Small-sample properties of tests for heteroscedasticity in the conditional logit model

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Abstract

This paper compares the small-sample properties of several asymptotically equivalent tests for heteroscedasticity in the conditional logit model. While no test outperforms the others in all of the experiments conducted, the likelihood ratio test and a particular variety of the Wald test are found to have good properties in moderate samples as well as being relatively powerful.

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1 Introduction

In most applications of the conditional logit model the error term is assumed to be homoscedastic. Recently, however, there has been a growing interest in testing the homoscedasticity assumption in applied work (Hensher et al., 1999; DeShazo and Fermo, 2002). This is partly due to the well-known result that heteroscedasticity causes the coefficient estimates in discrete choice models to be inconsistent (Yatchew and Griliches, 1985), but also reflects a behavioural interest in factors influencing the variance of the latent variables in the model (Louviere, 2001; Louviere et al., 2002). This paper compares the small-sample properties of several asymptotically equivalent tests for heteroscedasticity in the conditional logit model using simulated data. While no test outperforms the others in all of the experiments conducted, the likelihood ratio test and a particular variety of the Wald test are found to have good properties in moderate samples as well as being relatively powerful.

Section 2 presents the heteroscedastic logit model, section 3 describes the various tests for heteroscedasticity and section 4 presents the simulation results. Section 5 offers some concluding remarks.

2 The heteroscedastic logit model

I assume a sample of N consumers with the choice of J discrete alternatives. Let U_{nj} be the utility individual n derives from choosing alternative j. It is assumed that the utility can be partitioned into a systematic component, $X_{nj}\beta$, and a random component, ε_{nj}:

$$U_{nj} = X_{nj}\beta + \varepsilon_{nj} \tag{1}$$

where X_{nj} is a vector of attributes relating to alternative j and β is a vector of coefficients reflecting the desirability of the attributes. The random component ε_{nj} represents characteristics and attributes unknown to the researcher, measurement error and/or heterogeneity of tastes in the sample. The probability that individual n chooses alternative i is the probability that the utility of choosing i is higher than the utility of choosing any other alternative in the individual's choice set:

$$P_{ni} = P(X_{ni}\beta + \varepsilon_{ni} > X_{nj}\beta + \varepsilon_{nj}) = P(\varepsilon_{nj} - \varepsilon_{ni} < X_{ni}\beta - X_{nj}\beta)\ \forall\ j \neq i \in J \tag{2}$$

Assuming that the random terms are IID extreme value type I distributed we get the conditional logit model (McFadden, 1974) in which the probability that alternative i is chosen by respondent n is given by:

$$P_{ni} = \frac{\exp(\mu X_{ni}\beta)}{\sum_{j=1}^{J} \exp(\mu X_{nj}\beta)} \tag{3}$$

where μ is a positive scale parameter which can be shown to be inversely proportional to the error variance, σ_ε^2:

$$\mu = \frac{\pi}{\sqrt{6\sigma_\varepsilon^2}} \tag{4}$$

Since the scale parameter cannot be identified it is usually normalised to unity.

The conditional logit model assumes that the error variance is constant across individuals. This assumption has been called into question in several recent papers (Hensher et al. 1999; Louviere, 2001; DeShazo and Fermo, 2002; Louviere et al., 2002). Following DeShazo and Fermo (2002) and Hensher et al. (1999) an alternative to the conditional logit model which allows for unequal variances across individuals is given by:

$$P_{ni} = \frac{\exp(\mu_n X_{ni}\beta)}{\sum_{j=1}^{J} \exp(\mu_n X_{nj}\beta)} \tag{5}$$

where μ_n is a function of individual characteristics that influence the magnitude of the scale parameter and therefore the error variance.[1] μ_n is conveniently parametrised as $\exp(Z_n\gamma)$ where Z_n is a vector of individual characteristics and γ is a vector of parameters reflecting the influence of those characteristics on the error variance. This model is referred to as the heteroscedastic logit model by DeShazo and Fermo and the parametrised heteroscedastic multinomial logit model by Hensher et al. Note that when $J = 2$, eq. (5) is algebraically equivalent to the heteroscedastic binary logit model

[1] This formulation can be extended to incorporate heteroscedasticity across alternatives by introducing a j subsript for μ.

suggested by Davidson and McKinnon (1984).[2] The $\exp(Z_n\gamma)$ parametrisation has the desirable property that μ_n is positive for all n as well as ensuring that the heteroscedastic logit collapses to the conditional logit when $\gamma = 0$. A test for $\gamma = 0$ is therefore a test for the error variance being constant across respondents.

The parameter vector $\theta = (\beta', \gamma')'$ is estimated using maximum likelihood methods. The log-likelihood function is given by $LL = \sum_{n=1}^{N}\sum_{j=1}^{J} y_{nj} \ln P_{nj}$, where $y_{nj} = 1$ if alternative j is chosen by individual n and zero otherwise.

3 Heteroscedasticity tests

All the usual tests for parameter restrictions in models estimated by maximum likelihood - the likelihood ratio, Wald and Lagrange multiplier tests (see e.g. Greene, 2003) - can be employed to test the null hypothesis of homoscedasticity. The Lagrange multiplier (LM) test has the advantage that only the restricted (conditional logit) model needs to be estimated, while the Wald and likelihood ratio (LR) tests require estimation of the unrestricted (heteroscedastic logit) model. This difference is not of major practical importance, however, since the heteroscedastic logit model is straightforward to estimate despite being highly non-linear.[3]

The LM and Wald tests depend on an estimate of the covariance matrix for the restricted/unrestricted coefficient estimates, respectively. While the most common approach is to use the inverse of the negative Hessian as an estimate of the covariance matrix, the use of the robust estimator (White, 1982) is becoming widespread in applied work. It should be noted, however, that the use of the robust estimator does not remedy the presence of heteroscedasticity in conditional logit models since the heteroscedasticity causes the coefficient estimates to be inconsistent.[4] A third alternative is the outer product of gradients (OPG) estimator, which is a convenient choice for the LM test from a computational point of view. When the model is correctly

[2] Davidson and McKinnon specify μ_n to be an inverse function of $\exp(Z_n\gamma)$ but this difference is qualitatively unimportant since $1/\exp(Z_n\gamma) = \exp(-Z_n\gamma)$.

[3] A sample Stata program for estimating the heteroscedastic logit model is available from the author upon request.

[4] This contrasts from the linear case in which the OLS estimates are consistent (but inefficient) in the presence of heterosceadsticity.

specified and $N \to \infty$ the three estimators of the covariance matrix are equal due to the information identity (Train, 2003).

In total seven alternative heteroscedasticity tests are considered: the Wald and LM tests based on the three different covariance estimators and the LR test. While the tests are asymptotically equivalent they can give different results in finite samples. Since there are no substantial differences between the tests in terms of ease of computation, small-sample performance is deemed to be the most important selection criterion.

4 The Simulation study

4.1 The data generating process

The data generation process is the heteroscedastic logit model in eq. (5), with X_{nj} and Z_n specified to consist of a single variable which is standard normally distributed over attributes and individuals, respectively. The degree of heteroscedasticity is reflected in the true value of γ, which is specified to take four values (0, 0.1, 0.25 and 0.5) in the experiments. The true value of β, which is an important determinant of the explanatory power of the model, is specified to take three values (0.5, 1 and 1.5). Four sample sizes (100, 250, 500 and 1000) are considered along with three choice set sizes (2, 3 and 5). The range of values are chosen to be reasonably representative of the sample and choice set sizes typical of applied work. Combining the parameter values and sample/ choice set sizes result in 144 sampling experiments, of which heteroscedasticity is present in 108.

4.2 Simulation results

Firstly the performance of the tests are considered under the null of homoscedasticity ($\gamma = 0$). The simulated rejection rates of the various tests for heteroscedasticity at the nominal 5% significance level are reported in table 1. The results are based on 10.000 trials.[5] It can be seen from the table that nearly all the tests have rejection rates which are insignificantly

[5]In a small number of trials the heteroscedastic logit model did not converge. These trials were replaced.

different[6] from the nominal level for $N = 1000$ and $J = 5$ for all values of β. The only exception is the LM test based on the robust covariance matrix which is significantly oversized for $\beta = 0.5$. There is a substantial difference in the performance of the tests for smaller sample sizes/ choice sets, however. On the whole the LR test and the LM tests tend to be oversized along with the Wald test based on the robust covariance matrix, while the remaining Wald tests tend to be undersized. The extent to which the tests are over-/undersized vary substantially. While the LM tests based on the Hessian and robust covariance matrices have rejection rates of about twice and three times the nominal level in some cases, the LR test is about 40% oversized and the Wald test based on the Hessian about 25% undersized at worst.

All the tests generally improve in performance as N and J increase. The influence of changes in β is less clear-cut, however. As mentioned previously the true value of β is an important determinant of the explanatory power of the model. Table 2 reports the percentage of correctly predicted choices as a function of β and J. It can be seen from the table that the fit of the model increases with both β and J, using $P_{ni} = 1/J$ as a benchmark. Since the explanatory power is almost completely unaffected by changes in γ and N, only the results for $\gamma = 0$ and $N = 100$ are reported. The performance of the LR test along with the LM tests based on the Hessian and robust covariance matrix generally improves when β increases, in contrast to the performance of the LM test based on the OPG matrix which deteriorates. The effect of changes in β on the Wald tests is less systematic. On balance these results suggest that, conditional on J, good predictive power is not necessarily an indicator of good test performance; the most important determinant is the sample size, along with the size of the choice set.

The power of the tests is compared in three different cases: $\gamma = 0.1$, 0.25 and 0.5, with the degree of heteroscedasticity increasing in γ. The simulated rejection rates at the 5% level of significance are reported in tables 3-5. Again the results are based on 10.000 trials. In all three cases the power of the tests are similar for $N = 1000$ and $J = 5$ for all values of β. As expected the power of the tests increases substantially with the degree of of heteroscedasticity present in the data. The true value of β is also found to be an important determinant of test power; the greater the explanatory power of the model the greater the power of the tests. Unsurprisingly, the most powerful tests in the

[6] These tests are based on 95% normal approximation confidence intervals (Conover, 1999).

scenarios with smaller sample sizes/ choice sets are the LM tests based on the Hessian and robust covariance matrices, which were found to be substantially oversized in these cases. Among the tests with better size properties the LR test is found to be more powerful than the Wald test based on the Hessian, although the difference is only notable when the sample/ choice set size is low, in which case the LR test is oversized and Wald test undersized.

The simulation results for $J = 2$ can be compared to those reported by Davidson and MacKinnon (1984). They find that for a binary logit model with relatively good predictive power, the LR test and the LM test based on the Hessian matrix outperform the LM test based on the OPG matrix.[7] This is consistent with the findings in the present paper. When the fit of the model is less good, however, the LM test based on the OPG matrix is found to perform better than the test based on the Hessian matrix in this study.

To summarize, these results suggest that in moderate samples both the LR tests and the Wald test based on the Hessian are likely to perform fairly satisfactory and better than the remaining tests considered here, although none of them outperforms the other tests in all the experiments. The LR test is the slightly more conservative choice as it is more powerful but tends to over-reject the null in small samples, while the Wald test has less power and rejects the null too infrequently. As expected there is little to separate between the tests when the sample size/ choice set are both relatively large.[8] The simulation results suggest that for $N \geq 1000$ and $J \geq 5$ the performance of the tests is almost identical.

5 Concluding remarks

Recently there has been a growing interest in testing the assumption of homoscedasticity in conditional logit models. This paper compares the small-sample properties of several asymptotically equivalent tests for heteroscedasticity in the conditional logit model using simulated data. While the performance of the tests is similar when the sample size and choice set are rela-

[7] Davison and MacKinnon do not consider any of the Wald tests or the LM test based on the robust covariance matrix. On the other hand they devise a number of alternative regression based LM tests which are not considered here. The authors find that the latter tests have worse small-sample properties than the standard Hessian/OPG based LM tests.

[8] It should also be noted that the results show some evidence that increasing the sample size can compensate for a small choice set; the performance of the tests is somewhat better overall for $N = 250$ and $J = 2$ than for $N = 100$ and $J = 5$ for instance.

tively large, there is substantial variation in performance for smaller samples/ choice sets. On the whole it is found that the best performing tests are the likelihood ratio test and the Wald test based on the Hessian matrix.

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Table 1. Simulated rejection rates at the 5% significance level for the null hypothesis of homoscedasticity. Results for $\gamma = 0$.

		J=2				J=3				J=5			
		N=100	N=250	N=500	N=1000	N=100	N=250	N=500	N=1000	N=100	N=250	N=500	N=1000
β =0.5	LR	0.0691	0.0545	0.0539*	0.0519*	0.0600	0.0567	0.0481*	0.0530*	0.0583	0.0516*	0.0510*	0.0502*
	WH	0.0375	0.0438	0.0513*	0.0508*	0.0400	0.0515*	0.0470*	0.0525*	0.0463*	0.0485*	0.0507*	0.0499*
	WOPG	0.0193	0.0405	0.0501*	0.0488*	0.0296	0.0498*	0.0472*	0.0506*	0.0389	0.0496*	0.0538*	0.0511*
	WR	0.0707	0.0553	0.0534*	0.0528*	0.0656	0.0546	0.0492*	0.0532*	0.0676	0.0547	0.0528*	0.0506*
	LMH	0.1044	0.0777	0.0629	0.0567	0.0977	0.0732	0.0564	0.0568	0.0944	0.0639	0.0567	0.0538*
	LMOPG	0.0630	0.0515*	0.0543*	0.0521*	0.0544*	0.0513*	0.0480*	0.0524*	0.0549	0.0513*	0.0486*	0.0489*
	LMR	0.1586	0.1109	0.0809	0.0642	0.1479	0.0975	0.0697	0.0636	0.1441	0.0869	0.0706	0.0610
β =1	LR	0.0597	0.0513*	0.0501*	0.0503*	0.0545	0.0546	0.0467*	0.0491*	0.0514*	0.0561	0.0495*	0.0495*
	WH	0.0377	0.0444	0.0467*	0.0480*	0.0403	0.0488*	0.0446	0.0478*	0.0425	0.0520*	0.0478*	0.0490*
	WOPG	0.0250	0.0382	0.0439	0.0462*	0.0326	0.0450	0.0431	0.0477*	0.0359	0.0500*	0.0464*	0.0498*
	WR	0.0630	0.0542*	0.0532*	0.0518*	0.0586	0.0543*	0.0475*	0.0500*	0.0565	0.0556	0.0495*	0.0512*
	LMH	0.0696	0.0537*	0.0522*	0.0509*	0.0618	0.0565	0.0477*	0.0498*	0.0571	0.0573	0.0499*	0.0497*
	LMOPG	0.0747	0.0597	0.0561	0.0527*	0.0641	0.0568	0.0487*	0.0508*	0.0545	0.0561	0.0487*	0.0517*
	LMR	0.0749	0.0557	0.0513*	0.0515*	0.0707	0.0596	0.0502*	0.0499*	0.0676	0.0617	0.0521*	0.0523*
β =1.5	LR	0.0577	0.0497*	0.0528*	0.0465*	0.0559	0.0499*	0.0496*	0.0561	0.0506*	0.0540*	0.0501*	0.0504*
	WH	0.0374	0.0433	0.0502*	0.0453	0.0449	0.0446	0.0475*	0.0550	0.0426	0.0504*	0.0478*	0.0497*
	WOPG	0.0227	0.0354	0.0471*	0.0431	0.0331	0.0412	0.0459*	0.0515*	0.0350	0.0450	0.0472*	0.0496*
	WR	0.0746	0.0588	0.0579	0.0514*	0.0677	0.0576	0.0519*	0.0583	0.0596	0.0573	0.0537*	0.0518*
	LMH	0.0589	0.0489*	0.0528*	0.0460*	0.0561	0.0501*	0.0499*	0.0560	0.0508*	0.0536*	0.0498*	0.0501*
	LMOPG	0.0943	0.0648	0.0626	0.0526*	0.0722	0.0595	0.0521*	0.0593	0.0612	0.0586	0.0543*	0.0514*
	LMR	0.0545	0.0470*	0.0520*	0.0449	0.0583	0.0502*	0.0502*	0.0545	0.0553	0.0528*	0.0520*	0.0516*

Note: the asterisks denote the nominal 0.05 rejection rate being contained by a normal approximation 95% confidence interval. LR = likelihood ratio test, WH = Wald test based on Hessian, WOPG = Wald test based on OPG matrix, WR = Wald test based on robust covariance matrix, LMH = Lagrange multiplier test based on Hessian, LMOPG = Lagrange multiplier test based on OPG matrix, LMR = Lagrange multiplier test based on robust covariance matrix.

Table 2. Percentage of correctly predicted choices for $\gamma = 0$ and N=100.

	J=2	J=3	J=5
β=0.5	63.2	47.7	33.1
β=1	72.5	59.6	46.3
β=1.5	78.7	68.2	56.7

Table 3. Simulated rejection rates at the 5% significance level for the null hypothesis of homoscedasticity. Results for γ = 0.1.

		J=2				J=3				J=5			
		N=100	N=250	N=500	N=1000	N=100	N=250	N=500	N=1000	N=100	N=250	N=500	N=1000
β =0.5	LR	0.0781	0.0825	0.1119	0.1630	0.0786	0.0942	0.1227	0.2049	0.0794	0.1008	0.1476	0.2569
	WH	0.0448	0.0705	0.1048	0.1597	0.0530	0.0869	0.1218	0.2038	0.0628	0.0962	0.1456	0.2559
	WOPG	0.0234	0.0604	0.0991	0.1567	0.0385	0.0813	0.1198	0.2016	0.0491	0.0908	0.1443	0.2545
	WR	0.0805	0.0843	0.1107	0.1656	0.0810	0.0962	0.1262	0.2082	0.0864	0.1059	0.1514	0.2600
	LMH	0.1155	0.1091	0.1269	0.1730	0.1159	0.1167	0.1373	0.2133	0.1159	0.1195	0.1577	0.2659
	LMOPG	0.0726	0.0791	0.1106	0.1640	0.0728	0.0896	0.1246	0.2052	0.0734	0.0969	0.1450	0.2553
	LMR	0.1718	0.1444	0.1461	0.1841	0.1680	0.1466	0.1576	0.2281	0.1683	0.1502	0.1782	0.2781
β =1	LR	0.0820	0.1124	0.1762	0.2901	0.0882	0.1461	0.2438	0.4193	0.1043	0.1872	0.3228	0.5663
	WH	0.0548	0.1001	0.1682	0.2850	0.0715	0.1367	0.2378	0.4161	0.0883	0.1800	0.3174	0.5650
	WOPG	0.0367	0.0872	0.1619	0.2823	0.0570	0.1282	0.2273	0.4117	0.0761	0.1699	0.3134	0.5602
	WR	0.0829	0.1158	0.1762	0.2903	0.0906	0.1477	0.2465	0.4217	0.1079	0.1875	0.3239	0.5667
	LMH	0.0958	0.1178	0.1799	0.2922	0.0977	0.1508	0.2453	0.4205	0.1130	0.1913	0.3257	0.5679
	LMOPG	0.1015	0.1227	0.1821	0.2947	0.0977	0.1506	0.2486	0.4224	0.1064	0.1867	0.3226	0.5661
	LMR	0.1021	0.1186	0.1804	0.2917	0.1104	0.1532	0.2476	0.4216	0.1280	0.1983	0.3309	0.5688
β =1.5	LR	0.0913	0.1236	0.2041	0.3439	0.0992	0.1728	0.2907	0.5116	0.1236	0.2274	0.4133	0.6870
	WH	0.0636	0.1121	0.1962	0.3382	0.0822	0.1619	0.2835	0.5084	0.1079	0.2177	0.4077	0.6847
	WOPG	0.0402	0.0956	0.1824	0.3281	0.0610	0.1486	0.2696	0.4990	0.0894	0.2051	0.3970	0.6771
	WR	0.1021	0.1303	0.2075	0.3477	0.1110	0.1802	0.2982	0.5140	0.1311	0.2338	0.4186	0.6881
	LMH	0.0909	0.1218	0.2036	0.3433	0.1001	0.1717	0.2902	0.5115	0.1229	0.2269	0.4131	0.6864
	LMOPG	0.1286	0.1448	0.2154	0.3507	0.1224	0.1839	0.3011	0.5154	0.1343	0.2332	0.4181	0.6882
	LMR	0.0847	0.1175	0.1982	0.3368	0.0980	0.1708	0.2827	0.5088	0.1265	0.2300	0.4102	0.6841

LR = likelihood ratio test, WH = Wald test based on Hessian, WOPG = Wald test based on OPG matrix, WR = Wald test based on robust covariance matrix, LMH = Lagrange multiplier test based on Hessian, LMOPG = Lagrange multiplier test based on OPG matrix, LMR = Lagrange multiplier test based on robust covariance matrix.

Table 4. Simulated rejection rates at the 5% significance level for the null hypothesis of homoscedasticity. Results for $\gamma = 0.25$.

		J=2				J=3				J=5			
		N=100	N=250	N=500	N=1000	N=100	N=250	N=500	N=1000	N=100	N=250	N=500	N=1000
β =0.5	LR	0.1372	0.2375	0.4057	0.6766	0.1616	0.3066	0.5551	0.8292	0.1940	0.3916	0.6562	0.9138
	WH	0.0824	0.2084	0.3954	0.6724	0.1184	0.2919	0.5479	0.8274	0.1581	0.3825	0.6541	0.9137
	WOPG	0.0405	0.1842	0.3830	0.6683	0.0795	0.2718	0.5418	0.8274	0.1259	0.3632	0.6447	0.9153
	WR	0.1393	0.2317	0.4019	0.6725	0.1618	0.3070	0.5496	0.8230	0.1969	0.3915	0.6525	0.9095
	LMH	0.1766	0.2813	0.4338	0.6882	0.2040	0.3442	0.5786	0.8363	0.2397	0.4309	0.6728	0.9171
	LMOPG	0.1304	0.2294	0.4000	0.6721	0.1483	0.3000	0.5460	0.8207	0.1787	0.3748	0.6448	0.9077
	LMR	0.2329	0.3262	0.4668	0.7054	0.2659	0.3975	0.6105	0.8498	0.3067	0.4765	0.6989	0.9250
β =1	LR	0.2027	0.4148	0.6895	0.9333	0.2812	0.5901	0.8777	0.9914	0.3832	0.7479	0.9603	0.9999
	WH	0.1456	0.3869	0.6792	0.9320	0.2398	0.5717	0.8739	0.9913	0.3478	0.7377	0.9595	0.9999
	WOPG	0.0988	0.3527	0.6574	0.9252	0.1910	0.5436	0.8641	0.9907	0.3025	0.7196	0.9568	0.9997
	WR	0.1906	0.4146	0.6859	0.9352	0.2810	0.5888	0.8729	0.9908	0.3766	0.7442	0.9584	0.9998
	LMH	0.2250	0.4256	0.6933	0.9336	0.2986	0.5973	0.8792	0.9915	0.3964	0.7521	0.9605	0.9999
	LMOPG	0.2268	0.4288	0.6946	0.9353	0.2920	0.5965	0.8752	0.9910	0.3784	0.7404	0.9600	0.9998
	LMR	0.2296	0.4207	0.6871	0.9318	0.3129	0.5950	0.8771	0.9915	0.4122	0.7594	0.9608	0.9998
β =1.5	LR	0.2329	0.4847	0.7752	0.9709	0.3384	0.6833	0.9333	0.9976	0.4713	0.8517	0.9883	1.0000
	WH	0.1791	0.4576	0.7644	0.9702	0.2970	0.6687	0.9309	0.9975	0.4371	0.8438	0.9879	1.0000
	WOPG	0.1209	0.4056	0.7374	0.9639	0.2333	0.6330	0.9188	0.9975	0.3785	0.8254	0.9855	1.0000
	WR	0.2392	0.4965	0.7775	0.9719	0.3465	0.6875	0.9342	0.9980	0.4766	0.8518	0.9886	1.0000
	LMH	0.2331	0.4822	0.7743	0.9707	0.3370	0.6814	0.9327	0.9975	0.4689	0.8505	0.9884	1.0000
	LMOPG	0.2818	0.5129	0.7853	0.9716	0.3651	0.6948	0.9355	0.9981	0.4834	0.8546	0.9892	1.0000
	LMR	0.2167	0.4578	0.7576	0.9659	0.3272	0.6683	0.9266	0.9978	0.4612	0.8437	0.9868	1.0000

LR = likelihood ratio test, WH = Wald test based on Hessian, WOPG = Wald test based on OPG matrix, WR = Wald test based on robust covariance matrix, LMH = Lagrange multiplier test based on Hessian, LMOPG = Lagrange multiplier test based on OPG matrix, LMR = Lagrange multiplier test based on robust covariance matrix.

Table 5. Simulated rejection rates at the 5% significance level for the null hypothesis of homoscedasticity. Results for γ = 0.5.

		J=2				J=3				J=5			
		N=100	N=250	N=500	N=1000	N=100	N=250	N=500	N=1000	N=100	N=250	N=500	N=1000
β =0.5	LR	0.3468	0.6879	0.9271	0.9983	0.4688	0.8423	0.9871	0.9999	0.5861	0.9355	0.9972	1.0000
	WH	0.2281	0.6468	0.9205	0.9983	0.3783	0.8270	0.9861	0.9999	0.5255	0.9310	0.9973	1.0000
	WOPG	0.1195	0.6050	0.9184	0.9984	0.2778	0.8115	0.9872	0.9999	0.4586	0.9280	0.9977	1.0000
	WR	0.3166	0.6542	0.9165	0.9978	0.4383	0.8176	0.9842	0.9999	0.5560	0.9230	0.9962	1.0000
	LMH	0.3436	0.6944	0.9317	0.9984	0.4676	0.8443	0.9869	0.9999	0.5846	0.9333	0.9972	1.0000
	LMOPG	0.3277	0.6644	0.9159	0.9978	0.4297	0.8147	0.9837	0.9999	0.5355	0.9172	0.9959	1.0000
	LMR	0.4062	0.7336	0.9429	0.9987	0.5373	0.8674	0.9900	1.0000	0.6465	0.9457	0.9982	1.0000
β =1	LR	0.5532	0.9124	0.9971	1.0000	0.7406	0.9872	1.0000	1.0000	0.8787	0.9989	1.0000	1.0000
	WH	0.4444	0.8986	0.9969	1.0000	0.6850	0.9857	1.0000	1.0000	0.8552	0.9986	1.0000	1.0000
	WOPG	0.3195	0.8716	0.9952	1.0000	0.6074	0.9809	0.9999	1.0000	0.8152	0.9981	1.0000	1.0000
	WR	0.5089	0.9041	0.9970	1.0000	0.7126	0.9867	1.0000	1.0000	0.8610	0.9984	1.0000	1.0000
	LMH	0.5692	0.9142	0.9972	1.0000	0.7512	0.9876	1.0000	1.0000	0.8825	0.9989	1.0000	1.0000
	LMOPG	0.5766	0.9165	0.9969	1.0000	0.7384	0.9864	1.0000	1.0000	0.8647	0.9984	1.0000	1.0000
	LMR	0.5669	0.9087	0.9965	1.0000	0.7544	0.9863	1.0000	1.0000	0.8893	0.9986	1.0000	1.0000
β =1.5	LR	0.6198	0.9467	0.9992	1.0000	0.8135	0.9945	1.0000	1.0000	0.9339	0.9999	1.0000	1.0000
	WH	0.5313	0.9390	0.9989	1.0000	0.7745	0.9935	1.0000	1.0000	0.9221	0.9999	1.0000	1.0000
	WOPG	0.3873	0.9057	0.9981	1.0000	0.6799	0.9901	1.0000	1.0000	0.8817	0.9998	1.0000	1.0000
	WR	0.6068	0.9496	0.9992	1.0000	0.8044	0.9948	1.0000	1.0000	0.9313	1.0000	1.0000	1.0000
	LMH	0.6178	0.9450	0.9990	1.0000	0.8082	0.9943	1.0000	1.0000	0.9326	0.9999	1.0000	1.0000
	LMOPG	0.6736	0.9545	0.9993	1.0000	0.8290	0.9955	1.0000	1.0000	0.9350	1.0000	1.0000	1.0000
	LMR	0.5728	0.9282	0.9985	1.0000	0.7808	0.9923	1.0000	1.0000	0.9252	0.9999	1.0000	1.0000

LR = likelihood ratio test, WH = Wald test based on Hessian, WOPG = Wald test based on OPG matrix, WR = Wald test based on robust covariance matrix, LMH = Lagrange multiplier test based on Hessian, LMOPG = Lagrange multiplier test based on OPG matrix, LMR = Lagrange multiplier test based on robust covariance matrix.