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Equilibrium Incentives and Accumulation of Relational Skills
in a Dynamic Model of Hold up *

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Abstract

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Key words: A Dynamic Model of Hold up, Relation-Specific Investments, Markov Perfect Equilibria, Strategic Effect.
JEL Classification: C73, D23, L14

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1. Introduction

Konishi *et al.* (1996) analyzed the Holdup problem and its solution in a framework of finite-horizon continuous-time capital accumulation games. In this paper, we construct an infinite-horizon model of Holdup, where the state variable is the relational skill at the beginning of a given period, and solve for a Markov perfect Equilibrium. Firms decide their individual investment levels taking into account their impact on strategic interactions from the next period onwards. By considering *hypothetically* the impact of firms' current investment decisions only on their strategic positions in the next period, in other words, ignoring effectively those in the periods subsequent to the next period, we gain a useful understanding between two-period and infinite horizon formulations.[1] We also compare the equilibrium incentives in both two-period and infinite horizon formulations, and investigate the equilibrium comparative statics and its implications.

2. A Dynamic Model of Hold up

2.1 Set-up

We consider a dynamic game involving relation specific skills and the "Hold up" problem. There are two parties: Buyer B and Seller S. The two parties meet, *ex post* leading to a bilateral monopoly. B invests e^B, and S invests e^S. The *ex post* renegotiation surplus is $R\left(e^B\right)-C\left(e^S\right)$, where $R'\left(e^B\right)>0,\ C'\left(e^S\right)<0$. *Ex post* they renegotiate efficiently under symmetric information, dividing the renegotiation surplus 50/50 (Nash Bargaining Solution). Given the current level x_t of the relation specific skill and the investment levels e_t^B and e_t^B by players B and S at time period t, the dynamics (the evolution of the state variable) is modeled by:

$$x_{t+1}=f\left(x_t+e_t^B+e_t^S\right) \qquad t=0,1,2,...$$

where we assume that $f\left(0\right)=0$ and it is monotone increasing. We interpret state x_t as the common relational skill (capital stock) level at time t, to which both parties can access, and the state in the next period x_{t+1} is given by the above time-independent

[1] Suzuki (2005) performs almost the same exercise in an infinite horizon international duopoly model with dumping behavior and anti-dumping laws.

function. Moreover, there exists $\bar{x}_t > 0$ such that for every $x > \bar{x}_t$, we have $f_t(x) < x$. Hence, the state space at time period $t+1$ is $X_{t+1} \in [0, \bar{x}_t]$, regardless of investment levels e_t^B and e_t^S. Moreover, let us assume that $M = \sup_t \bar{x}_t < \infty$ and we denote by $X = [0, M]$, the set of feasible states. Players have bounded maximum investment levels, i.e. $0 \le e_t^i \le K^i(x_t), i = B, S$

In each stage game, two players choose specific investments as follows:

$$e_o^B \in \arg\max_{e^B} U^B(x, e^B, e^S) = \frac{1}{2}\left[R(x, e^B) - C(x, e^S)\right] - e^B$$

$$e_o^S \in \arg\max_{e^S} U^S(x, e^B, e^S) = \frac{1}{2}\left[R(x, e^B) - C(x, e^S)\right] - e^S$$

We have *an underinvestment result*, since each party internalizes only 50% of its contribution to total surplus, while bearing all investments costs. In this set-up, we define a function $\phi : E \times X \to \mathbb{R}$ where $E, X \subset \mathbb{R}$, which has *Single Crossing Property (SCP)* if $\phi_e(e, x)$ exists and it is strictly increasing in $x \in X$, for all e. The intuition is that higher x induces the marginal benefits of raising e^B, e^S. This property is called *supermodularity*. A key result in monotone comparative statics is that when the objective function satisfies *Single Crossing Property (SCP)*, the maximizers are increasing in the parameter value. So, according to the theorem of Topkis (1978) and Edlin and Shannon (1998), supposing that ϕ has *SCP*, $x'' > x'$ and $E(x) = \arg\max_{e \in E} \phi(e, x)$, then for any $e' \in E(x')$ and $e'' \in E(x''), e'' > e'$. Given that by assumption $\phi_B(x, e^B) = R(x, e^B)/2 - e^B$ and $\phi_S(x, e^S) = -C(x, e^S)/2 - e^S$ have *Single Crossing Property (SCP)*, it results in the monotonicity properties of optimal solutions: $e_i' \in E(x') < e_i'' \in E(x'')$, where $x'' > x', i = B, S$.

Next, the payoffs for the Infinite Horizon Game are:

$$\sum_{t=0}^{\infty} \delta^t U_t^i(x_t, e_t^B, e_t^S) \qquad i = B, S$$

where $\delta \in [0,1)$ is a common discount factor.

2.2 Equilibrium Concept: Markov Perfect Equilibria

The equilibrium concept that we mainly adopt is a pure strategy Markov Perfect Equilibrium. The strategy for player $i = B, S$ is a sequence of maps of the form:

$$\left(e_t^B\left(x_t\right), e_t^S\left(x_t\right)\right), t = 0,1,..,$$

where $e_t^i\left(x_t\right)$ is the Markov strategy of player $i = B, S$ in that strategies depend only on specified state variables x_t.

<u>Definition</u> *A pair of strategies* $\left(e_t^{B*}\left(x_t\right), e_t^{S*}\left(x_t\right)\right), t = 0,1,..,$ *is called a* ***Markov Perfect Equilibrium*** *(MPE) of the dynamic game if* ***for every feasible state*** $x_t \in S_t$ ***at time period*** t*, we have for every feasible pair* $\left(e_t^B\left(x_t\right), e_t^S\left(x_t\right)\right), t = 0,1,..,$

$$\sum_{k=t}^{\infty} \delta^k \cdot U_k^B\left(e_k^{B*}\left(x_k\right), e_k^{S*}\left(x_k\right)\right) \geq \sum_{k=t}^{\infty} \delta^k \cdot U_k^B\left(e_k^B\left(x_k\right), e_k^{S*}\left(x_k\right)\right)$$
$$\sum_{k=t}^{\infty} \delta^k \cdot U_k^S\left(e_k^{B*}\left(x_k\right), e_k^{S*}\left(x_k\right)\right) \geq \sum_{k=t}^{\infty} \delta^k \cdot U_k^S\left(e_k^{B*}\left(x_k\right), e_k^S\left(x_k\right)\right)$$

In summary, $\left(e_t^{B*}\left(x_t\right), e_t^{S*}\left(x_t\right)\right), t = 0,1,..$ is said to be a MPE if and only if for every player $i = B, S$ at every state x_t at time period $t = 0,1,\cdots$, the player would find no incentive to deviate from the equilibrium strategies, as far as the other player follows them. In this equilibrium concept, the play to follow after every state x_t prescribes a Nash equilibrium for the game that starts at x_t, which is commonly referred to as a *subgame*. In that sense, since the play off the equilibrium path is *credible*, this solution concept is time consistent. Hence, we can say that a MPE is a subgame perfect Nash equilibrium, where strategies depend only on specified state variables.

On the other hand, in "Nash Equilibrium" of the dynamic game, each player $i = B, S$ commits himself to a future path once at the beginning of the game, and no player has an incentive to deviate by playing another feasible path from the initial state x_0, as long as the other player follows. However, the play prearranged after some state other than initial state x_0 may not constitute a Nash equilibrium for the subgame that starts at such a state. Each player *ignores the evolution of the state variable* in the game and does not optimally respond to each state x_t. Thus, in order to avoid *non-credible equilibria* that may not prescribe equilibrium play after a subsequent state x_t, we use MPE as the equilibrium concept.

2.3 Dynamics and Parameter p

We assume that the dynamics is stationary and indexed by a parameter $p \in F$. Moreover, the indexing is such that it satisfies the following monotonicity property:

$p > q \Leftrightarrow f^p(x) > f^q(x)$ In words, *the higher the parameter, the higher the accumulation.* One interpretation is that the actual shape of the dynamics can vary for example, depending on whether or not the skill accumulation system is efficient.

3. Analysis of the Game

3.1 Equilibria in the Stage Game

The indexing satisfies the *single crossing property*, in the sense that

$$x'' > x' \Leftrightarrow \frac{\partial \phi_B(x'')}{\partial e^B} = \frac{1}{2}R'(x'', e^B) - 1 > \frac{1}{2}R'(x', e^B) - 1 = \frac{\partial \phi_B(x')}{\partial e^B} \qquad \forall e^B$$

$$x'' > x' \Leftrightarrow \frac{\partial \phi_S(x'')}{\partial e^S} = -\frac{1}{2}C'(x'', e^S) - 1 > -\frac{1}{2}C'(x', e^S) - 1 = \frac{\partial \phi_S(x')}{\partial e^S} \qquad \forall e^S$$

In words, the payoff function $U^i(x, e^B, e^S), i = B, S$ satisfies the *Single Crossing Property (SCP)* in x, since the marginal payoff $\partial U^i / \partial e^i, i = B, S$ is monotonically increasing in the parameter x. Then, the best response $BR^i(e^j, x), i, j = B, S, i \neq j$ is monotonically increasing in x for all e^j, and thus the equilibrium is also monotonically increasing in x for all p. Thus, we obtain the monotonicity of the equilibrium outcomes: $x'' > x' \Rightarrow e^i_{o,p}(x'') > e^i_{o,p}(x'), i = B, S$

3.2 Two-Period Formulation

In the two-period version of the model, Player B's problem (for $t=0$) can be defined as:

$$V_0^B\left(x_0,p\right)=\max_{e_0^B} U\left(x_0,e_0^B,e_0^S\left(x_0\right)\right)+\delta V_1^B\left(f^p\left(x_0+e_0^B+e_0^S\left(x_0\right)\right)\right)$$

where

$$V_1^B\left(f^p\left(x_0+e_0^B+e_0^S\left(x_0\right)\right)\right)=\max_{e_1^B} U^B\left(x_1,e_1^B,e_1^S\left(x_1\right)\right):=\frac{1}{2}\left[R\left(x_1,e_1^B\right)-C\left(x_1,e_1^S\left(x_1\right)\right)\right]-e_1^B$$

and $x_1=f^p\left(x_0+e_0^B+e_0^S\right)$ denotes the level of the state variable in period $t=1$.

Differentiating V_0^B with respect to e_0^B yields:

$$\partial V_0^B \Big/ \partial e_0^B=\frac{1}{2}R'\left(x_0,e_0^B\right)-1$$

$$+\delta f_p'\left(x_0+e_0^B+e_0^S\left(x_0\right)\right)\left\{\frac{1}{2}\left[R'\left(x_1,e_1^B\left(x_1\right)\right)-C'\left(x_1,e_1^S\left(x_1\right)\right)\right]-\frac{1}{2}C'\left(x_1,e_1^S\left(x_1\right)\right)\frac{de_1^S\left(x_1\right)}{dx_1}\right\}=0 \quad (1)$$

The envelope theorem was used in the derivation. The rationale is as follows. An increase in Player B's current investment e_0^B increases the relational skill (state variable) in the next period x_1, which brings about a positive *direct effect*, corresponding to the first term $\frac{1}{2}\left[R'\left(x_1,e_1^B\left(x_1\right)\right)-C'\left(x_1,e_1^S\left(x_1\right)\right)\right]$. Second, an increase in e_0^B similarly increases x_1 in the next period, which induces in equilibrium *less aggressive (passive)* behavior by Player S, which in turn will increase the profit of Player B. This is a positive *strategic effect*, corresponding to the second term $-\frac{1}{2}C'\left(x_1,e_1^S\left(x_1\right)\right)\frac{de_1^S\left(x_1\right)}{dx_1}$.

Note that this strategic effect does not exist in the "Nash equilibrium" of the two-period (more generally, dynamic) game. Player S's problem in the Two Period Formulation can be analyzed similarly. See Appendix 1.

Thus, we have the following proposition:

Proposition 1:

In the Two-Period Formulation, the first period investments in the Markov Perfect Equilibrium are greater than those in the Nash Equilibrium, due to the positive strategic effects.

3.3 Infinite Horizon Formulation

Let $V = \left(V^i\right)_{i=B,S}$ a $2-\text{tuple}$ of value function $V^i : X \to R$ assigning a value to each state x of the game. First, we look at Player B's **recursive formulation** of his decision problem.

$$V^B\left(x, p\right) = \max_{e^B} U\left(x, e^B, e^S\left(x\right)\right) + \delta V^B\left(f^p\left(x + e^B + e^S\left(x\right)\right)\right)$$

where V^B is the continuation value function for Player B, which should be the same across time, and should be written without a time script. Given the *continuity* of the value function $V^i, i = B, S$, which we refer to as the *continuity* of the game, the first order condition for the maximization is given by:

$$\frac{1}{2} R'\left(x, e^B\right) - 1 + \delta V^{B\prime}\left(x\right) f_p'\left(x + e^B + e^S\left(x\right)\right) = 0$$

which gives us the function $e^B\left(x\right)$.Then, it follows from the "envelope theorem" that

$$\begin{aligned}\partial V^B / \partial x &= dU\left(x, e^B\left(x\right), e^S\left(x\right)\right) / dx + \delta \cdot d V^B\left(f^p\left(x + e^B\left(x\right) + e^S\left(x\right)\right)\right) / dx \\ &= \frac{1}{2}\left[R'\left(x, e^B\left(x\right)\right) - C'\left(x, e^S\left(x\right)\right)\right] - \frac{1}{2} C'\left(x, e^S\left(x\right)\right) \cdot \frac{de^S\left(x\right)}{dx} \\ &\quad + \delta V^{B\prime}\left(f^p\left(x + e^B\left(x\right) + e^S\left(x\right)\right)\right) \times f_p'\left(x + e^B\left(x\right) + e^S\left(x\right)\right) \times \left(1 + \frac{de^S\left(x\right)}{dx}\right)\end{aligned}$$

The last term on the right hand side of this equation shows that the current value of the state variable x affects the continuation value from the next period through its own increase in x and the other Player S 's investment level e^S.

Now, suppose *hypothetically* that the current value of the state variable did not directly affect the valuation from the next period so that the second term would disappear. Thus, we have

$$\begin{aligned}\partial V^B / \partial x &= dU\left(x, e^B\left(x\right), e^S\left(x\right)\right) / dx \\ &= \frac{1}{2}\left[R'\left(x, e^B\left(x\right)\right) - C'\left(x, e^S\left(x\right)\right)\right] - \frac{1}{2} C'\left(x, e^S\left(x\right)\right) \frac{de^S\left(x\right)}{dx}\end{aligned}$$

which only captures the effects of the state variable on strategic interactions in the next period, that is, the direct effect and strategic effect in the IO literature ala Tirole (1988).

Then, letting x' denote the level of the state variable in the next period, we have from the above equations

$$\frac{1}{2}R'\left(x,e^{B}\left(x\right)\right)-1+\delta V^{B\prime}\left(x'\right)f_{p}'\left(x+e^{B}\left(x\right)+e^{S}\left(x\right)\right)=0 \Leftrightarrow$$

$$\frac{1}{2}R'\left(x,e^{B}\left(x\right)\right)-1$$

$$+\delta f_{p}'\left(x+e^{B}\left(x\right)+e^{S}\left(x\right)\right)\left\{\frac{1}{2}\left[R'\left(x',e^{B}\left(x'\right)\right)-C'\left(x',e^{S}\left(x'\right)\right)\right]-\frac{1}{2}C'\left(x',e^{S}\left(x'\right)\right)\frac{de^{S}\left(x'\right)}{dx'}\right\}=0$$

which is nothing but equation (1) of the Two-Period model. Player S's problem in Infinite Horizon Formulation can also be analyzed similarly. See Appendix 2

In the Infinite Horizon framework, the dynamic effects, consisting of the positive direct and strategic effects, are monotonically strengthened. Hence, we have a proposition on the comparison between the equilibrium incentives in Two-Period Framework $e_{2,p}^{i*}\left(x\right), i=B,S$ and those in Infinite Horizon Framework $e_{p}^{i*}\left(x\right), i=B,S$.

Proposition2: *As for the equilibrium investments,* $e_{p}^{i*}\left(x\right)>e_{2,p}^{i*}\left(x\right), i=B,S$ *hold.*

Now, we can see that the increase in p will have *positive* effects on the "envelopes" of V^{B} in x. This is exactly the *complementarities in the value functions*. This argument holds also for Player S's decision problem. Thus, we can order the gradients of the equilibrium function $e_{p}^{i*}\left(x\right)$ for $i=B,S$ as p changes. The equilibrium incentives will be monotonically increasing in p for all x. Hence, we have the following conjecture.

Conjecture: *In the stationary Markov Perfect Equilibrium, the equilibrium incentives are monotonically increasing in* $p\in F$*, i.e.,* $p>q \Rightarrow e_{p}^{i*}\left(x\right)>e_{q}^{i*}\left(x\right), i=B,S$.

One interpretation is that we can view p as an efficient skill accumulation system, such as in Toyota, while q as another less efficient one, and that as the accumulation of relational skill is more efficient: that is $p>q$, the equilibrium specific investments and the relational skill will become greater, in the stationary Markov Perfect Equilibrium.

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<u>Appendix 1</u> Player S’s problem in the Two Period Formulation

Player S’s problem can be written similarly, for some arbitrary period, $t=0$, as:

$$V_0^S\left(x_0,p\right)=\max_{e_0^S} U\left(x_0,e_0^S,e_0^B\left(x_0\right)\right)+\delta V_1^S\left(f^p\left(x_0+e_0^S+e_0^B\left(x_0\right)\right)\right)$$

where

$$V_1^S\left(f^p\left(x_0+e_0^S+e_0^B\left(x_0\right)\right)\right)=\max_{e_1^S} U^S\left(x_1,e_1^S,e_1^B\left(x_1\right)\right):=\frac{1}{2}\left[R\left(x_1,e_1^B\left(x_1\right)\right)-C\left(x_1,e_1^S\right)\right]-e_1^S$$

and $x_1=f^p\left(x_0+e_0^B+e_0^S\right)$ denotes the level of the state variable in period $t=1$.

Differentiating V_0^S with respect to e_0^S yields:

$$\partial V_0^S/\partial e_0^S=-\frac{1}{2}C'\left(x_0,e_0^S\right)-1$$

$$+\delta f_p'\left(x_0+e_0^S+e_0^B\left(x_0\right)\right)\left\{\frac{1}{2}\left[R'\left(x_1,e_1^B\left(x_1\right)\right)-C'\left(x_1,e_1^S\left(x_1\right)\right)\right]+\frac{1}{2}R'\left(x_1,e_1^B\left(x_1\right)\right)\frac{de_1^B\left(x_1\right)}{dx_1}\right\}=0 \quad (2)$$

The envelope theorem was made use of in the derivation. The rationale is as follows. First, an increase in Player S’s current investment e_0^S increases the relational skill in the next period x_1, which brings about a <u>positive</u> *direct effect*, corresponding to the first term $\frac{1}{2}\left[R'\left(x_1,e_1^B\left(x_1\right)\right)-C'\left(x_1,e_1^S\left(x_1\right)\right)\right]$. Second, an increase in e_0^S similarly increases x_1 in the next period, which induces in equilibrium *less aggressive (passive)* behavior by Player B, which will increase the profit of Player S. This is a positive *strategic effect*, which corresponds to the second term $\frac{1}{2}R'\left(x_1,e_1^B\left(x_1\right)\right)\frac{de_1^B\left(x_1\right)}{dx_1}$.

Appendix 2 Player S's decision problem in the Infinite Horizon Formulation

Then, we look at Player S's **recursive formulation** of his decision problem.

$$V^S\left(x,p\right)=\max_{e^S}U\left(x,e^S,e^B\left(x\right)\right)+\delta V^S\left(f^p\left(x+e^S+e^B\left(x\right)\right)\right)$$

where V^S is the continuation value function for Player S. Given the *continuity* of the value function $V^i, i=B,S$, which we refer to as the *continuity* of the game, the first order condition for the maximization is given by:

$$-\frac{1}{2}C'\left(x,e^S\right)-1+\delta V^{S\prime}\left(x\right)f_p'\left(x+e^S+e^B\left(x\right)\right)=0$$

which gives us the function $e^S\left(x\right)$.Then, it follows from the "envelope theorem" that

$$\begin{aligned}\partial V^S/\partial x&=dU\left(x,e^S\left(x\right),e^B\left(x\right)\right)/dx+\delta\cdot dV^S\left(f^p\left(x+e^S\left(x\right)+e^B\left(x\right)\right)\right)/dx\\&=\frac{1}{2}\left[R'\left(x,e^B\left(x\right)\right)-C'\left(x,e^S\left(x\right)\right)\right]+\frac{1}{2}R'\left(x,e^B\left(x\right)\right)\cdot\frac{de^B\left(x\right)}{dx}\\&\quad+\delta V^{S\prime}\left(f^p\left(x+e^S\left(x\right)+e^B\left(x\right)\right)\right)\times f_p'\left(x+e^S\left(x\right)+e^B\left(x\right)\right)\times\left(1+\frac{de^B\left(x\right)}{dx}\right)\end{aligned}$$

The last term in the right-hand side of this equation shows that the current value of the state variable x affects the continuation value from the next period through its own increase in x and the other Player B's investment level e^B.

Now, suppose *hypothetically* that the current value of the state variable did not directly affect the valuation from the next period so that the second term would disappear. That is, we have

$$\partial V^S / \partial x = dU\left(x, e^S(x), e^B(x)\right) / dx$$

$$= \frac{1}{2}\left[R'\left(x, e^B(x)\right) - C'\left(x, e^S(x)\right)\right] + \frac{1}{2}R'\left(x, e^B(x)\right)\frac{de^B(x)}{dx}$$

which only captures the effects of the state variable on strategic interactions in the next period, in other words, the direct effect and strategic effect in the standard IO literature ala Tirole (1988).

Then, letting x' denote the level of the state variable in the next period, we have from the above equations

$$-\frac{1}{2}C'\left(x, e^S(x)\right) - 1 + \delta V^{S\prime}(x') f_p'\left(x + e^S(x) + e^B(x)\right) = 0 \Leftrightarrow$$

$$-\frac{1}{2}C'\left(x, e^S(x)\right) - 1$$

$$+\delta f_p'\left(x + e^S(x) + e^B(x)\right)\left\{\frac{1}{2}\left[R'\left(x', e^B(x')\right) - C'\left(x', e^S(x')\right)\right] + \frac{1}{2}R'\left(x', e^B(x')\right)\frac{de^B(x')}{dx'}\right\} = 0$$

which is nothing but equation (2) of the model.