Luxury-based Growth

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Abstract

Assuming that there exists a preference for luxury goods and a knowledge spillover from luxury goods production to goods production, this paper constructs an endogenous economic growth model. The model predicts two steady states: one is a steady positive growth state with regard to luxury goods production, and the other is a zero growth state in the absence of luxury goods production. Thus, this study examines the polarization of economies based on luxury goods consumption.

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1 Introduction

Although consumption for subsistence has been a principal concern in the history of mankind, goods that have non-subsistence properties have also been in demand since the dawn of human culture and civilization, and the demand for such goods has contributed to various technological developments. The consumption of luxury goods, for example, gorgeous accessories and luxuriant handicrafts, is distinct from the consumption of subsistence goods. In addition, the production of luxury goods results in various types of technological spillovers; therefore, technological levels are being promoted by the supply of luxury goods. For example, nowadays, technologies promoted by the post-World War II space race-which involved colossal expenditures that were inessential for human survival-are applied to the production of non-military consumer goods and are a source of technological progress. In broader terms, alchemy in the medieval era developed the knowledge of chemistry. Thus, production activities for non-subsistence goods, which appear to be one of the properties of a modern, mass consumption society, have spillover effects on the technology level of the economy. In this paper, we refer to consumption activities that are different from those that relate to consumption for survival as "luxury" and analyze its effects on economic growth. In this short paper, in order to describe such an economic phenomenon, the following two assumptions are made and the effects of these two properties on economic growth are analyzed.

Assumption I *Mankind obtains utility from both consumption and luxury goods.*

Assumption II *Production of luxury goods improves the social capacity of production.*

The relationship between desire and the development of production described above was classically shown by Sombart (1914);[1] however, since then, this idea has not been analyzed sufficiently. However, modern economic growth depends significantly on consumption goods with properties of luxury goods as described above; this indicates the importance of analyzing the role of luxury goods in economic growth and is the main focus of the present short paper. A work with similar concerns as the present study is Matsuyama (2002), which models (1) the rise of a mass consumption society by assuming that utility can be derived from both essential and non-essential goods and (2) technological progress promoted by the learning-by-doing practice

[1] Sombart (1914) furthermore relates love with desire in addition to the relationship between desire and the development of manufacturing capacity. The other important study with regard to the difference between normal and luxury consumptions was Veblen (1899).

followed in non-essential goods production. Matsuyama (2002) focuses on the "flying geese" phenomenon, which refers to the continuous rise of mass consumption societies. On the other hand, the present study focuses on the disadvantages of the rise of mass consumption societies because many economies fail to grow constantly and the polarization of the world economy (see, for example, Quah 1996, 1997). The main results obtained are summarized in the following two statements. First, the model provides two types of steady growth states. One is the steady state that is accompanied by positive luxury goods production and perpetual growth. The other is distinguished by an absence of luxury goods production, and a no growth state. We refer to the former as "luxury-based growth" and the latter as "no-growth traps". Second, luxury-based growth is realized by a strong preference for luxury and high efficiency of luxury goods production; on the other hand, a no-growth trap is realized by a weak preference for luxury and low efficiency of luxury goods production. For an economy with a medium preference for luxury and average efficiency of luxury goods production, a low population growth rate and a high subjective discount rate are necessary to realize luxury-based growth. In this study, the endowment of capital, labor, and knowledge are no effect on the determination of the long-run growth phase.

This paper is organized as follows. In Section 2, the model is set up and the dynamic equations that describe the behavior of an economy are derived. The properties of dynamics and steady states are analyzed in Section 3.

2 The Model

It is assumed that two types of goods exist: final goods and luxury goods. The following three factors are used: labor, capital, and knowledge. Knowledge affects the efficiency of labor inputs. Final goods are consumed as consumption goods or are invested as physical capital. These goods are produced using labor and capital. Luxury goods are produced by using only labor; thus, labor can be used in the production of final and luxury goods. The labor employed in the production of final goods and that employed in the production of luxury goods are denoted as $L_Y(t) := u(t)L(t)$ and $L_X(t) := (1-u(t))L(t)$, respectively, where $L(t)$ denotes the aggregate labor supply and $u \in [0,1]$ denotes the share of labor employed in the final goods sector. The population of the economy grows at an exogenous constant rate n and each agent inelastically supplies one unit of labor. Therefore, the labor supply can be identified with the population, and the dynamic equation of $L(t)$ can be given as

$$\dot{L}(t) = nL(t). \tag{1}$$

2.1 The Final Goods Production

Final goods production is assumed to be competitive, and the firms produce goods by using capital and labor. The profit of this sector π^F is given as

$$\pi(t)^F = \underbrace{K(t)^\alpha (A(t)L_Y(t))^{1-\alpha}}_{Y(t)} - r(t)K(t) - w(t)L_Y(t), \qquad 0 < \alpha < 1, \quad (2)$$

where K, A, r, and w denote capital, knowledge level, capital rental price, and wage, respectively. The first order conditions of the final goods sector are given as

$$r(t) = \alpha \frac{Y(t)}{K(t)}, \quad \text{and} \quad w(t) = (1-\alpha)\frac{Y(t)}{u(t)L(t)}. \qquad (3)$$

2.2 Luxury Goods Production

Luxury goods are produced by using only labor and are only used for consumption, i.e., they are not durable or cannot be used in any production activity. One unit of luxury goods is assumed to be produced by ηA units of labor. Therefore, the aggregate supply of luxury goods, X, is given as

$$X(t) = \eta A(t) L_X(t). \qquad (4)$$

Luxury goods are also assumed to be competitive and are produced by using only labor. The profit accruing from luxury goods production is given as

$$\Pi(t)^X = p(t)X(t) - w(t)L_X(t). \qquad (5)$$

(4), (5), and the zero-profit condition yield the price of the luxury goods, which is denoted as follows:

$$p(t) = \frac{w(t)}{\eta A(t)}. \qquad (6)$$

2.3 Technological Spillover

Based on Assumption II, the production of luxury goods is assumed to improve the knowledge level of the economy. We assume that the increment rate of the knowledge level linearly depends on the share of labor employed in the production of luxury goods $L_X(t)/L(t) = 1 - u(t)$ as follows:

$$\frac{\dot{A}(t)}{A(t)} = \delta(1 - u(t)), \quad 0 < \delta < 1, \qquad (7)$$

where δ is an efficiency parameter. If the entire labor supply is employed in luxury goods production, the knowledge accumulation rate equals δ. Therefore, it defines the upper bound of the growt rate of TFP (total factor productivity).

2.4 Households with Utility for Luxury Goods

The central assumption of the present study is that households derive utility from both consumption goods c and luxury goods x. We further consider two additional assumptions.

Assumption Ia *Luxury goods have non-essential properties; in other words, the marginal utility of luxury goods does not diverge to ∞ if the consumption of luxury goods converges to 0, i.e.,*

$$\lim_{x(t)\to 0} u(c(t), x(t)) < \infty, \qquad \text{for} \quad \forall\, c(t) > 0,$$

where $u(c, x)$ is the utility function of the representative household.

Assumption Ib *Luxury goods have conspicuous*[2] *effects; in other words, an increase in the supply of luxury goods decreases their consumption utility, i.e., the utility function depends on $x(t)/A(t)$ rather than $x(t)$.*

We further specify the utility function of a representative agent in this economy as follows:

$$U = \int_0^\infty u(c(t), x(t)) \exp(-\rho t) dt = \int_0^\infty \left\{ \log c(t) + \beta \frac{x(t)}{A(t)} \right\} e^{-\rho t} dt \qquad (8)$$

where ρ, c, and $u(c, x)$ are a subjective discount rate, the per capita consumption, and the utility function of the representative utility function, respectively. $\beta \geq 0$ is the parameter that captures the intensity of the preference for luxury goods. When $\beta = 0$, there exists no preference for luxury goods, and when β is positive, the agent concerned has a preference for luxury goods; a greater β implies a stronger preference for luxury goods consumption.

$$\dot{k}(t) = r(t)k(t) + w(t) - c(t) - p(t)x(t) - nk(t), \qquad (9)$$

where k, r, w, and n denote the capital holding, interest rate, wage rate, and population growth rate, respectively.

[2] Consupicuous consumption is analyzed by, for example, Corneo and Jeanne (1997), and Yamada (2005).

From the optimal conditions resulting from the above maximization problem for $u^* \in (0,1)$, we have the Euler equation $\sigma\dot{c}(t)/c(t) = r(t) - n - \rho$ and optimal consumption $c(t) = w(t)/(\beta\eta)$. Both these equations are summarized in the following equation:

$$\frac{\dot{w}(t)}{w(t)} = r(t) - n - \rho. \tag{10}$$

From the optimal conditions resulting from the above maximization problem for $u^* = 1$, we have the Euler equation $\sigma\dot{c}(t)/c(t) = r(t) - n - \rho$ and optimal consumption $c(t) < w(t)/(\beta\eta)$. The detailed derivation of this equation is given in Appendix A.

3 Properties of Dynamics and Steady States

3.1 Dynamics and Steady States

Dynamic properties are analyzed in this section. From (3) and (9), the equilibrium of the final goods market yields the following resource constraint of the final goods:

$$\dot{K}(t) = Y(t) - L(t)c(t). \tag{11}$$

3.1.1 Luxury-based Growth

Dynamic equations (1), (7), (10), and (11) describe the complete dynamic behavior of this economy in $u \in (0,1)$. The dynamics of this economy are reduced to the following two equations:

$$\dot{\kappa}(t) = \kappa(t)^{\alpha} u(t)^{1-\alpha}\left(1 - \frac{1-\alpha}{\beta\eta u(t)}\right) - (n + \delta(1-u(t)))\kappa(t), \tag{12}$$

$$\dot{u}(t) = \frac{1-\alpha}{\alpha}\left\{n - \frac{\alpha\kappa(t)^{\alpha-1}}{\beta\eta}u(t)^{-\alpha} + \frac{\rho}{1-\alpha} + \delta(1-u(t))\right\}u(t), \tag{13}$$

and one condition $\frac{1-\alpha}{\beta\eta} < 1$ (see Appendix B for the derivation). From (12), the dynamics of κ is dominated as

$$\dot{\kappa}\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} 0 \Longleftrightarrow \kappa \left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} \mathcal{K}(u),$$

where

$$\mathcal{K}(u) \equiv \begin{cases} \left[\frac{1-\frac{1-\alpha}{\beta\eta u}}{n+\delta(1-u)}\right]^{\frac{1}{1-\alpha}} u(t), & \text{for} \quad 1 \geq u > \frac{1-\alpha}{\beta\eta} \\ 0, & \text{for} \quad 0 \leq u \leq \frac{1-\alpha}{\beta\eta} (< 1) \end{cases}.$$

$\mathcal{K}(u)$ has the following properties:

$$\mathcal{K}\left(\frac{1-\alpha}{\beta\eta}\right)=0,\quad \mathcal{K}(1)=\left[\frac{\left(1-\frac{1-\alpha}{\beta\eta}\right)}{n}\right]^{\frac{1}{1-\alpha}}\quad \text{and}\quad \mathcal{K}'(u)>0\quad \text{for}\quad u\in\left[\frac{1-\alpha}{\beta\eta},1\right].$$

From (13), the dynamics of u is dominated by

$$\dot{u}\left\{\begin{array}{c}>\\=\\<\end{array}\right\}0\Longleftrightarrow u\left\{\begin{array}{c}>\\=\\<\end{array}\right\}\mathcal{U}(u)\equiv\left[\frac{\alpha}{\beta\eta\left(n+\frac{\rho}{1-\alpha}+\delta(1-u)\right)}\right]^{\frac{1}{1-\alpha}}u^{\frac{-\alpha}{1-\alpha}},$$

where $\mathcal{U}(u)$ has the following properties:

$$\lim_{u\to 0}\mathcal{U}(u)=\infty,\quad \mathcal{U}(1)=\left[\frac{\alpha}{\beta\eta\left(n+\frac{\rho}{1-\alpha}\right)}\right]^{\frac{1}{1-\alpha}}\quad \text{and}\quad \mathcal{U}'(t)>0\quad \text{for}\quad u\in[0,1].$$

Thus, phase diagrams drawn on the (u,κ) plane are given as Fig. 1 (a) and (b), which are respectively related to $\mathcal{K}(1)>\mathcal{U}(1)$ and $\mathcal{K}(1)<\mathcal{U}(1)$.

As shown above, $\mathcal{K}(u)$ is a monotonically increasing function with the property $\mathcal{K}(0)=0$, and $\mathcal{U}(u)$ is a monotonically decreasing function with the property $\lim_{u\to 0}\mathcal{U}(u)=\infty$. Therefore, an equation $\mathcal{K}(u)=\mathcal{U}(u)$ has a unique solution u^* if $\mathcal{K}(1)>\mathcal{U}$ holds, and u^* yields κ^* through $\mathcal{K}(u)$ or $\mathcal{U}(u)$. This combination of solutions (u^*,κ^*) gives a steady state that is related with positive luxury goods production, which implies long-run positive growth. It is denoted as E^L, and the phase diagram Fig. 1 (a) shows that E^L has saddle stability. However, when $\mathcal{K}(1)<\mathcal{U}(1)$ holds, the internal solution u^* does not exist; as a result, the equilibrium E^L does not exist. Whether $\mathcal{K}(1)>\mathcal{U}(1)$ or $\mathcal{K}(1)<\mathcal{U}(1)$ depends on the following condition:

$$\left.\begin{array}{c}\mathcal{K}(1)>\mathcal{U}(1)\\ \mathcal{U}(1)>\mathcal{K}(1)\end{array}\right\}\Longleftrightarrow \beta\eta\left\{\begin{array}{c}>\\<\end{array}\right\}\mathcal{N}(\nu)\equiv\frac{\nu+1}{\nu+\frac{1}{1-\alpha}},$$

where $\nu\equiv n/\rho$. $\beta\eta=\mathcal{N}(\nu)$ is the boundary for both the cases. The graph of $\mathcal{N}$ is shown in Fig. 2, and the upper region of the graph is a region where luxury-based growth is possible. Because β and η respectively represent the intensity of preference for and the production efficiency of luxury goods, an economy with a high $\beta\eta$ can be interpreted as an economy that places high emphasis on luxury goods.

3.1.2 No-growth Trap

When the equilibrium $u=1$ is realized, the system corresponds with the Solow-type neoclassical growth model under the condition $c(t)<w(t)/(\beta\eta)$.

Therefore, the system has a saddle stable equilibrium E^N on the (χ, κ) plane, where $\chi \equiv c/A$ is the quality-adjusted per capita consumption. In this case, the equilibrium values $(\bar{\chi}, \bar{\kappa})$ are denoted as follows:

$$\dot{\chi} = 0 \Rightarrow \bar{\kappa} = \left(\frac{\alpha}{n+\rho}\right)^{\frac{1}{1-\alpha}}, \quad \text{and} \quad \dot{\kappa} = 0 \Rightarrow \bar{\chi} = \mathcal{X}(\bar{\kappa})(\equiv \bar{\kappa}^{\alpha} - n\bar{\kappa}).$$

This equilibrium exists on the line of $u = 1$. From $\mathcal{K}(1)$, $\mathcal{U}(1)$, and $\bar{\kappa}$, we can easily obtain that $\mathcal{K}(1) > \bar{\kappa} > \mathcal{U}(1)$ and $\mathcal{U}(1) > \bar{\kappa} > \mathcal{K}(1)$. Therefore, the location of $\bar{\kappa}$ is drawn between $\mathcal{K}(1)$ and $\mathcal{U}(1)$ in Fig. 1 (a) and (b). However, for an equilibrium to exist, the condition $c(t) < w(t)/(\beta\eta)$ must be satisfied. This condition is rewritten as

$$\chi < \Omega(\kappa) \equiv \frac{1-\alpha}{\beta\eta}\kappa^{\alpha}.$$

The graph of $\Omega(\kappa)$ is shown by an upward sloping dashed line on the (χ, κ) plane in Fig. 1 (a) and (b). It can be verified that $\Omega(\kappa) = \mathcal{X}(\kappa)$ has the solution $\kappa = \mathcal{K}(1)$, and E^N satisfies $\chi < \Omega(\kappa)$ when $\mathcal{K}(1) < \mathcal{U}(1)$ and does not do so when $\mathcal{K}(1) > \mathcal{U}(1)$. Therefore, only the case of $\mathcal{U}(1) > \mathcal{K}(1)$ contains a saddle stable path converging on the no-growth trap equilibrium E^N. Thus, in the region below the $\mathcal{N}(\nu)$ line in Fig. 2, only a no-growth trap equilibrium is a feasible steady state.

3.2 Polarization

The phase diagrams described in Fig. 1 imply that both the equilibria, E^L and E^N, have a stable path. If the economy has a low preference for luxury goods, i.e., a low $\beta\eta$, then it exhibits a steady state with no growth in the long run (Fig. 1 (b)). In contrast, if the economy has a sufficiently high $\beta\eta$, it can exhibit the steady state equilibrium E^N, which is accompanied with a positive growth rate (Fig. 1 (a)). Thus, economies are polarized as luxury-based and no-growth countries, and the determination has the following three properties. First, luxury-based perpetual growth is realized by a strong preference for luxury and high efficiency of luxury goods production; on the other hand, a no-growth trap is realized by a weak preference for luxury and low efficiency of luxury goods production. Second, for an economy with a medium preference for luxury and average efficiency of luxury goods production, a low population growth rate and a high subjective discount rate are necessary to realize perpetual growth. Finally, the endowment of capital, labor, and knowledge are no effect on the determination of the long-run growth phase in this study.

Appendix

A. Optimization of Household

The optimization of the representative agent is to maximize (8) under the constraint of (9). The Hamiltonian is given as

$$\mathcal{H} = u(c(t), x(t)) + \lambda\{r(t)k(t) + w(t) - c(t) - p(t)x(t) - nk(t)\}, \tag{14}$$

and we obtain the following three first order conditions:

$$\frac{\partial \mathcal{H}}{\partial c(t)} = \frac{1}{c(t)} + \lambda(t)(-1) = 0, \tag{15}$$

$$\rho\lambda(t) - \dot{\lambda}(t) = \frac{\partial \mathcal{H}}{\partial k(t)} = \lambda(r(t) - n), \tag{16}$$

$$\frac{\partial \mathcal{H}}{\partial x(t)} = \frac{\beta}{A(t)} + \lambda(t)(-p(t)) = 0, \tag{17}$$

The transversality condition is given as follows:

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)k(t) = 0 \tag{18}$$

From these conditions, we obtain the Euler equation as follows:

$$\rho + \frac{\dot{c}(t)}{c(t)} = r(t) - n. \tag{19}$$

By using (6), (15), and (17), we obtain the optimal value of c through an optimal allocation for the consumption of c and x as follows:

$$c(t) = \frac{w(t)}{\beta\eta}. \tag{20}$$

Uniting (19) and (20), we obtain

$$\frac{\dot{w}(t)}{w(t)} = r(t) - n - \rho. \tag{21}$$

It should be noted that (17) holds if and only if luxury goods are consumed (i.e., $u^* \in (0, 1)$), and $\mathcal{H}_x < 0$ must hold if luxury goods are not consumed (i.e., $u^* = 1$). In such a case, $\mathcal{H}_x < 0$ yields the condition

$$c(t) < \frac{w(t)}{\beta\eta}. \tag{22}$$

Consequently, (19) and (22) are the necessary conditions under $u^* = 1$.

B. The Dynamic Properties

Dynamic equations (1), (7), (11), and (21) describe the complete dynamic behavior of this economy. For the convenient for description of the system, we define the quality-adjusted per capita capital as $\kappa := K/AL$. Using this variable, we can rewrite r and w as follows.

$$r(t) = \alpha\kappa(t)^{\alpha-1}u(t)^{1-\alpha}, \quad \text{and} \quad w(t) = (1-\alpha)\kappa(t)^{\alpha}u(t)^{-\alpha}A(t). \tag{23}$$

Therefore, the dynamics of r and w are given as

$$\frac{\dot{r}(t)}{r(t)} = (\alpha - 1)\left(\frac{\dot{\kappa}(t)}{\kappa(t)} - \frac{\dot{u}(t)}{u(t)}\right), \tag{24}$$

$$\frac{\dot{w}(t)}{w(t)} = \alpha\left(\frac{\dot{\kappa}(t)}{\kappa(t)} - \frac{\dot{u}(t)}{u(t)}\right) + \frac{\dot{A}(t)}{A(t)}. \tag{25}$$

Substituting (23) and (20) into (11), we obtain the dynamics of K as

$$\frac{\dot{K}(t)}{K(t)} = \frac{r(t)}{\alpha}\left(1 - \frac{1-\alpha}{\beta\eta u(t)}\right). \tag{26}$$

The definition of κ and (1) and (7) yield

$$\frac{\dot{\kappa}(t)}{\kappa(t)} = \frac{\dot{K}(t)}{K(t)} - n - \delta(1 - u(t)). \tag{27}$$

Substituting (23) and (27) into (26), we obtain the dynamics of κ as follows

$$\frac{\dot{\kappa}(t)}{\kappa(t)} = \kappa(t)^{\alpha-1}u(t)^{1-\alpha}\left(1 - \frac{1-\alpha}{\beta\eta u(t)}\right) - (n + \delta(1 - u(t))). \tag{28}$$

Substituting (7), (21), and (28) into (25) provides the dynamics of u as follows:

$$\dot{u}(t) = \frac{1-\alpha}{\alpha}\left\{n - \frac{\kappa(t)^{\alpha-1}}{\beta\eta}u(t)^{-\alpha} + \frac{\rho}{1-\alpha} + \delta(1 - u(t))\right\}u(t). \tag{29}$$

Therefore, the dynamics of this economy are reduced to two equations – (28) and (29).

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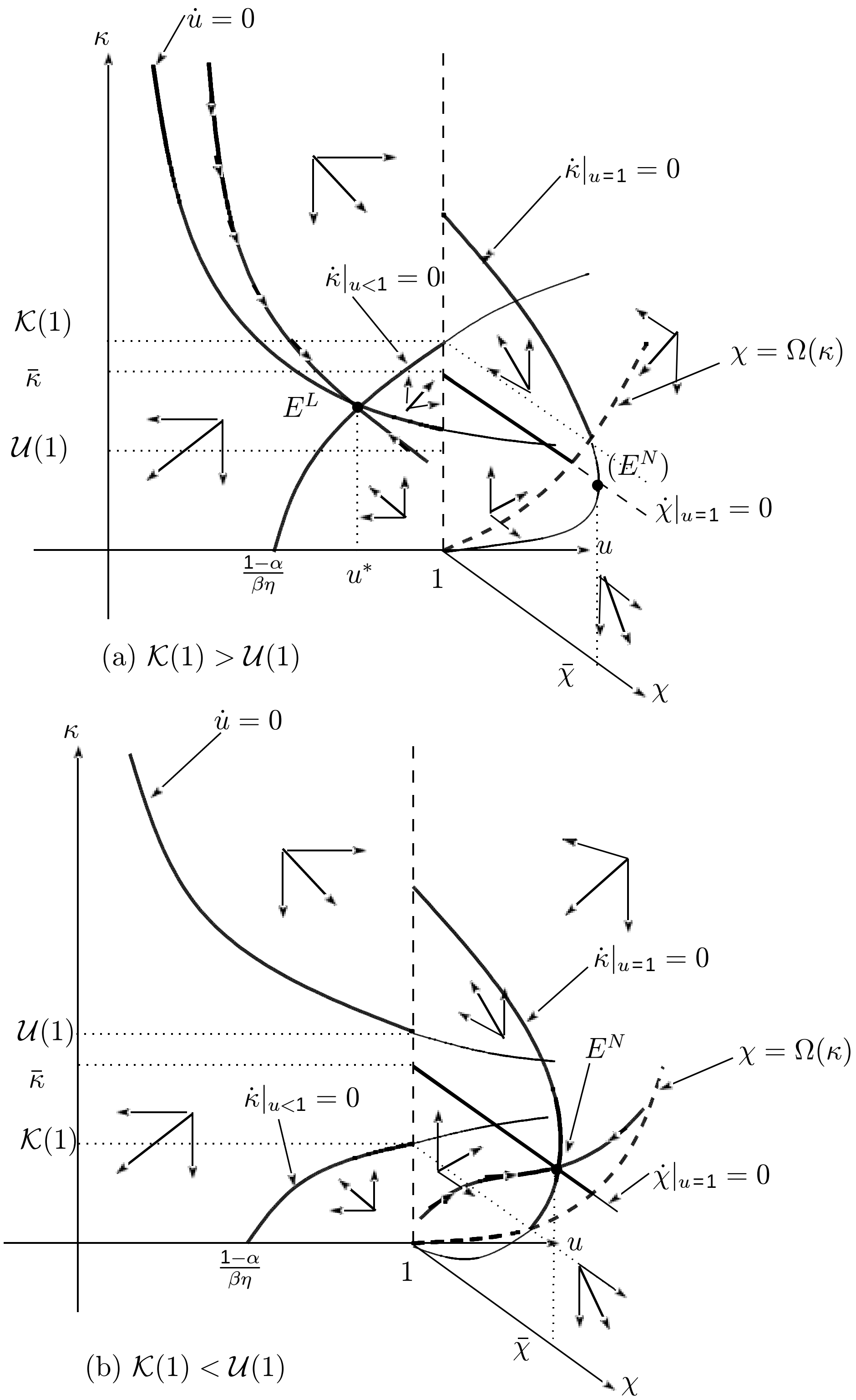

Figure 1: The Phase Diagram

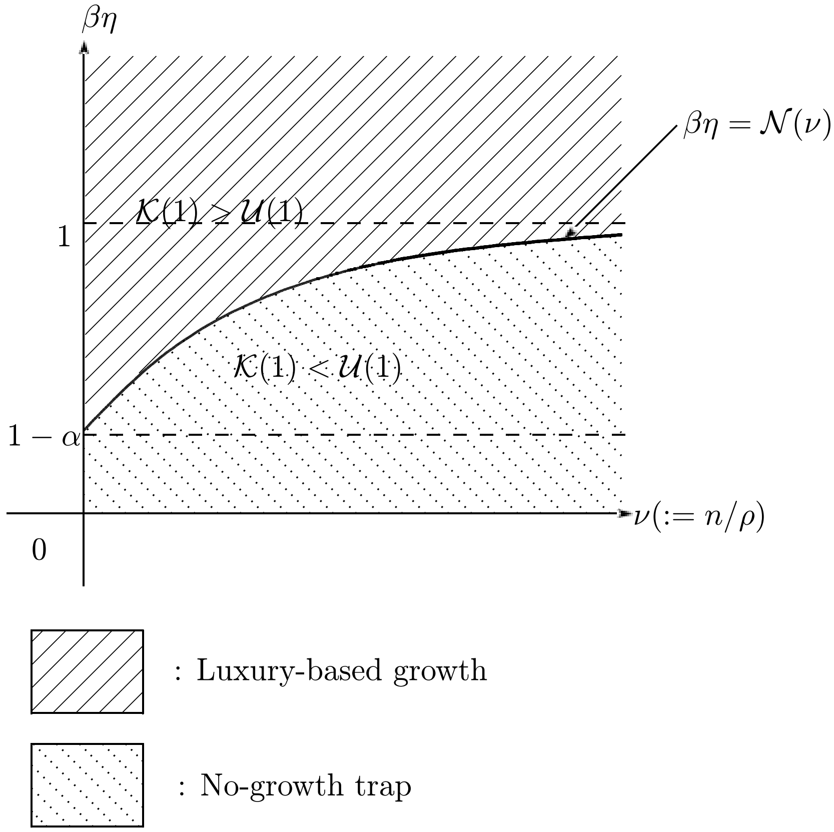

Figure 2: Growth Phases