E C O N O M I C S B U L L E T I N

A Stackelberg Game Model of Dynamic Duopolistic Competition with Sticky Prices

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Abstract

We develop the following Stackelberg game model of dynamic duopoly with sticky prices; the leader chooses its time profile of outputs to maximize the discounted sum of proftis, while the follower chooses the optimal output to maximize the instantaneous profit as a myopic profit maximizer at each point of time. Then, we compare the resulting outcomes with those in a Stackelberg model without price stickiness.

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1 Introduction

The last twenty years have witnessed a rapid growth in differential game theory and its application to economics. Differential games have been applied to broad fields of economics such as industrial organization, environmental economics, and trade theory. Given the fact that Cournot-Nash duopoly is the simplest and most useful tool that captures a two-player game, its extensions to differential games have been intensively carried out.

Fershtman and Kamien (1987) first formulate a differential Cournot-Nash game in which prices are so sticky that they adjust to the inverse demand only sluggishly. Regarding the current price as a state variable, Fershtman and Kamien (1987) examine the open-loop and closed-loop Nash equilibria. Instead of adopting a guessing method, Tsutsui and Mino (1990) find that there are uncountable nonlinear strategies that characterize the Markov perfect Nash equilibrium by making use of an auxiliary equation derived from the Hamilton-Jacobi-Bellman equation.[1]

On the other hand, there is little work involved in deriving and characterizing the Stackelberg equilibrium in a dynamic duopoly model. The purpose of this paper is to take a small step for it. When it comes to a Stackelberg equilibrium in differential games, it is natural to consider an open-loop and a feedback Stackelberg equilibria. However, such a task is extremely difficult since at least four differential equations arise even in a two-firm duopoly. In addition, the open-loop Stackelberg equilibrium in general suffers from the problem of time inconsistency.

Invoking these practical difficulties, we propose the following game. The follower chooses the output at each point of time to maximize its instantaneous profit. Then, its reaction function becomes a function of the leader's output. Taking this reaction function by the follower, the leader seeks to maximize its discounted sum of profits by choosing the time profile of outputs. The resulting equilibrium is time consistent since there is no costate variable that is associated with the follower's problem. Together with the tractability of our equilibrium concept, this facilitates the analysis and might have a

[1] Dockner *et al.* (2000) provide a concise overview of the dynamic duopoly model with sticky prices. They give a thorough explanation of Tsutsui and Mino's (1990) method of deriving nonlinear feedback strategies.

broad applicability.

After the saddle point stability in our Stackelberg game model of duopoly, we compare the dynamic and static Stackelberg equilibria to explore the long-run consequences of the leader-follower game. It is shown that the leader produces more in a dynamic Stackelberg equilibrium than in a static one and the opposite holds for the follower. Moreover, the equilibrium price in the dynamic Stackelberg equilibrium is lower than that in the static one. In this sense, dynamic behavior by the leader has an tendency toward a lower price and the consumer's benefit.

The rest of the paper is organized as follows. Section 2 lays out the base model and Section 3 derives the dynamic Stackelberg equilibrium. Then, Section 4 compares the dynamic and static Stackelberg equilibria. Section 5 offers a conclusion.

2 A model

The model to be employed is a dynamic duopoly model first developed by Fershtman and Kamien (1987). There are two firms (firms 1 and 2) both of which share the identical cost function specified by

$$cx_i + \frac{x_i^2}{2}, \quad c > 0, \tag{1}$$

where $x_i, i = 1, 2$ denotes the output of firm i. The market price derived from the consumer's utility maximization is linear and given by $a - x_1 - x_2$. Following Fershtman and Kamien (1987) and Tsutsui and Mino (1990), the price adjusts sluggishly and such an adjustment is captured by a differential equation:

$$\dot{p} = s(a - p - x_1 - x_2), \quad s > 0, \tag{2}$$

where p is the price at each point of time. Then, each firm's instantaneous profit is defined by

$$\pi_i \equiv px_i - cx_i - \frac{x_i^2}{2}. \tag{3}$$

Fershtman and Kamien (1987) and Tsutsui and Mino (1990) consider a differential game of the above-specified duopoly model. That is, letting $r > 0$ the common discount rate, both firms maximize

$$\int_0^\infty e^{-rt}\pi_i dt,$$

subject to the price dynamics (2). On the other hand, we consider the following 'specific' Stackelberg game. Assuming that firm 1 leads and that firm 2 follows, firm 2 solves a static profit maximization problem defined by

$$\begin{aligned} \max_{x_2} \quad & \pi_2 \\ \text{subject to} \quad & \{x_1(t)\}_{t=0}^{\infty} : \text{ given} \\ & p = a - x_1 - x_2. \end{aligned}$$

The resulting solution by firm 2 defines its reaction function as a function of x_1: $x_2 = r(x_1)$. Taking this into account, firm 1 solves the following dynamic optimization problem:

$$\begin{aligned} \max_{x_1} \quad & \int_0^{\infty} e^{-rt}\pi_1 dt, \quad r > 0 \\ \text{subject to} \quad & \dot{p} = s[a - p - x_1 - r(x_1)]. \end{aligned}$$

This is a standard optimal control problem with Bellman's principle of optimality satisfied. Therefore, the equilibrium in our game never suffers from the problem of time consistency.[2]

3 Dynamic and static Stackelberg equilibria

Having described a basic setting, this section solves the model and derives the equilibrium output and price. First of all, let us focus on firm 2's problem. Maximizing π_2 with respect to x_2 yields the following reaction function by firm 2:

$$x_2 = r(x_1) = \frac{a - c - x_1}{3}. \tag{4}$$

Firm 1, the leader, solves the above-specified intertemporal profit maximization by taking account of the follower's reaction function in (4). Substituting (4) into (2), firm 1's problem reduces to

$$\begin{aligned} \max_{x_1} \quad & \int_0^{\infty} e^{-rt}\left(px_1 - cx_1 - \frac{x_1^2}{2}\right) dt \\ \text{subject to} \quad & \dot{p} = \frac{s(2a + c - 3p - 2x_1)}{3}. \end{aligned}$$

[2]See Dockner *et al.* (2000) on the time consistency in an open-loop Stackelberg equilibrium.

To solve this problem, let us set up the current-value Hamiltonian:

$$H = px_1 - cx_1 - \frac{x_1^2}{2} + \lambda \frac{s(2a + c - 3p - 2x_1)}{3},$$

where λ is the costate variable associated with the constraint of price dynamics. Then, the optimality conditions are obtained as

$$\begin{aligned} 0 &= p - c - x_1 - \frac{2s\lambda}{3} && (5) \\ \dot{\lambda} &= \lambda(r + s) - x_1 && (6) \\ \dot{p} &= \frac{s(2a + c - 3p - 2x_1)}{3} && (7) \\ 0 &= \lim_{t\to\infty} e^{-rt}\lambda p. \end{aligned}$$

While there are three unknowns in the above system, one of them can be dropped. To this end, solve (5) for x_1 to get

$$x_1 = p - c - \frac{2s\lambda}{3}, \qquad (8)$$

and substitute (8) into (6) and (7), the present dynamic system becomes two-dimensional as follows.

$$\begin{aligned} \dot{\lambda} &= \lambda\left(r + \frac{5s}{3}\right) - (p - c) && (9) \\ \dot{p} &= \frac{s}{3}\left(\frac{4s\lambda}{3} - 5p + 2a + 3c\right). && (10) \end{aligned}$$

Before proceeding further, let us briefly address the stability of the steady state. This is summarized in:

Proposition 1. *The steady state associated with the dynamic system (9) and (10) is saddle point stable.*

Proof. The Jacobian determinant which is obtained by linearing (9) and (10) becomes

$$\begin{vmatrix} r + \frac{5s}{3} & -1 \\ \frac{4s^2}{9} & -\frac{5s}{3} \end{vmatrix} = -\frac{s(5r + 7s)}{3} < 0,$$

which immediately implies the saddle point stability.

Alternatively, one can obtain the phase diagram which is given by Figure 1, from which the saddle point stability is also verified geometrically. **Q. E. D.**

Having the saddle point stability of the steady state, we are now in a position to derive the steady state output and price. In the steady state in which $\dot{\lambda} = \dot{p} = 0$ and from (9) and (10), we have the following system:

$$\begin{bmatrix} r + \frac{5s}{3} & -1 \\ \frac{4s^2}{9} & -\frac{5s}{3} \end{bmatrix} \begin{bmatrix} \lambda \\ p \end{bmatrix} = \begin{bmatrix} -c \\ -\frac{s(2a+3c)}{3} \end{bmatrix}.$$

Solving for λ and p, their steady state values are explicitly obtained as

$$\lambda^L = \frac{2(a-c)}{5r+7s} \tag{11}$$

$$p^L = \frac{3r(2a+3c)+s(10a+11c)}{3(5r+7s)}, \tag{12}$$

where the superscript L denotes the Stackelberg equilibrium with price stickiness. In addition, further substitution of (11) into (8) and (4), each firm's equilibrium output becomes

$$x_1^L = \frac{2(r+s)(a-c)}{5r+7s}, \quad x_2^L = \frac{(3r+5s)(a-c)}{3(5r+7s)}. \tag{13}$$

This completes the derivation of the dynamic Stackelberg equilibrium in our game.

On the other hand, the static Stackelberg equilibrium is easily obtained by making use of (12) and (13). Setting $r = 0$ and $s = 1$ in them yields

$$p^S = \frac{10a+11c}{21} \tag{14}$$

$$x_1^S = \frac{2(a-c)}{7}, \quad x_2^S = \frac{5(a-c)}{21}. \tag{15}$$

One can verify that these price and outputs are also obtained by solving a standard Stackelberg game with firm 1 the leader and firm 2 the follower. That is, they are obtained by solving

$$\begin{aligned} \max_{x_1} \quad & px_1 - cx_1 - \frac{x_1^2}{2} \\ \text{subject to} \quad & x_2 = r(x_1) = \frac{a-c-x_1}{3} \\ & p = a - x_1 - x_2. \end{aligned}$$

Note that these results can also be obtained by setting $s \to \infty$ because the prices adjust instantaneously to the inverse demand in a static Stackelberg model.[3]

4 Comparison between static and dynamic equilibria

The preceding sections have concentrated on the derivation and characterization of the dynamic and static Stackelberg equilibria.[4] Based on them, we are now ready to undertake the main task: comparison between the dynamic and static Stackelberg equilibria. They are stated in:

Proposition 2. *On the comparison between the dynamic and static Stackelberg equilibria, we have* $p^L < p^S$, $x_1^L > x_1^S$ *and* $x_2^L < x_2^S$.

Proof. Subtracting p^S (resp. x_1^S and x_2^S) from p^L (resp. x_1^L and x_2^L) yields

$$\begin{aligned} p^L - p^S &= -\frac{8r(a-c)}{21(5r+7s)} < 0 \\ x_1^L - x_1^S &= \frac{4r(a-c)}{7(5r+7s)} > 0 \\ x_2^L - x_2^S &= -\frac{4r(a-c)}{21(5r+7s)} < 0, \end{aligned}$$

which immediately leads to the proposition. **Q. E. D.**

In our game, firm 1 maximizes the discounted stream of profits. This dynamic behavior makes its output larger than that obtained in a static Stackelberg game. This, in turn, makes the follower supply less and the price lower. That is, the leader's position in a whole game is stronger in a dynamic game than in a static game. This results in a lower price, from which the consumer benefits more in a dynamic equilibrium than in a static one. In other words, the existence of a long-farsighted firm is likely to benefit the consumer.

[3]For this point, see Dockner *et al.* (2000).

[4]The term 'dynamic' refers to the situation in which firm 1 solves an intertemporal profit maximization problem, whereas 'static' indicates dynamic optimization by neither firm.

Remark. It is well-known that the leader supplies more than the follower in Stackelberg output competition. In fact, in a static version of the present model, we see

$$x_1^S - x_2^S = \frac{a-c}{21} > 0.$$

The same applies to our dynamic game. This is easily seen because

$$x_1^L - x_2^L = \frac{(3r+s)(a-c)}{3(5r+7s)} > 0.$$

In this sense, the well-known feature on the inter-firm output differential in the static Stackelberg game carries over to the present dynamic framework. In other words, in both cases, the leader has an incentive to a larger output than the follower.

5 Concluding remarks

We have formulated a tractable Stackelberg game model of dynamic duopoly with sticky prices. Although we recognize that the feedback Stackelberg equilibrium is more a desirable and appropriate concept to fully explore the dynamic aspects of oligopolistic competition, our approach might have a role of connecting two extreme equilibria: static Stackelberg equilibrium and feedback Stackelberg equilibrium. As has been stressed repeatedly, our concept of the dynamic Stackelberg equilibrium passes time consistency as well as tractability. In addition, it is saddle point stable, which makes comparative statics easy. We hope that our equilibrium concept would be applied to various fields including public economics, environmental economics, and international trade.

References

[1] Dockner, E., S. Jorgensen, N. V. Long and G. Sorger (2000), *Differential games in economics and management science*, New York: Cambridge University Press.

[2] Fershtman, C. and M. Kamien (1987), 'Dynamic duopolistic competition with sticky prices', *Econometrica*, 55, 1151-1164.

[3] Tsutsui, S. and K. Mino (1990), 'Nonlinear strategies in dynamic duopolistic competition with sticky prices', *Journal of Economic Theory*, 52, 136-161.

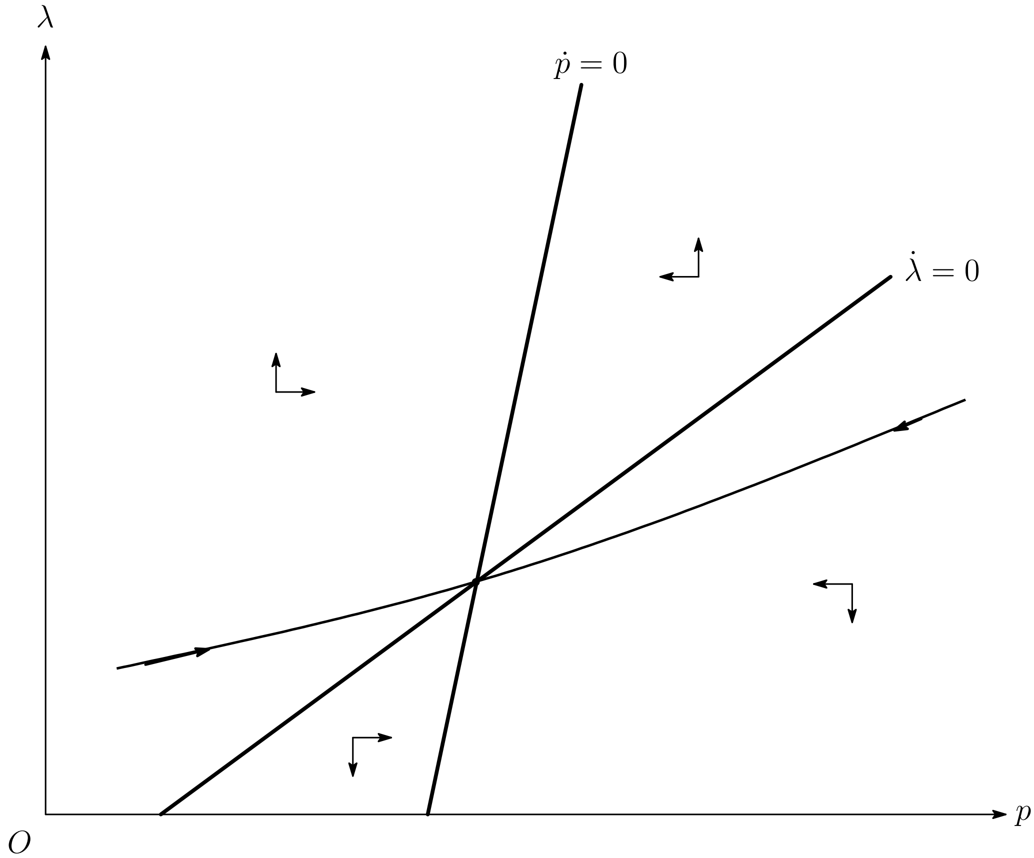

Figure 1: The phase diagram