i, using the simplification given in (15), and after assuming a uniform increase of \$2 in the quantity taxes for all goods. This last magnitude was chosen since it represented an increase from 0% to about 10% in the case of the average price of non-processed food and dairy products. Note that such a price change was taken to be the same across goods to make a fair comparison with the results obtained using (7), but, of course, one of the advantages of (15) over (7) is that now the goods could be accorded a differential tax treatment.

Table 2
Marginal and Approximate Social Welfare Costs

		Degree of inequality aversión			
		e=0	*e*=1	*e*=2	*e*=3
Non-processed food and dairy products	λ_1	1.018	0.125	0.051	0.040
	Λ_1	1.062	0.129	0.053	0.041
Processed food, clothing and appliances	λ_2	1.096	0.055	0.008	0.004
	Λ_2	1.088	0.054	0.008	0.004
Alcoholic beverages and tobacco	λ_3	*0.869*	0.098	0.044	0.036
	Λ_3	*0.851*	0.096	0.043	0.035
Medicines	λ_4	1.022	0.059	0.011	0.006
	Λ_4	0.988	0.057	0.010	0.006
Education	λ_5	1.021	*0.035*	*0.004*	*0.001*
	Λ_5	0.992	*0.034*	*0.003*	*0.001*

Now suppose that the Mexican government decides to increase tax revenue at the lowest social welfare cost.[2] According to Table 2, when there is no inequality aversion, both methods suggest that the excise tax to be raised is the one on alcoholic beverages and tobacco. However, once *e* is increased, the preferred choice becomes education for both approaches. Therefore, as the concern about the poorest takes more importance, one has to shift from "sin taxes" to taxes on education. This last result should be interpreted with extreme care, however, since in this type of models education is wrongly put on the same footing as mere consumption, while in fact the social return on education is certainly larger than its private return.

It should also be noted that even though both variants suggest, in this example, the same optimal way to raise revenue, the ranking of social costs is not always the same for both methods. For instance, when there is no inequality aversion, the A-S approach would suggest that imposing a tax on education would have a lower social welfare cost than taxing medicines, while our method suggests precisely the contrary. Finally, it almost goes without saying that the main reason for the numerical similarity between the two approaches is the use of the rough approximation given in (15), instead of the direct use of (14).

5. Conclusion

This paper has presented an extension to the marginal tax analysis put forward by Ahmad and Stern two decades ago. The variant recognizes that welfare weights do depend on prices, that tax changes are usually more than marginal, and that tax reforms typically include a differential treatment across goods.

[2] Actually, since the end of 2000, the government has made several attempts to impose non-zero tax rates on non-processed food, medicines and/or education, without any success. One of the reasons for that failure could be that Congress has found the social welfare consequences of the tax reform to be lacking.

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