ECONOMICS BULLETIN

Common and almost common knowledge of credible assignments in a coordination game

Laura Bangun
Department of Economics, University of Auckland

Ananish Chaudhuri
Department of Economics, University of Auckland

Parapin Prak
Department of Economics, University of Auckland

Chenan Zhou
Department of Accounting and Finance, University of Auckland

Abstract

We build on Van Huyck, Gillette and Battalio (1992) and examine the efficacy of credible assignments in a stag−hunt type coordination game with two Pareto−ranked equilibria, one payoff dominant and the other risk dominant. The majority of our subjects fail to coordinate to the payoff dominant outcome when no assignment is made. However, the majority of them always coordinate to the payoff dominant outcome when an assignment is made. This happens regardless of whether the assignment is "almost common knowledge" or "common knowledge".

We are grateful to the University of Auckland Research Council for providing the funds to conduct the experiments for this study. We thank Professor Charles Holt for providing us access to the Veconlab website at the University of Virginia for carrying out the experiments.
Citation: Bangun, Laura, Ananish Chaudhuri, Parapin Prak, and Chenan Zhou, (2006) "Common and almost common knowledge of credible assignments in a coordination game." *Economics Bulletin*, Vol. 3, No. 1 pp. 1−10
Submitted: December 18, 2005. **Accepted:** January 6, 2006.
URL: http://www.economicsbulletin.com/2006/volume3/EB−05C90025A.pdf

1. Introduction

A number of prior experimental studies have documented that in stag-hunt type coordination games with multiple Pareto-ranked equilibria, subjects often fail to coordinate to the payoff dominant outcome. Payoff dominance, therefore, is often not a good predictor of the equilibrium selected in these games. Cooper, Dejong, Forsythe and Ross (1990) study a number of coordination games with two Nash equilibria - a payoff dominant equilibrium and a risk dominant equilibrium. They find that while the majority of players select strategies that lead to a Nash equilibrium, it is also the case that the most frequent outcome is the risk dominant equilibrium rather than the payoff dominant one. Van Huyck, Battalio and Beil (1990) study a minimum effort coordination game where they also find this tendency on the part of the subjects to converge to the secure outcome rather than the payoff dominant one. This latter result is replicated in Chaudhuri, Schotter and Sopher (2004).

Cooper, Dejong, Forsythe and Ross (1990) argue that players' beliefs about other players' strategies are the key to the outcome of this type of coordination games. They posit that beliefs can be inferred from the strategy selected by players even though such beliefs are inherently unobservable. This suggests that if we can strengthen a player's beliefs that his pair member will choose the strategy commensurate with the payoff dominant outcome, then we should observe an increase in the proportion of payoff dominant outcomes in the game.

A number of studies look at interventions that might help players in stag-hunt games to coordinate to the payoff dominant outcome. Van Huyck, Gillette and Battalio (1992) study "credible assignments" where an assignment is a non-binding pre-game announcement made by an external arbiter instructing the players to adopt a particular strategy.

One of the coordination games studied by Van Huyck, Gillette and Battalio (1992) is shown in Table 1. They find that assignments to the payoff dominant outcome of {1, 1} are credible to most players. When players are instructed to choose strategy 1, 98% of the pairs managed to coordinate to the {1, 1} equilibrium following the assignment. At the same time, they also find that assignments to other equilibria – namely {2, 2} and {3, 3} - are not credible to over half of the players. When players deviate from the strategy they are asked to choose, it is always to strategy 1, i.e. to the strategy commensurate with the payoff dominant outcome.

Cooper, Dejong, Forsythe and Ross (1992) study the role of one-way and two-way communication in games with two Pareto-ranked equilibria – one payoff dominant and the other risk dominant. One-way communication allows only one player to make a non-binding announcement about the strategy he will choose in the game while in two-way communication both players are allowed to make non-binding announcements. They find that one-way communication raises the proportion of players who play the payoff dominant strategy. However, it is not strong enough to facilitate consistent coordination to the payoff dominant outcome. Two-way communication, on the other hand, solves the coordination problem in their simple 2X2 coordination game, i.e. the majority of players coordinate to the payoff dominant outcome. Two-way communication, however, does not solve the coordination problem in the cooperative coordination game which is a 3X3 game that includes a dominated strategy for each player. Considerable coordination failures are still observed.

In this paper we wish to re-examine the efficacy of an external assignment such as the one in Van Huyck, Gillette and Battalio (1992). It is possible that the structure of the payoff matrix in their game might be a factor behind the efficacy of the assignment. In their game there is no obvious conflict between payoff dominance and risk dominance. As they also point out in their paper (p. 611) the payoff dominant outcome {1, 1} is also the outcome which requires the smallest "minimum sufficient degree of credibility". Therefore, the {1, 1} outcome is the most credible outcome in this game, which is followed by {2, 2} and then by {3, 3}. It would be instructive to see how well a similar assignment works in a different game where there is a sharp distinction between the payoff dominant and risk dominant outcomes.

There is another issue that we wish to address here. In Van Huyck, Gillette and Battalio (1992), the assignments are "common knowledge" in that the message given to the subjects are projected on the lab wall and also read aloud by the experimenter. Thus each player knows that all other players have received the same message and they have also heard this information read out loud. What will happen when the assignment is only "almost common knowledge", i.e. the assignment is only handed over to the players on a sheet of paper and *not* read aloud? In the latter case not every player may be convinced that every other player has actually read the message on the sheet of paper or even paid attention to it.

Chaudhuri, Schotter and Sopher (2004) study an inter-generational version of the Van Huyck, Battalio and Beil (1990) minimum effort game where subjects in generation *t* can leave free-form advice for their generation *t+1* successors. Chaudhuri et al. find that this process of leaving advice does facilitate coordination to the payoff dominant outcome but only when such advice is both *public and common knowledge*, i.e. advice from all players in generation *t* is made public to all players in generation *t+1* and this advice is also *read aloud* by the experimenter. They find that even small deviations from such "common knowledge" – for instance when advice is made public to all members of a generation on a sheet of paper or projected on the lab wall but not read aloud – leads to seriously sub-optimal outcomes with massive coordination failures.

Rubinstein (1989) studies an electronic mail game where he shows that there is a dramatic discontinuity in behavior when subjects move from having "almost common knowledge" to "common knowledge" of the payoffs of the game. Here two players, player 1 and player 2, are involved in a coordination game. Each can take one of two actions, "A" or "B", and there are two possible states of nature, "a" or "b" which occur with probability (1-p) and p respectively with $p < 1/2$, making state "a" more likely. See Table 2. Initially only player 1 knows the true state of nature. Suppose that the two players can only communicate via electronic mail. There is a small probability, ε, that due to technical problems the message does not arrive at its destination. Rubinstein assumes that when player 1 gets the information that the state of nature is "b", his computer automatically sends a message to player 2 and then player 2's computer confirms the message and then player 1's computer re-confirms the message and so on. If the message does not arrive, the communication stops. No message is sent if the state of nature is "a". If the machines exchanged messages infinitely, then there would be "common knowledge" about the state of nature and players would find it easy to coordinate their actions. Rubinstein demonstrates, however, that for any finite number of exchanges, the only equilibrium is for both players to choose A in each period. Hence, even if a large number of messages are exchanged back and forth, a situation where players should have at least "almost common knowledge" of the true state of nature if not common knowledge, Rubinstein shows that the players can not obtain an expected payoff in equilibrium higher than what they could achieve without any information exchange.

Consequently, the second question we investigate is what will happen when the assignment is only "almost common knowledge" and not "common knowledge" as in Van Huyck, Gillette and Battalio (1992). What happens if the assignment is only handed over to players on a sheet of paper but is not read aloud as opposed to a "common knowledge" assignment where each player receives the assignment on a sheet of paper and this information is also read aloud to them? We should point out that in our game common knowledge or the lack thereof applies to the assignment only and not to the payoffs of the game itself. Thus we do not claim any formal similarity between our results and those of Rubinstein (1989).

We find that in our study only around 38% of the pairs manage to coordinate to the payoff dominant outcome in the absence of an assignment. This result is similar to that in the simple coordination game in Cooper, Dejong, Forsythe and Ross (1992). However, when an assignment is made, whether it is an "almost common knowledge" or a "common knowledge" assignment, the majority of the pairs manage to coordinate to the payoff dominant outcome.

We proceed as follows. In Section 2, we explain the design of our experiment. In Section 3, we report our main findings. We make concluding remarks in Section 4.

2. Experimental Design and Procedures

The experiments were conducted in a computer laboratory at the University of Auckland using the Veconlab on-line software developed by Charles Holt at the University of Virginia. (http://veconlab.econ.virginia.edu/admin.htm). We use the on-line instructions and do not provide a copy here. The participants are primarily first year students in Microeconomics and Macroeconomics at the University of Auckland.

There are two sessions with 20 subjects (10 pairs) in each session for a total of 40 subjects. Each session consists of 12 rounds of play of the stage game. In each session subjects first play 4 rounds with no assignment. Then an assignment is made prior to the beginning of round 5. This assignment is either "almost common knowledge" or "common knowledge" for the players. After this the players play for 8 more rounds consecutively in that particular treatment with no further announcements made. In session 1, the 4 rounds with no assignment are followed by 8 rounds of "common knowledge" assignment while in session 2 this is followed by 8 rounds of "almost common knowledge" assignment. Subjects are randomly assigned into row and column participants. The participants are randomly re-matched at the end of every round.[1]

The actual game that the subjects played is shown in Table 3.[2]

Row players can choose to play either {Top} or {Bottom}. Similarly, column players can choose to play either {Left} or {Right}. There are three Nash equilibria in this game. {Top, Left} is the payoff dominant equilibrium while the risk dominant equilibrium is {Bottom, Right}. There is also a mixed strategy equilibrium where the

[1] It is possible for two players to interact more than once. But subjects never get to see the identification number of the player they are paired with and there is no scope for reputation building.
[2] The payoff matrix is taken from Rydval and Ortmann (2004).

Row player plays {Top} with probability 4/7 and {Bottom} with probability 3/7 while the Column player plays {Left} with probability 4/7 and {Right} with probability 3/7.

In the first four rounds of each session, subjects play the game shown in Table 3 without any assignment being made by the experimenter. In the "almost common knowledge" treatment, prior to beginning round 5, the experimenter hands out a sheet of paper to all the players with the following message:

> ROW PARTICIPANT: CHOOSE **TOP**.
>
> COLUMN PARTICIPANT: CHOOSE **LEFT**.
>
> IF THE ROW PARTICIPANT CHOOSES TOP THEN THE BEST THE COLUMN PARTICIPANT CAN DO IS TO CHOOSE LEFT. IF THE COLUMN PARTICIPANT CHOOSES LEFT THEN THE BEST THAT THE ROW PARTICIPANT CAN DO IS TO CHOOSE TOP.
>
> NOTICE, from the payoff matrix, that if both the Row and Column participants follow the message then they both earn 8 experimental dollars. However, if one of the participants follows the message and the other does not, then both participants will earn a smaller amount.

The language and presentation of the announcement is virtually identical to the one used in Van Huyck, Gillette and Battalio (1992). The subjects then play 8 more rounds with no further announcements being made.

For the "common knowledge" treatment, again, after the first 4 rounds and immediately prior to the beginning of round 5, the experimenter hands out the same message shown above on a piece of paper but in addition the message is also *read aloud in front of all subjects*. Again the subjects play 8 more rounds after this with no further announcements being made.

We should point out that in our study the assignment to a strategy is done only once prior to the beginning of round 5 of the 12 rounds of play as opposed to Van Huyck, Gillette and Battalio (1992) where the authors make an announcement prior to each round of play.

Since we have 10 pairs in each session we get 10 plays of the game in each round. Thus we get 80 plays of the game in the no assignment treatment (10 plays of the game for each of the first 4 rounds over two sessions), 80 plays of the game with a "common knowledge" assignment (10 plays of the game for each of 8 rounds in Session 1) and 80 plays of the game with an "almost common knowledge" assignment (10 plays of the game for each of 8 rounds in Session 2).

The payoffs are denoted in experimental dollars and are converted into cash at the rate of NZ $0.10 per experimental dollar. Subjects also get a $5 show-up fee for being on time. They are paid privately at the end of the session. Each session lasts about 45 minutes and subjects make NZ $13 on average including the $5 show up fee.[3]

[3] At the time when the experiments were carried out the exchange rate was approximately NZ $1 = US $0.72.

3. Results

We begin by looking at the actual strategies adopted by the players in the three different treatments. After that we undertake a careful analysis of the actual outcomes in the different treatments.

We will refer to a choice of Top or Left (i.e. a strategy choice commensurate with the payoff dominant outcome) as Strategy 1 and Bottom or Right as Strategy 2. In the absence of an assignment, 58% of players play Strategy 1 while 42% play Strategy 2. There is a dramatic change once we make the assignment with players in both the “almost common” and “common” knowledge treatments choosing strategy 1 in 90% cases or more. See Table 4.

Next, we look at the proportion of outcomes under each treatment (see Table 5 and Figure 1). We find that in the absence of an assignment 37.5% of the plays result in the payoff dominant outcome while 21.3% plays lead to the risk dominant outcome. The highest proportion (41.3%) is of disequilibrium outcomes of either {Top, Right} or {Bottom, Left}. These results are similar to that in Cooper, Dejong, Forsythe and Ross (1992) when no communication is allowed among players.

When we move to the “almost common knowledge” assignment, where each subject gets a sheet of paper with the message, 92.5% of the player pairs manage to coordinate to the payoff dominant outcome. Now only 7.5% plays result in the disequilibrium outcome and there are no risk dominant outcomes. Under the “common knowledge” assignment, the proportion of payoff dominant outcomes is 82.5%. As a whole, the majority of players in our game can coordinate to the payoff dominant outcome when an assignment is made, regardless of whether this assignment is almost common knowledge or common knowledge. This suggests that assignments in our game help players to form optimistic beliefs that their pair members in the game will choose a strategy commensurate with the payoff dominant equilibrium.

Using a sample proportions test[4] we find that compared to the no assignment treatment, the proportions of payoff dominant outcomes are significantly higher in the almost common knowledge treatment ($z = 8.93$, $p = 0.00$) as well as the common knowledge treatment. ($z = 6.52$, $p = 0.00$).

However, contrary to our expectation we find that the proportion of payoff dominant outcomes turns out to be higher under the “almost common knowledge” assignment (92.5%) than under the “common knowledge” assignment (82.5%). Upon closer examination we find that this result is caused by a particular row player - player number 9 in Session 1 - who consistently chooses “Bottom” every round even after receiving the message to choose “Top”. It is not clear to us whether this is due to a misunderstanding of the assignment or some other factor. It is possible that this player

---

[4] If the two relevant sample proportions are p_1 and p_2 and the two samples have n_1 and n_2 observations, then the corresponding test-statistic is $z = \dfrac{p_1 - p_2}{\sqrt{\dfrac{p_1 * (1 - p_1)}{n_1} + \dfrac{p_2 * (1 - p_2)}{n_2}}}$.

cares less about absolute earnings and more about relative earnings. Suppose player 9 believes that, after the assignment is made, every column player will be convinced by the assignment and will consistently choose “Left” for the rest of the session. Then player 9, by choosing “Bottom”, can ensure that he gets $5 while his pair member gets $1. This, of course, does not maximize player 9’s own earnings and given that players are re-matched at the end of every round player 9’s power to depress the earnings of others is limited.

Next, we exclude this player and re-analyse the results. That is for each round of Session 1 we exclude the pair which includes player 9. This means that for each round of this session we lose one play of the stage game. This reduces the observations under no assignment from 80 to 76 (since there are 4 rounds of no assignment) and under “common knowledge” assignment from 80 to 72 (since there are 8 rounds of “common knowledge” assignment). As one can see from Table 6 and Figure 2, once we exclude this particular player the outcomes in the “almost common knowledge” treatment and the “common knowledge” treatment are virtually identical. We carry out a sample proportions test to see if the proportion of payoff dominant outcomes in the “common knowledge” treatment is different from that in the “almost common knowledge” treatment after we exclude player 9. We get a z-statistic of 0.19 with a corresponding p-value of 0.85. Thus the null hypothesis that there is a difference in the proportion of payoff dominant outcomes under the two assignments can be comfortably rejected at conventional significance levels. This suggests that in a simple stag-hunt type coordination game, it is sufficient but not necessary to make common knowledge assignments in order to induce optimistic beliefs among players. Even a weaker form of assignment can help the majority of players coordinate to the Pareto-efficient outcome.

4. Conclusion

There are three main conclusions arising from this study. First, without an assignment we observe significant coordination failures similar to those found in previous studies such as Cooper et al. (1990). Introduction of an assignment along the lines of Van Huyck, Gillette and Battalio (1992) facilitates coordination to the payoff dominant outcome. Thus a credible assignment made by an external arbiter facilitates coordination to the payoff dominant outcome even in a game where there is a clear distinction between the payoff dominant and risk dominant outcomes. Second, coordination to the payoff dominant outcome is achieved with both “almost common knowledge” and “common knowledge” of the assignment and there is no significant difference between the two. This result is in contrast to Chaudhuri, Schotter and Sopher (2004) who find that only “common knowledge” of advice, which is a form of assignment, helps subjects reach the payoff dominant outcome in the minimum effort game. Third, in our study the assignment is made only once with the players proceeding to play 8 more rounds afterwards compared to Van Huyck, Gillette and Battalio (1992) where the assignment is made prior to each round. In spite of the one-time nature of the assignment, it still managed to generate optimistic enough beliefs that enabled players to coordinate to the payoff dominant outcome consistently. Thus in this simple 2X2 coordination game at least, a weaker form of assignment works well in enabling players to coordinate to the payoff dominant outcome.

References:

Chaudhuri, Ananish, Andrew Schotter, and Barry Sopher (2004), "Talking Ourselves to Efficiency: Coordination in Inter-Generational Minimum Games with Private, Almost Common and Common Knowledge of Advice", Working Paper, Department of Economics, New York University, New York, NY.

Cooper, Russell W., Douglas V. Dejong, Robert Forsythe, and Thomas Ross (1990), "Selection Criteria in Coordination Games: Experimental Results", *American Economic Review*, 80, 218-233.

Cooper, Russell W., Douglas V. Dejong, Robert Forsythe, and Thomas Ross (1992), "Communication in Coordination Games", *Quarterly Journal of Economics* 107, 739-771.

Rubinstein, Ariel (1989), "The Electronic Mail Game: Strategic Behavior Under "Almost Common Knowledge"", *American Economic Review*, 79, 385-391.

Rydval, Ondrej and Andreas Ortmann (2004), "Loss Avoidance as Selection Principle: Evidence from Simple Stag-Hunt Games", Working Paper, CERGE-EI, Prague, Czech Republic.

Van Huyck, John., Raymond Battalio, and Richard Beil (1990), "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure", *American Economic Review* 80, 234-248.

Van Huyck, John., Ann B. Gillette, and Raymond Battalio (1992), "Credible Assignments in Coordination Games", *Games and Economic Behavior* 4, 606-626.

Table 1

Payoff matrix in Van Huyck, Gillette and Battalio (1992)

	Column Player			
Row Player		1	2	3
	1	9,9	0,0	0,0
	2	0,0	5,5	0,0
	3	0,0	0,0	1,1

Table 2

Payoff matrix in Rubinstein (1989)

Game G_a in State "a" (Probability 1-p)

	Player 2		
Player 1		A	B
	A	M, M	0, - L
	B	- L, 0	0, 0

Game G_b in State "b" (Probability p)

	Player 2		
Player 1		A	B
	A	0, 0	0, - L
	B	- L, 0	M, M

Table 3

Payoff matrix in our study

	Column Player		
Row Player		Left	Right
	Top	8, 8	1, 5
	Bottom	5, 1	5, 5

Table 4

Proportion of players choosing Strategy 1 or Strategy 2 under different treatments

	Strategy 1	Strategy 2
No assignment	58%	42%
Almost Common Knowledge Assignment	96.3%	3.7%
Common Knowledge Assignment	90.6%	9.4%

Table 5

Proportion of outcomes in each treatment

	{Top, Left}	{Bottom, Right}	{Top, Right} / {Bottom, Left}
No Assignment	37.5%	21.3%	41.3%
Almost Common Knowledge Assignment	92.5%	---	7.5%
Common Knowledge Assignment	82.5%	1.3%	16.3%

Table 6

Proportion of outcomes in each treatment - excluding Player 9

	{Top, Left}	{Bottom, Right}	{Top, Right} / {Bottom, Left}
No Assignment	39.5%	21%	39.5%
Almost Common Knowledge Assignment	92.5%	---	7.5%
Common Knowledge Assignment	91.7%	---	8.3%

Figure 1

Proportion of Outcomes in Each Treatment

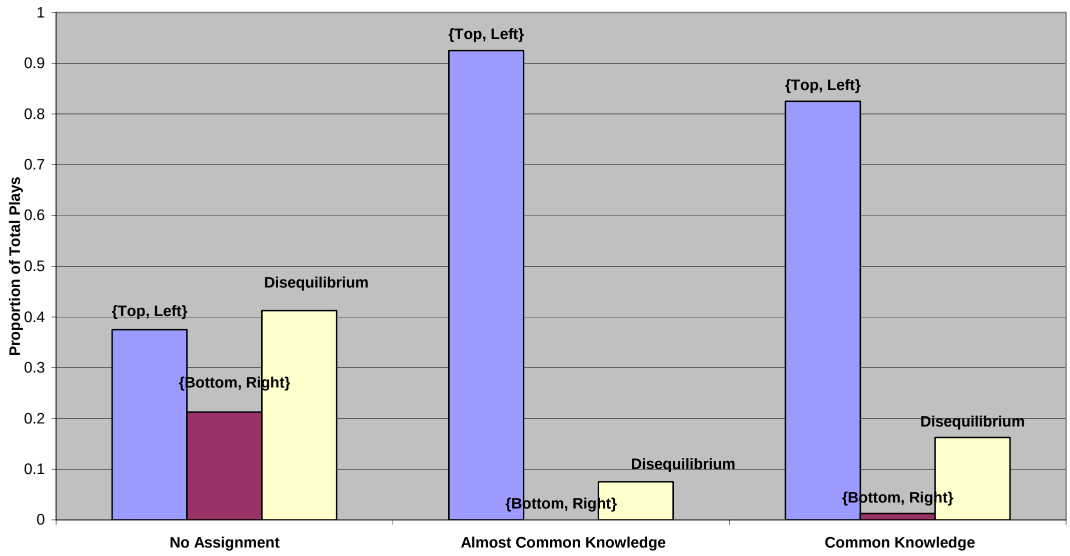

Figure 2

Proportion of Outcomes in Each Treatment excluding Player 9