How reliable are Taylor rules? A view from asymmetry in the U.S. Fed funds rate

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Abstract

This note raises the issue of whether asymmetry in estimated monetary-policy rules for the U.S. can be a spurious result due to model specification, rather than a robust feature of the estimated rules themselves. I estimate standard - linear - Taylor rules, and test for conditional symmetry using the procedures presented in Bai and Ng (2001a). The results cast doubt on Taylor rules providing a consistent description of the conduct of the Fed.

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“Pending a more careful and convincing appraisal of the loss function, the goal of monetary policy ought to be to make approximately symmetrical errors. That is harder than a more one-sided approach, but whoever said that macroeconomic policy would be easy?”


1 Introduction

The celebrated rule for monetary policy of Taylor (1993) proposes a linear relationship between the short-term interest rate and a set of aggregate variables including inflation and the output gap. The rule is often used both for describing the historical conduct of monetary policy (e.g. see Clarida, Galí and Gertler, 2000), and for producing policy advice (e.g. see Gerlach and Schnabel, 1999).

Some recent contributions have proposed asymmetric formulations for the Taylor rule. Bec, Salem and Collard (2002) and Surico (2002) argue that a nonlinear conditional mean in the Taylor rule generates a plausible description of the conduct of monetary policy in the U.S. These studies suggest that monetary policy is more aggressive during recessions than during cyclical expansions.

This paper investigates the extent to which asymmetric Taylor rules are able to capture the nonlinear features of the Federal funds rate. In particular, I concentrate on whether the alleged asymmetry in estimated policy rules is based on features of the data that are independent from the econometric specification of the rules themselves. Hence, I consider the evidence for conditional asymmetry, namely the asymmetric shape of the residuals from alternative Taylor rules. After estimating different models for the conditional mean, I apply the test developed by Bai and Ng (2001). These tests are asymptotically distribution-free, retain power in small samples, and can be applied irrespective of the degree of dependence in the data.

The monetary policy literature proposes different functional forms for the Taylor rule. Each of them can be derived from theoretical models of optimizing central bank behavior. The true structural model is unknown to the researcher. However, since both inflation and output are strongly autocorrelated, there are alternative specifications for the conditional mean of the Taylor rule that are considered equally valid proxies for the true structural model. In other words, there are different empirical specifications for the Taylor rule that are observationally equivalent.
The reasoning outlined above suggests that the estimated policy rules should provide evidence of asymmetry in a way that is uniform across different specifications of the conditional mean. The results from the tests for conditional asymmetry show that this does not happen. Alternative specifications of the Taylor rule generate conflicting pictures of asymmetry in the Federal funds rate. These considerations indicate that Taylor rules are unable to provide a valid historical account of monetary policy in the U.S.

The paper is organized as follows. Section 2 reviews the statistical tests of Bai and Ng (2001). Section 3 describes the models for conditional mean based on Taylor rules. Section 4 deals with the dataset, and section 5 explains the implications of the test results. Section 6 proposes some final remarks.

2 A short overview of tests for conditional symmetry

The null hypothesis of conditional symmetry is tested on an auxiliary parametric model of the form:

\[ y_t = \mathcal{H}(\Omega_t, \alpha) + \sigma(\Omega_t, \alpha)\varepsilon_t \]

where \( \mathcal{H} \) is the conditional mean, \( \alpha \) is the parameter vector, and \( \sigma^2(\Omega_t, \alpha) \) is the conditional variance of \( y_t \) based on the information set \( \Omega_t := \{y_{t-1}, \ldots, x_t, x_{t-1}, \ldots\} \). There are no assumptions on either the persistence or the i.i.d. behavior of \( y_t \) and \( x_t \).

Testing for conditional symmetry of \( y_t \) is equivalent to testing for symmetry of \( \varepsilon_t \) around zero. The test compares the empirical distribution function of the standardized residuals \( \hat{\varepsilon}_t \) with that of \( -\hat{\varepsilon}_t \):

\[ \hat{\varepsilon} = \frac{y_t - \mathcal{H}(\hat{\Omega}_t, \hat{\alpha})}{\sigma(\hat{\Omega}_t, \hat{\alpha})} \]

The strategy proposed by Bai and Ng (2001) relies on the following:

\[ W_T(z) := \frac{1}{\sqrt{T}} \sum_{t=1}^{T} [\mathcal{I}(\hat{\varepsilon}_t \leq z) - \mathcal{I}(-\hat{\varepsilon}_t \leq z)] \]

where \( \mathcal{I}(\cdot) \) is an indicator function. Since \( W_T(z) \) depends on the difference between the number of \( \varepsilon_t \) and the number of \( -\varepsilon_t \) less than or equal to \( z \), it should take small values for all \( z \) under the null of symmetry. This justifies the fact that two types of test statistics can be constructed, namely one from the positive values of \( z \) and the other from the negative
values.

In the computation of the test statistics, the unobserved error $\varepsilon$ is replaced with the estimated residuals $\hat{\varepsilon}$. This implies that the estimated $\hat{W}_T(z)$ should be used. However, the limiting distribution of $\hat{W}_T(z)$ is not asymptotically distribution-free. In order to avoid this problem, Bai and Ng (2001) construct a martingale transformation. For $z \leq 0$, the following process is computed:

$$S_T = W_T(z) - W_T(0) + \int_z^0 H_T^-(y) dy$$

and for $z \geq 0$:

$$S_T = W_T(z) - W_T(0) + \int_z^0 H_T^+(y) dy$$

The operators $H(\cdot)$ take the form:

$$H_T^- := g_T(y) f_T(y) \left[ \int_{-\infty}^y g_T(k)^2 f_T(k) dk \right]^{-1} \int_{-\infty}^y g_T(k) dW_T(k)$$

$$H_T^+ := g_T(y) f_T(y) \left[ \int_{y}^{\infty} g_T(k)^2 f_T(k) dk \right]^{-1} \int_{y}^{\infty} g_T(k) dW_T(k)$$

where $g_T$ is the estimated density of $\varepsilon_t$, $g_T$ is the estimated $g := \hat{f} / f$, and $\hat{f}$ is the derivative of $f$.

The test statistics are defined in the following way:

$$CS := \max_x |S_T(x)|$$

$$CS^+ := \max_{x \geq 0} |S_T(x)|$$

$$CS^- := \max_{x \leq 0} |S_T(x)|$$

Each test statistic is asymptotically distributed as $\sup_{0 \leq s \leq 1} |B(s)|$, where $B(s)$ is a Brownian motion over $[0,1]$. Bai and Ng (2001) suggest approximating the integrals with summations. Both the densities and their derivatives are computed through a Gaussian kernel with bandwidth $1.06\sigma T^{-1/5}$. 

3
3 Taylor rules as models of the conditional mean

Taylor (1993) proposes a simple and yet powerful way of describing the historical behavior of the Federal funds rate in the U.S.:

\[ rf_t = \Delta_4 p_t + \alpha_1 (y_t - y^*) + \alpha_2 (\Delta_4 p_t - \pi^*) + rf^* + \varepsilon_t \]

This equation states that the Federal Reserve Board sets the policy rate \( r_t \) as a function of the quarterly change in the price level \( \Delta_4 p_t \) and of the difference between current output \( y_t \) and potential output \( y^* \). The central bank responds also to the movements of both the inflation target \( \pi^* \) and the long-run real rate of interest \( r^* \). Several refinements have followed, the main one being the forward-looking specification of Clarida, Galí and Gertler (2000):

\[ rf_t = (1 - \alpha_R) [\alpha_1 (E_t[\Delta_4 p_{t+4}] - \pi^*) + \alpha_2 E_t([y_{t+4}] - y^*) + rf^*] + \alpha_R \beta (L) rf_{t-1} + \varepsilon_t \]

where \( E \) is the expectation operator conditional on the information set at time \( t \). Both policy rules have a linear functional form. They predict changes of the Federal funds rate that are symmetric around the long-run targets.

The specification proposed by Clarida, Galí and Gertler (2000) can be derived from a reduced-form model (e.g. see Svensson, 1997) with a loss function of the central bank that is quadratic in both inflation and the output gap, an aggregate supply and an IS curve that are linear in each determinant. Although analytically more tractable, the linear-quadratic framework has been criticized lately.

In the model of Cukierman (2001), the political accountability of the central bank makes monetary policy decisions vulnerable to the influence of the government. For the political authority, the welfare costs of the recessions are larger than the benefits of the expansions. Thus the government has an incentive to bargain for asymmetric monetary policy over the business cycle.

3.1 The auxiliary models

This section describes the econometric models that are estimated in order to compute the fitted residuals \( \hat{\varepsilon} \). In the empirical exercise I ignore the issues raised by the fact that the long-run targets of inflation and the Federal funds rate are not directly observable. The
first model takes the form of a standard ordinary-least squares (OLS) regression:

$$ffr_t = \alpha_0 + \alpha_{\pi,1}\Delta_4p_{t-1} + \alpha_{y,1}gap_{t-1} + \varepsilon_t$$  \hspace{1cm} (1)$$

This specification is generalized to account for both interest-rate inertia and autocorrelation in the other explanatory variables:

$$ffr_t = \alpha_0 + \sum_{i} \alpha_{\pi,i}\Delta_4p_{t-i} + \sum_{i} \alpha_{y,i}gap_{t-i} + \sum_{i} \alpha_{R,i}ffr_{t-i} + \varepsilon_t$$ \hspace{1cm} (2)$$

I also consider the nonlinear rule studied by English, Nelson, and Sack (2002), which is shown to account properly for the smooth behavior of the Federal funds rate:

$$\tilde{i}_t = \alpha_0 + \alpha_{\pi,1}\Delta_4p_t + \alpha_{y,1}gap_t$$

$$ffr_t = (1 - \alpha_{R,1})\tilde{i}_t + \alpha_{R,1}ffr_{t-1} + \varepsilon_t$$  \hspace{1cm} (3)$$

Equation 3 is estimated through nonlinear least squares (NLS). Finally I introduce a forward-looking variant of the policy of Clarida, Galí and Gertler (2000):

$$ffr_t = \alpha_0 + \alpha_{\pi,1}E[\Delta_4p_{t+4}|\Omega_{t-1}] + \alpha_{y,1}E[y_t|\Omega_{t-1}] + \alpha_{R,1}ffr_{t-1} + \varepsilon_t$$  \hspace{1cm} (4)$$

Models of this form are typically estimated through the generalized method of moments. However, the results from this type of estimators depend heavily on the set of instruments used. For this reason, I ignore the issues raised by the selection of instruments. I use a two-stage least-squares estimator with instruments based on time $t-1$ information.

4 The data

The dataset includes quarterly observations and covers the period 1954(7)-2004(5). The source is the FRED II Database of the Federal Reserve Bank of St. Louis. The series for the Federal funds rate is constructed by taking simple averages of monthly data. I compute the output gap as the percentage deviation of current output $y_t$ from potential $\bar{y}_t$: $gap_t = (y_t - \bar{y}_t)/\bar{y}_t$. The inflation rate is calculated as the four-quarter difference of the implicit price deflator of gross domestic product.

Before estimating the policy rules, all the series are tested for unit roots through the statistical procedures proposed by Perron and Ng (1996, 2001). These tests retain good power properties in small samples. Table 1 shows that the null of a unit-root is hard to
reject for the Federal funds rate. The evidence of non-stationarity of the Federal funds rate precludes the possibility of testing this series for unconditional symmetry.

5 Results

The Taylor rules described in section 3.1 are estimated through a general-to-specific approach for the choice of the lag structure. The statistical properties of the estimated policy rules are broadly consistent with those of previous studies. Most of the coefficients are significant at standard confidence levels, and are of a magnitude comparable to that of well-established estimates (see Clarida, Gali, and Gertler, 2000). The apparent heteroscedasticity of the residuals arises from outliers that are not captured by the conditional means. The normality assumption can be rejected for the residuals of most of the models.

Table 2 reports the results from the tests for conditional symmetry. As mentioned earlier, the Taylor rules used as conditional mean models reflect the idea of symmetric movements of the policy rates around the natural rate of interest. If the Federal funds rate was proved to be unconditionally asymmetric, the contrasting results for the full sample across rules would indicate which specification of the Taylor rule fails to capture properly the shape of the distribution of the Federal funds rate. However, we do not know whether the level of the Federal funds rate exhibits unconditional symmetry. And even if we did, we should consider that it could still be possible for a series to display unconditional symmetry and conditional asymmetry at the same time (see Bai and Ng, 2001). In the end, the only logical way of interpreting the outcome of the tests is that of searching for conditional asymmetry as a spurious result.

The conditional means of equations 1 and 2 are associated with evidence of conditional asymmetry only for the post-Volcker period. The forward-looking rule 4 produces mild support for conditional asymmetry over the full sample. The introduction of the lagged Federal funds rate as an explanatory variable does not change the asymmetric behavior of the residuals in the backward-looking model. The view on asymmetry from the forward-looking rule 4, instead, is strongly affected by the inclusion of a term of interest-rate smoothing. Unlike the other specifications, model 4 without lags of the Federal funds rate detects no conditional asymmetry for the post-Volcker period.
6 Conclusion

This note applies the tests of conditional symmetry proposed by Bai and Ng (2001) to the residuals of estimated monetary policy rules for the U.S. economy. The results are sensitive both to the type of explanatory variables included — backward-looking and forward-looking — and to the estimation method used — OLS/NLS and instrumental variables. The apparent conditional asymmetry of the Federal funds rate is not a robust feature of alternative formulations of the Taylor rule. These findings suggest that conditional asymmetry is a spurious result. This casts some serious doubt on the capability of Taylor rules to provide a consistent description of U.S. monetary policy.
References


Table 1: Unit-root tests

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<th>$f f t$</th>
<th>$gap_t$</th>
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<td><strong>GLS detrending</strong></td>
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<tr>
<td>Phillips-Perron</td>
<td>-3.074</td>
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<td>[-5.856]</td>
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<td>-2.071</td>
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<td></td>
<td>[0.265]</td>
<td>[0.147]*</td>
<td>[0.391]</td>
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<td>Point-optimal test</td>
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<td>1.133*</td>
<td>13.141</td>
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<tr>
<td></td>
<td>[16.527]</td>
<td>[3.958]*</td>
<td>[30.974]</td>
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<td>[15.578]</td>
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<td>[-1.229]</td>
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<td><strong>OLS detrending</strong></td>
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<tr>
<td>Phillips-Perron</td>
<td>-14.474*</td>
<td>-23.978*</td>
<td>-4.595</td>
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<td>-12.772</td>
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<td>Said-Dickey-Fuller</td>
<td>-2.499</td>
<td>-3.477</td>
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<td></td>
<td>[-2.359]</td>
<td>[-3.477]</td>
<td>[-1.391]*</td>
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Legend: Results without brackets are based on a constant only. The figures in brackets are computed from models with both a constant and a linear time trend. The Phillips-Perron test is described in Phillips and Perron (2000), the modified Phillips-Perron are all outlined in Perron and Ng (1996), the point-optimal test is from Elliott and Stock (1996) and is amended in Perron and Ng (2001) together with the test of Sargan and Bhargava (1983). The distinction between GLS and OLS detrending can be found in Perron and Ng (2001). All the tests: *significant at the 5% level.
Table 2: Test results for conditional symmetry

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<tr>
<td>CS</td>
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<td>0.777</td>
<td>3.887*</td>
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<tr>
<td>CS^+</td>
<td>1.545</td>
<td>0.548</td>
<td>2.103***</td>
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<tr>
<td>CS^-</td>
<td>1.149</td>
<td>0.777</td>
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<tr>
<td>CS</td>
<td>1.139</td>
<td>0.739</td>
<td>2.414**</td>
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<td>0.605</td>
<td>2.414**</td>
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<tbody>
<tr>
<td>CS</td>
<td>2.401**</td>
<td>0.789</td>
<td>2.885*</td>
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<td>CS^+</td>
<td>0.893</td>
<td>0.789</td>
<td>1.340</td>
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<td>CS^-</td>
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<td>0.788</td>
<td>2.885*</td>
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<tbody>
<tr>
<td>CS</td>
<td>2.420**</td>
<td>2.431**</td>
<td>1.283</td>
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<tr>
<td>CS^+</td>
<td>2.420**</td>
<td>1.444</td>
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<td>CS^-</td>
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<td>2.431**</td>
<td>1.205</td>
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<tr>
<td>CS</td>
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<td>1.408</td>
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<tr>
<td>CS^+</td>
<td>0.510</td>
<td>1.120</td>
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<td>CS^-</td>
<td>3.153*</td>
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Legend: The critical values for the test statistics are 2.78, 2.21 and 1.91 at the 1%, 5% and 10% significance levels respectively. The tests are based on the full of conditional symmetry. All the tests: *rejection at the 1% level; **rejection at the 5% level; ***rejection at the 10% level.