E C O N O M I C S B U L L E T I N

Simulations of fundamental tax reform with irrational households

Michael Williams
University of Saint Thomas

Abstract

Dynamic tax models have been devised to examine the effects of fundamental tax reform−−replacing the current U. S. federal tax system with a national retail sales tax. These models impose a constant and positive rate of time preference on households, in the tradition of the rational, time−consistent consumer. Evidence suggests, however, that households are impatient and time−inconsistent, questioning the validity of a constant rate of time preference. This paper modifies an existing dynamic life−cycle tax model so that it can incorporate this time inconsistency, using a construct known as hyperbolic discounting. We find a significant change in the model's predictions of the effects of fundamental tax reform, including smaller short term losses and smaller long term gains, when the standard assumption of a constant rate of time preference is replaced with the hyperbolic discounting assumption.

Thanks to George R. Zodrow for his collaboration on earlier versions of this research. Any errors are my own.
Citation: Williams, Michael, (2005) "Simulations of fundamental tax reform with irrational households." *Economics Bulletin*, Vol. 8, No. 3 pp. 1−11
Submitted: November 3, 2004. **Accepted:** January 26, 2005.
URL: http://www.economicsbulletin.com/2005/volume8/EB−04H20013A.pdf

1. Introduction

Interest in fundamental tax reform—replacing the current income tax system with one based on consumption—has been percolating in the United States recently, with considerable support in the U. S. Congress and lukewarm support from President Bush. Dynamic tax models have been devised to examine the effects of such reform. These models include four life-cycle models (Altig *et al*. 2001; Engen and Gale 1996; Fullerton and Rogers 1993, 1996, and 1997; Zodrow and Williams 1998a, 1998b) and one infinite horizon intertemporal model (Jorgenson and Wilkoxen 1997a, 1997b). As noted by Engen *et al.* (1997) the predictions of dynamic tax models depend crucially on behavioral specifications of economic agents, including the specification of intertemporal decision-making by households. All of the aforementioned models, for example, impose a constant and positive *rate of time preference* on households, in the tradition of Samuelson's (1937) rational, time-consistent consumer.

Evidence suggests, however, that households are impatient and time-inconsistent, questioning the validity of a constant rate of time preference. This paper modifies the Zodrow-Williams dynamic life-cycle tax model so that it can incorporate this time inconsistency, using a construct known as *hyperbolic discounting*. We find a significant change in the model's predictions of the effects of fundamental tax reform, including smaller short term losses and smaller long term gains, when the standard assumption of a constant rate of time preference is replaced with the hyperbolic discounting assumption.

2. The Standard Discounted Utility Model: Exponential Discounting

Despite Paul Samuelson's reservations concerning its descriptive validity, his discounted utility model (Samuelson 1937) is widely used, including in the life-cycle tax models mentioned above. Its general form is widely known:

$$U_t = \sum_{k=0}^{T-t} \left(\frac{1}{1+\rho} \right)^k u_k \tag{1}$$

Rest-of-life utility at time period t, U_t, is a discounted sum of the utilities obtained in time periods t through T, $u_t \ldots u_T$. ρ is the household's constant pure rate of time preference. Typically (as in the dynamic life cycle tax models mentioned above) ρ is assigned a positive constant value, consistent with exponential discounting over the household's lifetime.

3. Criticism of the Discounted Utility Model: Time Inconsistency

As discussed in Frederick *et al.* (2004), exponential discounting as exhibited in equation (1) above means that a household's intertemporal preferences are *time consistent*; intuitively, this means that preferences later in life confirm the choices made earlier in life. More formally, suppose that in time period t, a household prefers outcome X_τ occurring in time period τ to outcome $Y_{\tau+\delta}$ occurring in time period $\tau + \delta$. Constant discounting implies that the household should always prefer X_τ to $Y_{\tau+\delta}$ in *any* time period $t = 0 \ldots T$.

Empirical evidence does not support constant discounting. Instead, several experimental studies suggest that a household has a declining rate of time preference. (See Kirby 1997, Loewenstein and Prelec 1992, and Frederick *et al*. 2004 for surveys of some of these studies.) For example Thaler (1981) found that when subjects were asked to compare a smaller-sooner award to a larger-later award, the implied rate of time preference was lower over longer time periods than the implied rate over shorter time periods.

4. Hyperbolic Discounting: An Alternative to the Standard Discounting Model

The term *hyperbolic discounting* has come to mean that a person has a declining rate of time preference—longer time horizons mean lower rates of time preference (even if the rate of decline does not precisely follow a hyperbolic path). A simple functional form consistent with hyperbolic discounting was introduced by Phelps and Pollack (1968), popularized by Laibson (1997, 1998) and discussed at length by Angeletos *et al*. (2001). Let $D_H(k)$ represent a discount function such that:

$$D_H(k) = \begin{cases} 1 & if \quad k = 0 \\ \beta\delta^k & if \quad k > 0 \end{cases} \tag{2}$$

We can define a hyperbolic discounted utility function as follows:[1]

$$U_t = \sum_{k=0}^{T-t} D(k) u_k \tag{3}$$

Angeletos *et al.* contend that applying the values $\beta = .7$ and $\delta = .957$ best replicates the *self-control* problem characteristic of human subjects in laboratory experiments. Not only are these subjects time inconsistent but they are also impatient, preferring, for example, “a 15-minute break right now, rather than a 20-minute break tomorrow” (Angeletos *et al*., p. 48).

5. Exponential vs. Hyperbolic Discounting in the Modified Zodrow-Williams Tax Model

It is reasonable to expect differences in a dynamic tax model’s predictions of the effects of fundamental tax reform, depending upon the discounting assumption applied to households—exponential or hyperbolic. We will test this expectation using a modification of the dynamic life-cycle tax model developed by Zodrow and Williams (hereafter referred to as the ZW model). The ZW model posits a time separable lifetime utility function in which during each year of life k:

$$u_k = \left[\alpha^{\frac{1}{\sigma^2}} C_k^{1-\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma^2}} E_k^{1-\frac{1}{\sigma}} \right]^{\frac{1}{1-\sigma}} \tag{4}$$

[1] Since the discount path implied by this function is not precisely hyperbolic, it is also known as *quasi-hyperbolic*.

Where C_k is consumption in year k, E_k is leisure in year k, α is a share parameter and σ is a substitution elasticity.

In its original form the ZW model specified exponential discounting of future utility consistent with equation (1). In it new form, the ZW model can still incorporate exponential discounting, but alternatively this assumption can be replaced with the hyperbolic discounting formulation of equation (2).

6. Other Attributes of the ZW Model

The ZW model is in the tradition of dynamic overlapping generations tax models. A closed economy is represented by a compendium of households, a representative firm, and two representative governments.

- Fifty-five coexisting Households, differentiated by age, act to maximize their rest-of-life utilities in each year of their 55-year lifetimes, where rest-of-life utility is calculated according to the description in section 5 above. Each household earns labor income during the first 45 years of its life and then retires for 10 years. The model can be constructed to incorporate either exponential discounting or hyperbolic discounting by households.
- A national government raises sufficient tax revenue to fund expenditures. Debt finance is prohibited.
- In each year t, a representative firm acts to maximize its value, V_t:

$$V_t = \sum_{s=t}^{\infty} \prod_{u=t}^{s} \frac{1}{\left(1+\theta_u\right)} \Gamma \tag{5}$$

where:

$$\theta_u = [(1-\tau_i)\, i_u\,] / (1-\tau_g) \tag{6}$$

$$\begin{aligned}\Gamma = {} & \Omega(1-\tau_{bs})(\, F(K_s,L_s) - w_s L_s) - K_s[\Omega(1-\tau_{bs} f_2) i_s b - d\{\, 1\text{-}b\text{-}\,\Omega(1-\tau_{bs} f_4 b)\}] \\ & - I_s \{\, 1 - b - \Omega[\tau_{bs}(f_1 - f_3 b) + ITC] - f_5 Z_t + (1\text{-}\,\Omega\tau_{bs})\Phi_t \,\}\end{aligned} \tag{7}$$

$$\Omega = [\zeta\,(1-\tau_d) + (1\text{-}\,\zeta)(1-\tau_g)] \;\;/\;\; (1-\tau_g) \tag{8}$$

$$F(K_s,L_s) = K_s^{a}\, L_s^{(1-a)} \tag{9}$$

$$Z_t = [\Omega\tau_{bs}\, d_\tau\,] / [\theta_u + d_\tau] \tag{10}$$

$$\Phi'_s = \frac{d}{d\,\frac{I_t}{K_t}} [(B/2)(I_t/K_t - \mu)^2 / (I_t/K_t)] \tag{11}$$

a, b, B, d, d_τ, and μ are constant terms

f_1-f_4 are constant terms that equal 1 under conditions simulating the current tax regime and equal zero when the current tax system is replaced with a single retail sales tax under fundamental tax reform.

i_u is the interest rate.

I is investment

ITC is the investment tax credit (positive under the current tax system, zero under fundamental tax reform).

K is units of capital.

L is units of labor

τ_d, τ_i, τ_g, and τ_{bs} are tax rates on dividends, interest, capital gains, and corporate income (non-zero under the current tax system, zero under fundamental tax reform).

w is the wage per unit of labor.

A *steady state equilibrium* is attained in which:

- All markets clear annually
- The economy grows at a constant rate (equal to the sum of the exogenous rate of population growth and rate of technological change)
    - Consumption, Investment, and Government Purchases grow at a constant rate and remain a constant fraction of GDP.
- Firm values grow at a constant rate
- Household utilities grow at a constant rate

A change in the tax structure (which in our case will be a replacement of personal and corporate income taxes with a national retail sales tax) causes the economy to veer from its former steady state path. Eventually, a new steady state is established. Analysis of the transition path to the new steady state, and analysis of the new steady state itself, provides insight into the effects of the tax reform over time.

7. Parameter Specification and Calibration of the modified ZW Model

Parameter specification and calibration of dynamic tax models is a complex process that cannot be fully addressed in this short space. Full parameterization and calibration details are available from the author. (A thorough description of the specification and parameterization of the Fullerton-Rogers model—a model similar to the ZW model—is contained in Fullerton and Rogers 1993.)

Parameterization and calibration is undertaken so that the model reflects the target real world economy—in our case, the modern U. S. economy—as accurately as is practicable.

- Production parameters and business tax rates are specified. In our case these include: the scale and share parameters in the Cobb-Douglas aggregate production function; adjustment cost parameters for new investment; a share parameter for the portion of investment funded by debt (vs. equity); a dividend distribution parameter; a corporation income tax rate.
- Household parameters are specified for two alternate household models—the exponential discounting model and the hyperbolic discounting model. In the exponential case, the

constant rate of time preference is specified. In the hyperbolic case, β and δ are specified. In both cases, tax rates on wages, interest income, and capital gains are specified, as is the substitution elasticity between consumption and leisure.

- U.S. economic data from the year 2000 regarding production, investment, capital stocks, labor supply, wealth, consumption, personal savings, government spending and tax revenues are used to complete the calibration of the ZW model

Table I lists and describes some important model parameter values.

Table I: Zodrow-Williams Model, Selected Parameters

Parameter	**Description**	**Benchmark Value**
Household Parameters		
β	Hyperbolic discounting parameter (hyperbolic discounting model)	0.7
δ	Hyperbolic discounting parameter (hyperbolic discounting model)	0.957
ρ	Constant rate of time preference (exponential discounting model)	0.03
σ	Elasticity of substitution between consumption and leisure	0.25
τ_c	Sales tax rate	0
τ_d	Dividend tax rate	0.235
τ_g	Effective tax rate on capital gains	0.091
τ_i	Interest income tax rate	0.195
τ_w	Wage tax rate	0.258
Production Parameters		
τ_b	Corporate income tax rate	0.383
a	Cobb-Douglas production function share parameter	0.25
d	Rate at which capital depreciates	0.08
d_τ	Accelerated depreciation allowed by income tax	0.24
B	Adjustment cost parameter for installing new capital	10
μ	Adjustment cost parameter for installing new capital	0.0353
ζ	Fraction of after-tax earnings paid as dividends	0.48
b	Debt-to-capital ratio	0.337

8. Two Benchmark Equilibria: Exponential and Hyperbolic

Two parameterization and calibration procedures were performed. The first benchmark equilibrium represents the U.S. economy assuming that households are accurately represented by the Samuelson-type exponential discounted utility function. The second benchmark equilibrium represents the same U.S. economy, but it assumes that households are accurately represented by the Phelps-Pollack-type hyperbolic discounted utility function.

9. Two Simulations of Fundamental Tax Reform: Exponential and Hyperbolic

We simulate the effects of fundamental tax reform by setting all benchmark tax rates to zero and replacing them with a single tax rate on consumption that yields equivalent (real) tax revenue. We perform this simulation twice—once for the exponential model and once for the hyperbolic model. In each case, the steady state equilibrium is disrupted for a number of years as households and firms adjust to changes wrought by the tax reform; aggregate consumption (C), aggregate investment (I), and GDP (Y) are disrupted from their steady state growth rates under the old tax regime. Eventually (after approximately 100 years) new steady state equilibria are attained, but the absolute levels of C, I, and Y differ from the levels that would have obtained had the old tax regime remained in place.[2]

10. Simulation Results

To ascertain the effects of fundamental tax reform, we trace the paths of consumption, investment, and GDP in the years following reform. For each simulation, we calculate the percentage by which the post-reform levels of C, I and Y deviate from the levels that would have occurred in the absence of tax reform, for each year subsequent to reform, for 100 years. These 100-year paths of the deviations of C, I and Y from their no-reform 100-year paths are plotted in figures 1-3 at the end of this paper. Figure 4 displays the 100-year path of the equal yield sales tax rate.[3]

11. Analysis of Results

It is clear from figures 1-3 that the differences between the exponential discounting simulation and the hyperbolic discounting simulation are nontrivial. As figure 1 indicates for example, the initial decline in aggregate consumption is smaller under hyperbolic discounting than under exponential discounting; in the first year after reform aggregate consumption deviates by -4.4% under exponential discounting, while under hyperbolic discounting the deviation is only -1.2%. However, the long term increase in aggregate consumption is also smaller in the hyperbolic case—a gain of .8%, compared to a long term gain of 2% in the exponential case. This means that a higher sales tax rate is ultimately required under the hyperbolic case than the exponential case, as shown in figure 4. If critics of exponential discounting are correct about its lack of validity, then the results of dynamic tax models that rely on it may be overstating the adverse effects of fundamental tax reform on the current generation of households, and also

[2] The model is translated to computer code and simulations solved using a variant of the Gauss-Seidel method.
[3] We assume that 100% of aggregate consumption is taxed; there are neither exemptions nor evasion.

overestimating the benefits that accrue to future generations (while understating the long term consumption tax rate required under fundamental tax reform).

12. Future Research

The results obtained above are sensitive to the values chosen for important parameters. Because it is our intention to contrast hyperbolic and exponential discounting by households, perhaps the most important parameter value are those chosen for ρ, β, and δ. Since there is contention surrounding the empirically valid values for these parameters, sensitivity analysis should be performed to ascertain the robustness of our results to changes in these values. Such sensitivity analysis is an ongoing part of our research.

13. Conclusion

Given the recent political success of those who advocate consumption tax reform, it behooves economists to refocus efforts at forecasting the effects of such reform, to add information to the political debate that may be forthcoming. Because of the devastating lack of empirical evidence to support the prevailing exponential discounting utility model, it is sensible to try alternative household specifications in dynamic tax models. Our preliminary results indicate that when the hyperbolic discounting specification replaces the standard methodology, it has a significant impact on the forecasts of the effects of fundamental tax reform, including a reduction in the short term negative consequences, a reduction in the long term benefits, and an increase in the long term revenue neutral tax rate.

References

Altig, D., Auerbach, A. J., Kotlikoff, L. J., Smetters, K. A., and J. Walliser (2001) "Simulating Fundamental Tax Reform in the United States" *American Economic Review* **91**, 547-595.

Angeletos, G-M., Laibson, D., Repetto, A., and J. Tabacman (2001) "The Hyperbolic Consumption Model: Calibration, Simulation, and Empirical Evaluation" *Journal of Economic Perspectives* **15**, 47-68.

Auerbach, A., and L. Kotlikoff (1987) *Dynamic Fiscal Policy*, Cambridge University Press: New York.

Engen, E. M., and W. G. Gale (1996) "The Effects of Fundamental Tax Reform on Savings," in H. J. Aaron and W. G. Gale, Eds., *Economic Effects of Fundamental Tax Reform*, Brookings Institution; Washington D. C., 91-122.

Engen, E. M., Gravelle, J., and K. Smetters (1997) "Dynamic Tax Models: Why They Do the Things They Do" *National Tax Journal* **50**, 657-82.

Frederick, S., Loewenstein, G., and T. O'Donoghue (2004) "Time Discounting and Time Preference: A Critical Review," in C. F. Camerer, G. Loewenstein, and M. Rabin, Eds., *Advances in Behavioral Economics*, Russell Sage Foundation: New York, 223-251.

Frey, B. S., and A. Stutzer (2002) "What Can Economists Learn from Happiness Research?" *Journal of Economic Literature* **40**, 402-435.

Fullerton, D., and D. L. Rogers (1993) *Who Bears the Lifetime Tax Burden*? Brookings Institution: Washington, D.C.

----- and ----- (1996) "Lifetime Effects of Fundamental Tax Reform," in H. J. Aaron and W. G. Gale, Eds., *Economic Effects of Fundamental Tax Reform*, Brookings Institution: Washington D. C., 321-352.

----- and ----- (1997) "Neglected Effects on the Uses Side: Even a Uniform Tax Would Change Relative Goods Prices" *American Economic Review* **87**, 120-125.

Jorgenson, D. W., and P. J. Wilkoxen (1997a) "Fundamental U. S. Tax Reform and Energy Markets" *The Energy Journal* **18**, 1-30.

----- and ----- (1997b) "The Effects of Fundamental Tax Reform and the Feasibility of Dynamic Revenue Estimation" prepared for the Joint Committee on Taxation's Symposium on Modeling the Macroeconomic Consequences of Tax Policy.

Kirby, K. N. (1997) "Bidding on the Future: Evidence Against Normative Discounting of Delayed Rewards" *Journal of Experimental Psychology* **126**, 54-70.

Laibson, D. (1997) "Golden Eggs and Hyperbolic Discounting" *Quarterly Journal of Economics* **62**, 443-478.

----- (1998) "Life-Cycle Consumption and Hyperbolic Discount Functions" *European Economic Review* **42**, 861-871.

Loewenstein, G., O'Donoghue, T., and M. Rabin (2003) "Projection Bias in Predicting Future Utility" *Quarterly Journal of Economics* **118**, 1209-1248.

Loewenstein, G., and D. Prelec (1992) "Anomalies in Intertemporal Choice: Evidence and an Interpretation" *Quarterly Journal of Economics* **107**, 573-597.

Loewenstein, G., and D. Schkade (1999) "Wouldn't It Be Nice? Predicting Future Feelings," in D. Kahneman, E. Diener, and N. Schwartz, Eds., *Well-Being: The Foundations of Hedonic Psychology*, Russell Sage Foundation: New York, 85-105.

O'Donoghue, T. and M. Rabin (2004) "Doing It Now or Later," in C. F. Camerer, G. Loewenstein, and M. Rabin, Eds., *Advances in Behavioral Economics*, Russell Sage Foundation: New York, 223-251.

Phelps, E. S., and R. A. Pollack (1968) "On Second-Best National Saving and Game-Equilibrium Growth" *Review of Economic Studies* **35**, 185-199.

Prelec, D., and Loewenstein, G. (1997) "Beyond Time Discounting" *Marketing Letters* **8**, 97-108.

Samuelson, P. A. (1937) "A Note on the Measurement of Utility" *Review of Economic Studies* **4**, 155-161.

Strotz, R. H. (1956) "Myopia and Inconsistency in Dynamic Utility Maximization" *Review of Economic Studies* **23**, 165-180.

Thaler, R. H. (1981) "Some Empirical Evidence on Dynamic Inconsistency" *Economic Letters* **8**, 201-207.

Zodrow, G. R., and M. F. Williams (1998a) "Intergenerational Transition Issues in the Implementation of a National Retail Sales Tax" prepared for Americans for Fair Taxation.

----- and ----- (1998b) "The Effects of a Value Added Tax on Consumption and Retail Sales" prepared for the Economic Policy Institute.

Figure 1

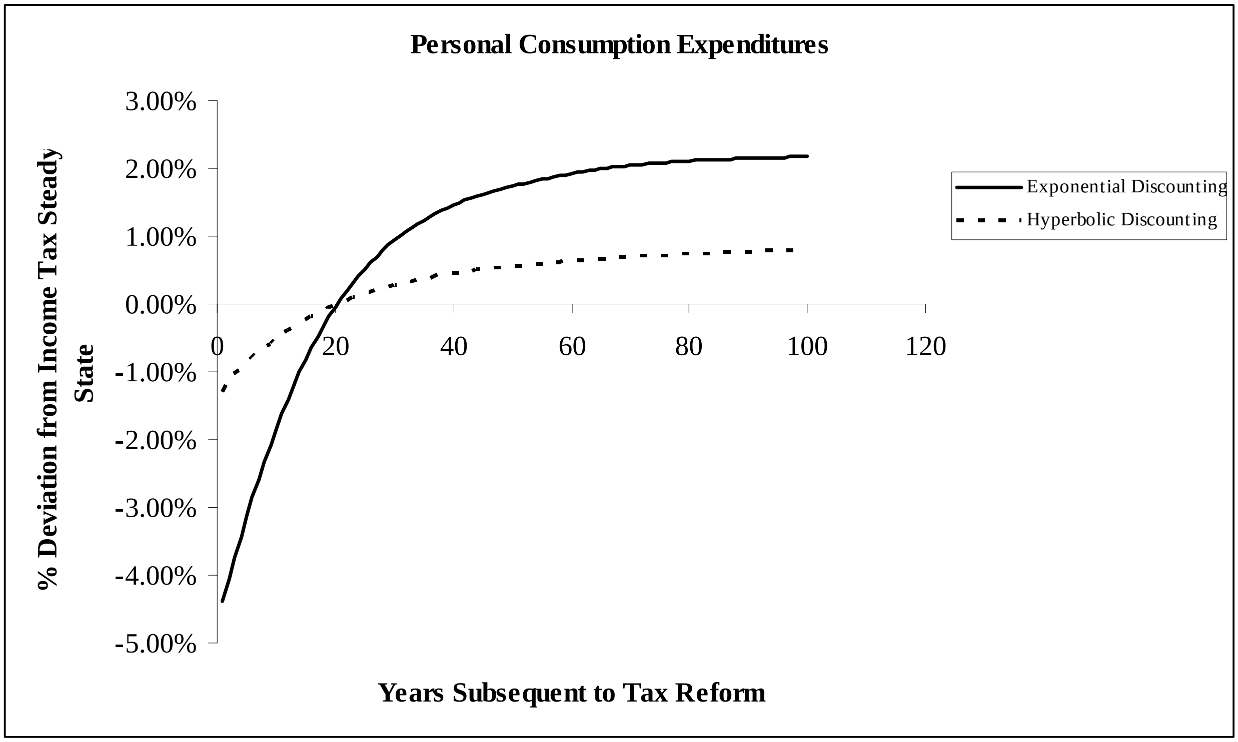

Figure 2

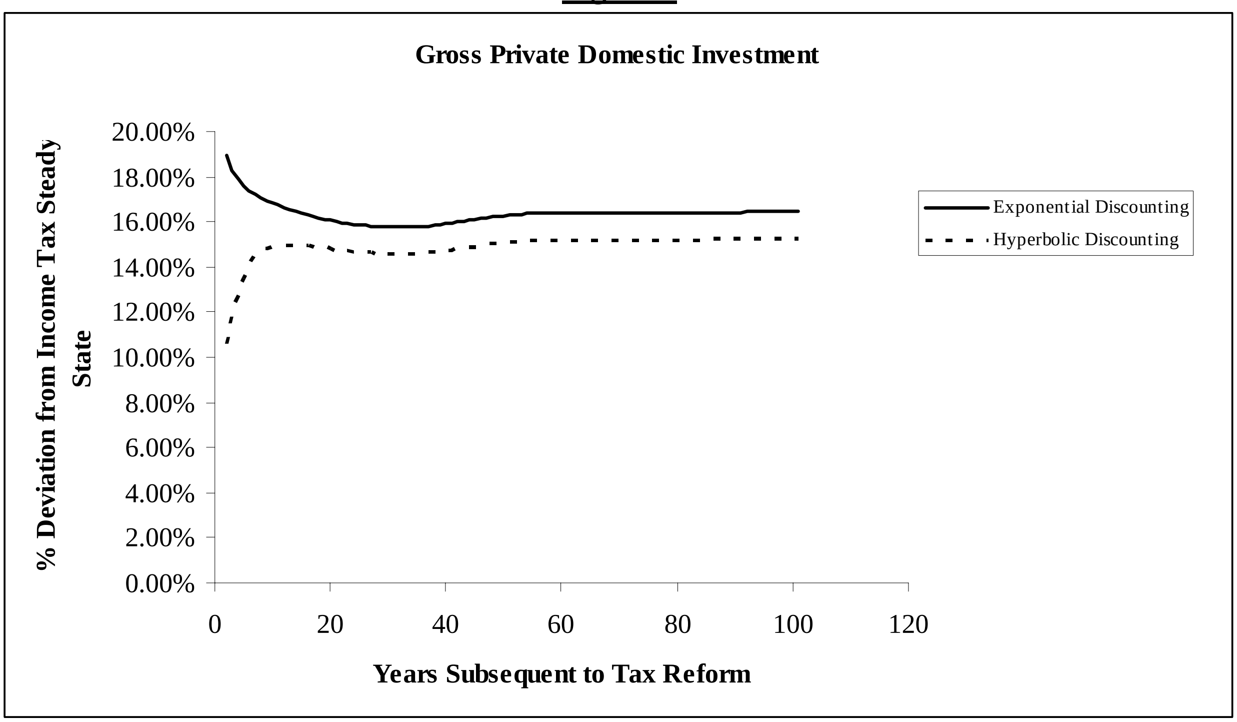

Figure 3

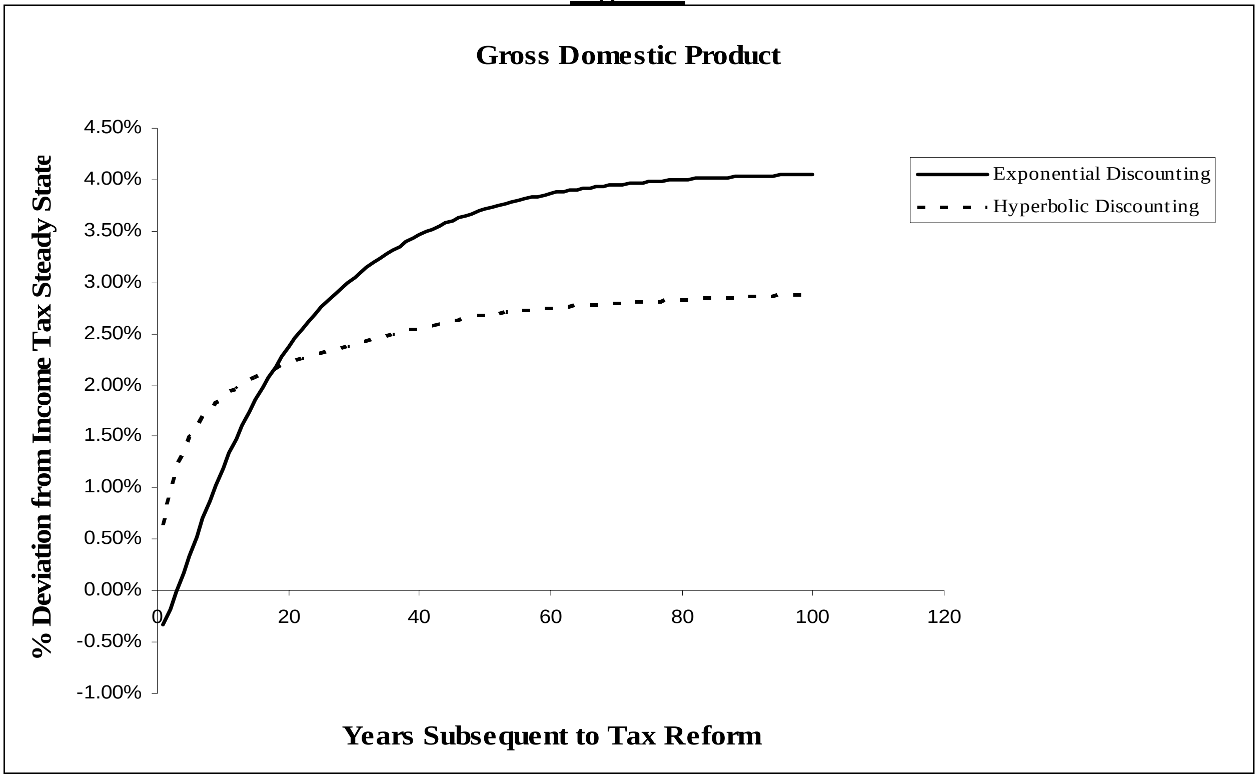

Figure 4