Mixed motives in a Cournot game

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Abstract

The paper analyzes a Cournot model with two types of firms: Maximizers of profits and maximizers of relative payoffs. It is shown that the equilibrium is located somewhere between the regular Cournot-Nash equilibrium and the competitive Walrasian (or Bertrand-) equilibrium.

1 Introduction

The Cournot model of simultaneous oligopolistic quantity choice is one of the classic workhorses of economic theory. Since its origin in 1838, countless variations of the original model have been brought up. More or less all aspects of the model have been changed, varied and re–organized, analyzed and re–analyzed.[1] What has very rarely been looked at is the aspect of the firms' behavioral motives: What happens if firms have aims other than the mere maximization of their profits? Apart from 'classical' profit maximization, there is another out-standing way of behavior: maximization of relative payoff, meaning that a firm aims to have higher profits than the competitors. Besides the fact that individuals may hold certain preferences about relative payoffs (Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000), there are many more reasons why a firm might concentrate on being better than the others, instead of just trying to be as profitable as possible. One frequently named reason is the firm's wish to increase its market share, which can serve as a means of pushing other firms out of the market or to prevent market entry. A second possible reason for maximization of relative payoffs is the fact that managers are paid due to relative performance of their firm: The manager of the largest firm in the market gets the highest pay. A third reason is a lack of information. Vriend (2000) shows that firms that cannot rely on private information and are forced to determine their production quantity by mimicking other firms' decisions are de–facto maximizers of relative payoff. Finally, it seems worthwhile noting that, in a somewhat broader sense, it is the Bertrand model of oligopoly that represents the most severe model of maximizing relative payoff by maximizing the market share.

Schaffer (1989) was probably the first author to analytically analyze firm behavior in a Cournot model in an evolutionary context. Referring to the concept of spite from evolutionary biology (Hamilton, 1970), he shows that there is a way of unilaterally deviating from a Cournot equilibrium that decreases the profit of the deviator, but at the same time decreases the other firms' profits even more. Given a force that 'selects for' the firm with the highest profit, the deviator will be better off than the others. The Cournot equilibrium is not 'stable' in an evolutionary sense. The evolutionarily stable solution results from a process of every firm trying to be ahead of every other one (maximizing relative payoff), which in the Cournot model results in the Walrasian (competitive market) equilibrium. Schaffer shows this for a model with zero costs.

For the basic evolutionary concept of 'being better than the others', it does not matter if a firm's 'relative payoff' means the ratio of its own payoff to the total payoff of all firms or if it means the difference between the firm's payoff and the

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[1]For an instructive survey on oligopoly theory, see Shapiro (1989).

average payoff of all firms. The latter concept, which should more aptly be named 'differential payoff' is the one used by Schaffer (1988, 1989).

In an important subsequent paper, Vega-Redondo (1997), using differential payoffs, replicates Schaffer's results in a dynamic framework, while Riechmann (2006) finds the same results for a more general class of Cournot games, again using differential payoffs. Relative payoffs in form of ratios are in frequent use in basic evolutionary dynamics, but turn into the differential formulation as soon as these dynamics take place in continuous time (see, e.g. Weibull 1995; Vega-Redondo 1996; Samuelson 1997; Fudenberg and Levine 1998).

Thus, the results of two extreme forms of Cournot models are quite clear: If all firms follow the classical motive of profit maximization, the result will be the Cournot equilibrium. If all firms maximize relative payoff, they will all end up in the Walrasian equilibrium of a competitive market (which, in turn, is identical to the oligopolistic Bertrand equilibrium). What has not been analyzed yet is the question of what happens if in the same market there are both types of firms, maximizers of absolute payoff as well as maximizers of relative payoff.

In the field of experimental economics, a finding common to most Cournot experiments is the one that the experimental outcome is 'usually more competitive than the Cournot prediction' (Holt, 1995, p. 367). This paper will show that this finding can be explained by a heterogeneity of individuals' motives in the game: As soon as both, maximizers of absolute *and* maximizers of relative payoff are active in the same market, the resulting equilibrium must necessarily be located somewhere in between the oligopolistic Cournot- and the competitive Walrasian outcome.

The paper proceeds as follows. In the second section, the oligopolistic model is introduced. Section three analyzes a duopolistic version of the model, before the fourth section presents the general model of mixed motives in a Cournot game. The paper ends with a summary.

2   The Model

The basic model is the following: Market demand is given by

$$D = 1 - p, \tag{1}$$

with p giving the (market) equilibrium price. Let s_i denote the quantity of firm i. Firms must supply non–negative quantities. Aggregate supply, S, is given as the sum of the supplied quantities of the n firms involved, $S = \sum_{i=1}^{n} s_i$. The equilibrium price, p, results as

$$p = 1 - \sum_{i=1}^{n} s_i. \tag{2}$$

Assume that the total of all firms' joint capacities are too low to reach or even exceed autonomous demand, $S < 1$, such that prices will be positive.

Let the n firms have identical quadratic cost functions $C(\cdot)$,

$$C(s_i) = \frac{1}{2} s_i^2. \tag{3}$$

Fixed costs are assumed to be zero.

The (absolute) profit of each firm i is given by

$$\pi_i(s_i, S_{-i}) = p\, s_i - C(s_i) = (1 - S)\, s_i - \frac{1}{2} s_i^2, \tag{4}$$

where S_{-i} gives the aggregate quantity of all firms except of i, $S_{-i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} s_j$.

Relative payoff to firm i will be defined in the tradition of evolutionary game theory (Samuelson 1997, p. 66; Weibull 1995, pp. 72–74) as the difference between i's absolute payoff and the average absolute payoff off all firms, $\overline{\pi} = \frac{1}{n} \sum_{j=1}^{n} \pi_j$:

$$\pi_i^r(s_i, S_{-i}) = \pi_i(s_i, S_{-i}) - \overline{\pi}(s_i, S_{-i}), \tag{5}$$

$$= \frac{n-1}{n}\left((1 - S)\, s_i - \frac{1}{2} s_i^2 \right) - \frac{1}{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \left((1 - S)\, s_j - \frac{1}{2} s_j^2 \right). \tag{6}$$

3 Duopoly

For a start, let us take a look at a duopolistic version of the model. For only two firms in the market, A and R, (4) becomes

$$\pi_i(s_i, s_{-i}) = (1 - s_{-i})\, s_i - \frac{3}{2} s_i^2, \quad i, -i \in \{A, R\}. \tag{7}$$

From this, the reaction function for firm A, aiming to maximize absolute payoffs, becomes

$$s_A^\star = \frac{1}{3}(1 - s_{-i}). \tag{8}$$

A maximizer of relative payoff, though, derives the reaction function from the duopolistic version of (5), which is

$$\pi_i^r(s_i, s_{-i}) = \frac{1}{2}\left[\pi_i(s_i, s_{-i}) - \pi_{-i}(s_i, s_{-i}) \right]. \tag{9}$$

Maximizing (9) with respect to s_i results in the reaction function for firm R, maximizing relative payoff:

$$s_R^\star = \frac{1}{3}. \tag{10}$$

This is in fact identical to the Walrasian (competitive market) equilibrium quantity: A firm neglecting its influence on the equilibrium price and consequently using the rule 'produce the quantity that equates the price to your marginal costs' to determine the output level will produce exactly $s_R^\star$. (A Bertrand model would of course result in the same equilibrium quantity.)

It is remarkable to see that the reaction function for relative payoffs (10) is a degenerate function. For a maximizer of relative payoffs in a duopoly, producing the quantity $s_R^\star$ is the optimum strategy as long as the opponent is restricted to producing quantities that keep prices positive (which is guaranteed by the assumption that $\sum_i s_i < 1$): No matter what the opponent does, a maximizer of relative payoffs should produce the Walrasian quantity. Note, thought, that this is a special trait of the duopolistic case. As soon as there are more than two firms involved, no type of firms has a constant best strategy any more. (See equation (15) below, which shows the general best response function for an R–type firm.)

The equilibrium is easily derived as

$$s_A^\star = \frac{2}{9}, \quad s_R^\star = \frac{1}{3}. \tag{11}$$

Obviously, $s_A^\star < s_R^\star$. The R–firm produces a higher quantity than the A–firm. Moreover, it can be shown that the relative–payoff–maximizer has higher absolute payoff. (Of course it has, because it maximized the difference.)

$$\pi_R\left(s_R^\star, s_A^\star\right) > \pi_A\left(s_A^\star, s_R^\star\right). \tag{12}$$

Thus, the maximizer of relative payoff does exactly this: She maximizes her relative payoff. All that is left to do for the maximizer of absolute payoff is to find his best response to the strategy of his opponent. The maximizer of absolute payoff does indeed maximize his payoff given the relative-payoff-maximizers quantity.

Considering this outcome, it might be asked why firm A does not switch to using R's strategy, too. The answer to this question is straightforward: By switching from $s_A^\star$ to $s_R^\star$, he reduces his (absolute) payoff to a level less than his previous payoff from playing s_A (This should of course be obvious from the reaction function (8).):

$$\pi_A\left(s_R^\star, s_R^\star\right) < \pi_A\left(s_A^\star, s_R^\star\right). \tag{13}$$

Moreover, following the usual definition of efficiency as a measure in absolute payoffs, the resulting equilibrium is inefficient for the firms.[2] Still, it should be

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[2]Of course, if we looked at a broader measure of efficiency like the sum of producers' and consumers' surpluses, efficiency would probably rise compared to the original Cournot situation. An efficient state for the producers would have both players use the standard Cournot-Nash quantity or even collude on the monopolistic quantity.

kept in mind that *absolute* payoff is not what firm R cares for, such that the regular measures of welfare might be inadequate in this model.

This result implies a structure that holds true for the general case of the n–player model. As will be shown further down in this paper, in the general case, too, maximizers of relative payoff will at the same time achieve higher payoffs than maximizers of absolute payoff.

All in all, the model subsumes at three different outcomes. It has been shown before (Riechmann, 2006) that, given both players aim to maximize absolute payoffs, the result will be the usual Cournot equilibrium, but if both players care for relative payoffs instead, the result will be Walrasian. If players hold different motives, the result will be the one presented in (11). A special case of this equilibrium bears a nice interpretation. For the case of no variable costs ($\delta = 0$), the result is equal to a Stackelberg equilibrium where the R–type firm is the Stackelberg leader and the A–type firm is the follower.[3] This outcome derives from the fact that in the duopolistic case, the R–type firm has an optimum strategy it needs not condition on what the A–type firm will do. In a strategic sense, this implicitly makes the R–firm the Stackelberg–leader, who (trivially) decides first. The A–type 'follows' by playing a best response.

4   The General Case

The derivation of respective results for the general n–player case is not complicated, but involves some rather tedious computation. In order to preserve readability, this section only gives the most important results, while the technical details are postponed to the appendix.

From (4), the reaction function for A–type firm number j derives as

$$s_{A,j}^{\star} = \frac{1}{3}\left(1 - S_{-j}\right),\tag{14}$$

with S_{-j} giving the total quantity minus (A–type) firm j's quantity, $S_{-j} = S - s_{A,j}$.

The respective function for R–type firm k derives from (6):

$$s_{R,k}^{\star} = \frac{1}{3}\left(1 - \frac{n-2}{n-1}S_{-k}\right),\tag{15}$$

with S_{-k} giving the total quantity minus firm k's quantity.

Note that the reaction function of R–types explicitly contains the number of firms while the reaction function of A–types does not. The reason for this is straightforward. A–types effectively play against the rest of the economy's supply side, while R–types effectively play against every single supplier.

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[3]I am thankful to a referee to point out this fact.

The equilibrium will be a semi–symmetric one (an equilibrium with all players of the same type behaving identically). With n_R giving the number of R–type firms, the equilibrium quantities result as

$$s_A^\star = \frac{2n - n_R - 1}{2(n^2 - 1) + 3(n - n_R)}, \tag{16}$$

$$s_R^\star = \frac{3n - n_R - 2}{2(n^2 - 1) + 3(n - n_R)}. \tag{17}$$

Notably, the equilibrium quantities are determined by both the size of the supply side of the market (i.e. the number of firms, n) and the composition of the supply side (measured by the number of R–types, n_R). There are no 'dominant' strategies any more.

All the other results remain true in the n–player game. Again, R–type firms produce a higher quantity than A–type firms, such that R–type firms are better off than A–type firms even in terms of absolute payoff. Again, switching from $s_A^\star$ to $s_R^\star$ is not worthwhile.

Again, the result in (16) and (17) includes two special cases, namely the Walrasian and the Cournot equilibrium. For a market with only A–type firms, the equilibrium quantity becomes

$$s_A^\star(n_R = 0) = \frac{1}{n + 2}, \tag{18}$$

the Cournot equilibrium quantity.

In a market with only R–type firms, the individual equilibrium quantity is

$$s_R^\star(n_R = n) = \frac{1}{n + 1}, \tag{19}$$

which is the Bertrand equilibrium quantity and the Walrasian competitive market equilibrium quantity.

These two special cases represent the limiting cases for the model. The more R–types there are in the market, the more the market tends to the Bertrand/Walrasian outcome. The more A–types there are, the closer the result will be to the Cournot outcome. Generally, the result will always fall into the range between (including) the Cournot and the Walrasian equilibrium.

5   Summary

This paper makes one short point: In a Cournot model of oligopolistic quantity choice with regular profit maximizers and maximizers of relative payoffs active at the same time, the resulting equilibrium quantity will generally be located in the range between (including) the Cournot and the Walrasian/Bertrand quantity.

Appendix: The General Case

In the general n–player case, there are n firms, n_R of them R–types and $n - n_R$ A–types. Let $s_{A,j}$ denote the quantity that A–type firm number j produces and use the respective notation $s_{R,k}$ for R–types.

S, the total quantity, is the sum of the individual quantities of A– and R–types:

$$S = \sum_{j=1}^{n-n_R} s_{A,j} + \sum_{k=1}^{n_R} s_{R,k}. \tag{20}$$

From (4), the reaction function for A–type firms number j derives as

$$s_{A,j}^{\star} = \frac{1}{3} - \frac{1}{3} S_{-j}, \tag{21}$$

with S_{-j} giving the total quantity minus (A–type) firm j's quantity, $S_{-j} = S - s_{A,j}$.

The respective function for R–type firms k derives from (6):

$$s_{R,k}^{\star} = \frac{1}{3} - \frac{1}{3} \frac{n-2}{n-1} S_{-k}, \tag{22}$$

with S_{-k} giving the total quantity minus firm k's quantity.

As the equilibrium will be a semi–symmetric one (an equilibrium with all players of the same type behaving identically, i.e. $s_{A,j} = s_{A,j}^{\star} = s_A^{\star}$ for all A–types and $s_{R,k} = s_{R,k}^{\star} = s_R^{\star}$ for all R–types), the equilibrium quantities can be derived from a simple system of two equations. We find that

$$s_A^{\star} = \frac{1}{2+n-n_R} - \frac{n_R}{2+n-n_R} s_R^{\star}, \tag{23}$$

$$s_R^{\star} = \frac{(n-1)}{2n-2n_R+nn_R-1} - \frac{(n-2)(n-n_R)}{2n-2n_R+nn_R-1} s_A^{\star} \tag{24}$$

The equilibrium quantities result as

$$s_A^{\star} = \frac{2n-n_R-1}{2(n^2-1)+3(n-n_R)}, \tag{25}$$

$$s_R^{\star} = \frac{3n-n_R-2}{2(n^2-1)+3(n-n_R)}. \tag{26}$$

From

$$s_A^{\star} = \frac{2n-n_R-1}{3n-n_R-2} s_R^{\star}, \tag{27}$$

it can be seen that

$$s_R^{\star}(n_R = i) > s_A^{\star}(n_R = i) \quad \forall 1 < i \leq n, \tag{28}$$

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and

$$\pi_R\left(n_R = i\right) > \pi_A\left(n_R = i\right) \quad \forall\, 1 < i \leq n. \tag{29}$$

Switching from $s_A^\star$ to $s_R^\star$ is not worthwhile:

$$\pi_R\left(s_A^\star, n_R = i\right) > \pi_R\left(s_R^\star, n_R = i+1\right) \quad \forall\, 1 < i \leq n-1. \tag{30}$$

For a market with only A–type firms, the equilibrium quantity becomes the Cournot quantity:

$$s_A^\star\left(n_R = 0\right) = \frac{1}{n+2}. \tag{31}$$

In a market with only R–type firms, the individual equilibrium quantity is Walrasian:

$$s_R^\star\left(n_R = n\right) = \frac{1}{n+1}. \tag{32}$$

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