The seasonal KPSS statistic

Johan Lyhagen
Uppsala University

Abstract

In this paper a seasonal version of the KPSS test for unit roots are proposed and its asymptotic distribution is stated. Further, a small Monte Carlo simulation is used to analyse some size and power properties.

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1 Introduction

The most common unit root test is of the Dickey-Fuller type, see Fuller (1976) and Dickey and Fuller (1979), which has a null of a unit root. For example Nelson and Plosser (1982) use this kind of tests on 14 annual U.S. economic time series not rejecting the null for all except one series. One possible explanation for this is that unit root tests often have low power against alternatives close to the null, see e.g. Diebold and Rudebusch (1991), DeJong et. al. (1992) and Rudebusch (1992). This implies that there would be an interest in unit root tests which has the null of no unit root. A popular test with this feature is the KPSS test of Kwaitkowski et. al. (1992). The purpose of this paper is to generalize the KPSS statistic to seasonal unit roots. Similar tests have been proposed by e.g. Canova and Hansen (1995) and Caner (1998) but the model that they base their tests on is different. One reason to consider seasonal variations is that it is a fundamental part of the data and should be modeled and analyzed accordingly, see e.g. Ghysels (1988). The next section presents the seasonal KPSS test and its asymptotic distribution while the paper ends with a Monte Carlo simulation investigating small sample properties of the size and power.

2 The seasonal KPSS

Consider the following process

$$A(L)\, y_t = e_t \tag{1}$$

where $A(L)$ is a polynomial in the lag operator L. Our interest is to test if $A(L)$ contains a unit root at frequency θ_m. First, difference the series so that there are no other unit roots, i.e. for quarterly data and we want to test for the unit root -1 then use the filter $(1-L)(1+iL)(1-iL)$ giving the residuals ε_t.

Let $S_t^{(m)} = \sum_{j=1}^{t} z_m^j \varepsilon_j$ where $z_m = e^{i\theta_m}$ then the seasonal KPSS statistic is

$$\eta^{(m)} = \frac{\sum_{t=1}^{T} S_t^{(m)} \bar{S}_t^{(m)}}{T^2 \sigma_\varepsilon^2} \tag{2}$$

where $\bar{S}_t^{(m)}$ is the complex conjugate of $S_t^{(m)}$. Further σ_ε^2 need to be replaced with an estimate, e.g. a consistent one as in Newey and West (1987), i.e.

$$s^2(l) = T^{-1}\sum_{t=1}^{T}\varepsilon_t^2 + 2T^{-1}\sum_{s=1}^{l}\left(1-\frac{s}{l+1}\right)\sum_{t=s+1}^{T}\varepsilon_t\varepsilon_{t-s}. \tag{3}$$

The asymptotic distribution of $\eta^{(m)}$ is easily found by using the continuous mapping theorem and Lemma 5 in Johansen and Schaumburg (1998). For unit roots at frequencies 0 and π the distribution is the same as in Kwaitkowski et. al. (1992):

$$\int_0^1 W(r)^2\,dr \tag{4}$$

where $W(r)$ is a standardized Brownian motion For the frequencies $0 < \theta_m < \pi$ the distribution is

$$\frac{\int_0^1 W_R(r)^2\,dr + \int_0^1 W_I(r)^2\,dr}{2} \tag{5}$$

where $W_R(r)$ and $W_I(r)$ are independent standardized Brownian motions. By regressing the filtered series on an intercept or an intercept and a trend the Brownian motions are replaced by a Brownian bridge or a second order Brownian bridge respectively. Note that the complex roots come in pairs which gives the same test statistic, hence the test can not distinguish between them.

3 A small Monte Carlo simulation

To investigate the size properties of the seasonal KPSS statistic a small Monte Carlo simulation is carried out with the seasonal roots of a quarterly process. For the root -1 the data generating process is

$$y_t + \rho y_{t-1} = e_t \tag{6}$$

while for the complex pairs of roots $(i, -i)$ it is

$$y_t + \rho y_{t-2} = e_t \tag{7}$$

for $\rho \in -0.8, -0.2, 0, 0.2, 0.8, 1$ and sample sizes $T \in 50, 100$. The number of replicates is 20000. Three choices of bandwidth are used, $l0 = 0$, $l4 =$ integer $\left[4\,(T/100)^{1/4}\right]$ and $l12 =$ integer $\left[12\,(T/100)^{1/4}\right]$. This is a subset of the Monte Carlo setup in Kwaitkowski et. al. (1992).

The results in Table 1 shows that decreasing ρ decreases the size and that increasing the sample size have small effects, results also found for the ordinary KPSS test. A surprising result is that, contrary to the ordinary KPSS test, $l4$ and $l12$ do not have better size properties than $l0$ and sometimes even much worse. The problem for the ordinary KPSS test is that by correcting the size most power is lost, a problem not shared by the seasonal KPSS test where the bad size properties induces substantial power.

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Tables

		$\eta^{(-1)}$			$\eta^{(i,-i)}$		
ρ	T	$l0$	$l4$	$l12$	$l0$	$l4$	$l12$
-0.8	50	0.000	0.000	0.000	0.000	0.000	0.000
	100	0.000	0.000	0.000	0.000	0.000	0.000
-0.2	50	0.016	0.013	0.024	0.011	0.016	0.028
	100	0.017	0.010	0.014	0.011	0.009	0.014
0	50	0.050	0.068	0.093	0.048	0.079	0.116
	100	0.052	0.061	0.077	0.051	0.067	0.095
0.2	50	0.113	0.199	0.248	0.135	0.243	0.327
	100	0.115	0.198	0.229	0.143	0.248	0.314
0.8	50	0.608	0.933	0.957	0.792	0.976	0.985
	100	0.629	0.954	0.980	0.826	0.988	0.996
1.0	50	0.969	0.999	1.000	0.992	1.000	1.000
	100	0.990	1.000	1.000	0.998	1.000	1.000

Table 1: Size and power of the seasonal KPSS test for the seasonal roots of a quarterly process. The 5% asymptotical critical values are 1.650 and 1.312 for the roots -1 and the complex roots (i,-i) respectively.