Price Competition in a Mixed Duopoly

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Abstract

We analyze sequential and simultaneous price setting under a mixed duopoly with homogeneous products and symmetric quadratic cost functions. When public firm is the follower, there exists the case that the equilibrium price is highest of all timings.

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1 Introduction

This paper analyzes the price competition in a homogeneous product market under a mixed duopoly. We consider the case that cost functions are symmetric between two firms and they are strictly convex.[1] In our model, one private firm and one public firm exist. The former maximizes its own profits. The latter maximizes a weighted average of social welfare and its own profits.[2] Since we do not understand well which firm is a first-mover, we compare three timings of price setting: (timing S) Both firms set those prices simulnateously. (timing V) First the private firm sets its price, and second the public firm does one. We call this situation "private price leadership". (timing B) First, the public firm sets its price, and second the private firm does one. We name this situation as "public price leadership".

From the seminal work of [4], mixed oligopoly becomes one of the major topic in the theory of industrial organization. Many studies consider quantity competition and deals with asymmetric linear cost function or quadratic cost function. However, only few analyzes about price competition. Thus this paper answers a part of left question.

[2] studies a price competition with homogeneous product markets under private oligopoly. They show that the equilibrium prices are multiple in a pure strategy if cost functions are symmetric between both firms.

We show that the equilibrium price under S have a range and it equals to [2] even though the public firm exists in the market. We also find that the equilibrium price under V is higher than the one under B and exceeds the range of the one under S under some condition.[3]

This paper has 4 sections. Section 2 builds the model. Section 3 solves the equilibrium. Section 4 concludes the paper.

2 The model

Suppose there are a homogeneous product market which consists of one private firm and one public firm. The demand function is given by $D(p) = a - p$, where a is positive and sufficiently large. The cost function is given by cq_i^2, where c is positive and q_i is the output of firm i, $i = 0, 1$.

We introduce the following assumptions:[4]

Assumption

1. *Firms have to supply all the demand it faces.*

[1] We consider a quadratic cost function.

[2] For a rationarization of the objective, see [1]. Using such a objective function, we can deal with several types of public firm.

[3] [2] focuses on a simultaneous price competition, and thus some of our main results are not compared with it.

[4] These assumptions are also used in [3].

2. *When both firms choose the same price, they share the demand equally, that is, when they choose the same price, p, each firm supplies $\frac{1}{2}D(p)$ respectively.*

The profits of firm i is given by

$$\pi_i = \begin{cases} p_i(a-p_i) - c(a-p_i)^2 & \text{if } p_i < p_j, \\ p_i\left\{\frac{1}{2}(a-p_i)\right\} - c\left\{\frac{1}{2}(a-p_i)\right\}^2 & \text{if } p_i = p_j, \\ 0 & \text{if } p_i > p_j. \end{cases} \tag{1}$$

Social welfare is given by

$$\begin{aligned} SW &= \text{consumer's surplus} + \text{producer's surplus} \\ &= \frac{1}{2}(q_0+q_1)^2 + \pi_0 + \pi_1 \end{aligned} \tag{2}$$

The objective function of the public firm U_0 and that of the private firm U_1 are given by

$$U_0 = \alpha SW + (1-\alpha)\pi_0, \tag{3}$$

$$U_1 = \pi_1. \tag{4}$$

3 Equilibrium

We consider the three types of the price competition; simultaneous (S), private price leadership (V), and public price leadership (B). We restrict our attention to the situation where each firm chooses pure strategies.

3.1 Simultaneous price competition

Proposition 1. *In the equilibrium, both firms choose the same price p^S within $\left[a\frac{c}{c+2}, a\frac{3c}{3c+2}\right]$.*

Proof. First we consider an undercut incentive. Since the public firm counts private firm's profits, the public firm's incentive is weaker than the private firm's one. Thus we focus on the private firm. Suppose the public firm sets the price $p_0 \leq a\frac{3c}{3c+2}$. If the private firm sets the price $p_1 = p_0 - \epsilon$, then the increase of revenue is less than the increase of cost. Second we consider pullup incentive. If $U_i < 0$ by setting the same price as the opponent's, firm i can increase U_i by pulling up the price.[5] Since the public firm counts the consumer's surplus, a price that causes negative U_0 is lower than the one that causes negative U_1. Thus we focus on the private firm. Calculating $U_1 = 0$, we have $p_1 = a\frac{c}{c+2}$. □

Note that this proposition is the same result and the similar intuition of [2] because the public firm have a weak incentive to set a different price from the one of the private firm and thus the incentive is not binding. In other words, the public firm is not beneficial or harmful for social welfare in simultaneous case.

[5]Then firm i supplies nothing.

3.2 Private price leadership

Proposition 2. *1. In the equilibrium, both firms choose*

$$p^V = \begin{cases} a\frac{c+1}{c+2} & \textit{(Case 1, 2)}, \\ a\frac{c(1+\alpha)+1-\alpha+\sqrt{(-\alpha^3+2c\alpha^2+3\alpha+2c\alpha-2)/(2c+2-\alpha)}}{c(1+\alpha)+2} & \textit{(Case 3)}, \\ a\frac{(3-\alpha)c}{2(1-\alpha)+(3-\alpha)c} & \textit{(Case 4, 5)}, \end{cases}$$

where

$$\begin{aligned} &\textit{Case 1:} \quad \alpha \leq \tfrac{\sqrt{17}-3}{2}, \ c \geq \tfrac{2(1-\alpha)}{\alpha+1}, \\ &\textit{Case 2:} \quad \alpha > \tfrac{\sqrt{17}-3}{2}, \ c \geq \tfrac{\alpha^2-9\alpha+2+\sqrt{\alpha^4+14\alpha^3+37\alpha^2-4\alpha+4}}{4\alpha}, \\ &\textit{Case 3:} \quad \alpha > \tfrac{\sqrt{17}-3}{2}, \ \tfrac{2(\alpha-1)^2}{3\alpha-1} \leq c < \tfrac{\alpha^2-9\alpha+2+\sqrt{\alpha^4+14\alpha^3+37\alpha^2-4\alpha+4}}{4\alpha}, \\ &\textit{Case 4:} \quad \alpha \leq \tfrac{\sqrt{17}-3}{2}, \ c < \tfrac{2(1-\alpha)}{\alpha+1}, \\ &\textit{Case 5:} \quad \alpha > \tfrac{\sqrt{17}-3}{2}, \ c < \tfrac{2(\alpha-1)^2}{3\alpha-1}. \end{aligned}$$

2. We note that p^V in case 1 and 2 exceeds p^S if $c < 2$.

Proof. As we mentioned at the proof of proposition 1, the public firm has only weak incentive to undercut. Hence, the private firm can choose p^V from wider range than the one of p^S. In case 1 and 2, profit-maximizing price subjected to the range is inner solution, $a\frac{3c}{3c+2}$. If $c < 2$, it exceeds $a\frac{3c}{3c+2}$. □

Note that the public firm may be harmful for social welfare in private price leadership case.

3.3 Public price leadership

Proposition 3. *1. In the equilibrium, both firms choose*

$$p^B = \begin{cases} a\frac{\{1-\alpha+(1+\alpha)c\}}{2+(1+\alpha)c} & \textit{if} \quad c > \frac{2(1-\alpha)}{1+\alpha}, \\ a\frac{3c}{3c+2} & \textit{if} \quad c \leq \frac{2(1-\alpha)}{1+\alpha}. \end{cases}$$

2. When $\alpha = 1$, $p^B = a\frac{c}{c+1}$, which coincides with the price under first best.

3. $\frac{\partial p^b}{\partial \alpha} < 0$, $\frac{\partial SW^b}{\partial \alpha} > 0$.

Proof. As we mentioned at the proof of proposition 1, the private firm chooses the same price as the public one if it is in $\left[a\frac{c}{c+2}, a\frac{3c}{3c+2}\right]$. The public firm knows it and maximizes U_0 subjected to the range because in the case that the public firm sets price outside the range, the private firm chooses a different price. In such a situation, the production cost increases dramatically and thus U_1 is damaged. If $c > \frac{2(1-\alpha)}{1+\alpha}$, the maximization problem has an inner solution. If not, it has a corner solution.

The part 2 is derived by a simple comparison. About part 3, the larger a weighted average of social welfare is, the more the public firm concerned with social welfare. As the production inefficiency does not occur as long as both firms supply, the public firm can decrease the price to increase the consumer surplus.

□

Note that the public firm may be beneficial for social welfare in public price leadership case.

Now, we compare p^B with p^V and find that $p^B < p^V$. The intuition behind this result is that the public firm has an incentive to decrease the price since the public firm has an incentive to enhance social welfare.

4 Concluding remarks

We analyze three types of price competition with homogeneous products and symmetric quadratic cost functions under mixed duopoly. We find that the equlibrium price in private price leadership case is higher than the one in simultaneous case under some condition of cost parameter, and always exceeds the one in public price leadership case. We also find that the public firm chooses the same price as the private firm chooses regardless of the timing of the price setting.

We have the following intuition from the results: The public enterprises are often justified by the reason that they are conscious of social welfare and enhance it. However, even if they acts for the improvement of social welfare, the existence of them may lead worse outcome because private firms would take advantage of such a behavior. Therefore, a price monitoring in mixed markets is quite important and privatization would be promoted when a highly marked-up price is sustained.

References

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