# Multiple Shareholder Control as a Signaling Mechanism

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# Abstract

We show that multiple shareholder control (MSC) can arise as a signaling mechanism. A controlling shareholder can sell her shares because of personal liquidity needs or because of bad fundamentals of the asset she owns. Because the market is unable to distinguish the motivation for sale and the seller's liquidity risk type, ex ante returns of investors with high liquidity risk will be adversely affected. With MSC, shocks to the fundamentals of the asset can be more easily disentangled from the liquidity shocks of the individual owners. As a result, ex ante returns will come closer to true returns and increase incentives of investors with high liquidity risk to acquire controlling shares.

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## 1. Introduction

Variations in firm ownership structures have been studied mostly along one dimension – variations in ownership concentration. Higher concentration is believed to give the owners more ability and willingness to discipline the managers (Vischny, Schleifer, 1986). Concentration is endogenously determined depending on the characteristics of the firm and the environment where it operates (Demsetz and Lehn, 1985, Bolton and Von Thadden, 1998, La Porta, et al, 2000).<sup>1</sup>

We consider another dimension - the number of shareholders who control the firm. What can explain variations in the number of controlling shareholders? Many firms, especially in markets with weaker corporate control, are controlled by multiple large shareholders. According to Laeven and Levine (2004), about one-third of 865 public firms in 13 Western European countries have two or more owners with 10 percent or more of the voting rights each. In addition, multiple shareholder control is common in private firms (Ball and Shivkumar, 2004).

There exists a (negative) correlation between ownership concentration and the number of controlling shareholders. However, these are not the same. For example, consider two firms. Firm 1 has one shareholder owning 51% of the firm and the remaining owners owning about 0.16% each. Firm 2 has two shareholders owning 36.06% each with the remaining owners owning about 0.29% each. Concentration is about the same for both, yet it takes only one owner to control firm 1 and two to control firm 2.<sup>2</sup>

There are a few theoretical papers studying the rationale behind ownership structures with multiple shareholder control.<sup>3</sup> In Zwiebel (1995) large investors invest their money across firms in a manner that maximizes benefits from control, understanding that others are acting likewise. Multiple blockholder structures are one of the resulting equilibria of this strategic behavior. In Pagano and Roel (1998), the initial owner who gives up part of his stake in return for financing and cares about his private benefits of control, may choose a multiple blockholder structure as an optimal (from his perspective) monitoring commitment mechanism. Gomes and Novaes (2005) argue that with a larger number of shareholders in the controlling group, ex post bargaining problems within the group may prevent decisions that benefit the controlling group at the expense of the minority shareholders.

In our model, multiple shareholder control (MSC) arises as a signaling mechanism aimed at alleviating the informational asymmetry between the firm and the market. The intuition is the following. There are two types of investors on the market – those with high ex ante liquidity needs and those with low ex ante liquidity needs. Type is private information. There is also information asymmetry about the asset fundamentals between the controlling shareholders and the market. A sale by the *only* controlling shareholder can be motivated by personal liquidity needs or private information about the asset she owns. Market's inability to distinguish the motivation for sale can adversely affect the sales price even if the sale is

<sup>&</sup>lt;sup>1</sup> For a review of the literature on ownership concentration see Short (1994) or Denis and McConell (2003).

<sup>&</sup>lt;sup>2</sup> The Herfindhal index is used to calculate concentration.

<sup>&</sup>lt;sup>3</sup> Few empirical papers look at the relationship between various measures of firm performance and the number of large shareholders (Volpin (2002), Faccio et al (2001), Lehman and Weigand (2000), Gutierrez and Tribo (2004). They report a positive relationship.

purely for liquidity needs. In addition, as the market is unable to distinguish the types of investors, the ex ante return of a high liquidity risk type will be adversely affected. In contrast, MSC can make market's inference problem easier. For example, when there are two controlling shareholders with independent liquidity shocks, a sale by only one of them will be interpreted by the market as liquidity motivated. Hence, in the presence of MSC, shocks to the fundamentals of the asset can be more easily disentangled from the liquidity shocks of the individual owners. MSC will bring ex ante returns closer to the true returns (i.e. when investor types are public information). This, in turn, will increase incentives to acquire controlling shareholders does not invalidate this result.

Firm and investor characteristics will determine the degree of information asymmetry and, hence, the number of controlling shareholders. Degree of information asymmetry will vary from firm to firm depending on factors such as the type of activity or the age of the firm. Investor identity and diversification opportunities can determine liquidity needs of the investors and, hence, the number of controlling shareholders. The efficiency of corporate control regulation will matter too. When there are tough and efficient insider trading rules, acting on private information would be harder and the benefits of MSC identified in this paper would be smaller or non-existent. MSC explanation of this paper would be more relevant for markets with weaker corporate control regulation and/or privately held firms.

The paper is organized the following way. In section 2 we present the model. Subsection 2.1 outlines the benefit of MSC structure arising from its signaling ability. Section 3 discusses the sensitivity of results to collusion. Section 4 concludes.

## 2. The Model

There are three periods: 0, 1 and 2. An investment at date 0 has a gross return of  $\mathbb{R}^{H}$  (good state) or  $\mathbb{R}^{L}$  (bad state) at date 2. Assume that  $\mathbb{E}(\mathbb{R}^{H}) = \mu > 0$  and  $\mathbb{E}(\mathbb{R}^{L}) = \alpha \mu$ ,  $\alpha < 1$ . The probability of  $\mathbb{R}^{H}$  at date 0 is p and the probability of  $\mathbb{R}^{L}$  at date 0 is (1-p). The true date 0 expected return of the investment is  $E^{O} = p \mu + (1-p) \alpha \mu$ . At date 1, the state is observed by the investors, but not by the market.

There are two types of investors at date 0: Type 1 (T1) and Type 2 (T2). With probability u T1 will have a liquidity shock at date 1 and will need to sell her share.<sup>4</sup> The probability of liquidity shock at date 1 for T2 is v, v < u. At date 0 the type of the investor is private information. Assume for simplicity that the investors' liquidity shocks are independent.<sup>5</sup> Every investor can be T1 with probability q and T2 with probability (1-q).

Assume that the market is competitive and parameters are public knowledge at date 0.

### 2.1. The Benefit of MSC

<sup>&</sup>lt;sup>4</sup> Allowing for borrowing will not make a difference. If the owner's share is used as a collateral, then there is asymmetric information between the owner and the lenders about the value of the collateral.

<sup>&</sup>lt;sup>5</sup> Allowing for a correlation of types with each other and/or with returns will weaken, but not destroy, the results.

Let the number of controlling shareholders be n. For now, assume away the possibility of collusion among the controlling shareholders. At date 1, unlike the market, a controlling shareholder knows the date 2 return. If at least one controlling shareholder is not selling at date 1, the market assumes the good state.<sup>6</sup> If everyone is selling at date 1, the return offered by the market is

$$E_{t=I}(R^{M}) = \operatorname{Prob}\left(G/\operatorname{Everyone Sells}\right)\mu + \operatorname{Prob}(B/\operatorname{Everyone Sells})\alpha\mu$$
(1)  
$$= \frac{\operatorname{Pr}ob(G)\operatorname{Pr}ob(All \ Sell/G)}{\operatorname{Pr}ob(All \ Sell)}\mu + \frac{\operatorname{Pr}ob(B)\operatorname{Pr}ob(All \ Sell/B)}{\operatorname{Pr}ob(All \ Sell)}\alpha\mu$$
(1)

where  $\operatorname{Prob}(G)=p$ ,  $\operatorname{Prob}(B)=1-p$ ,  $\operatorname{Prob}(\operatorname{All Sell}/G) = (qu+(1-q)v)^n$ ,  $\operatorname{Prob}(\operatorname{All Sell}/B)=1$ ,  $\operatorname{Prob}(\operatorname{All Sell}) = p(qu+(1-q)v)^n + (1-p)^7$ For notational simplicity, let A=qu+(1-q)v. After substitutions we have

$$E_{t=1}(R^{M}) = \frac{pA^{n} + (1-p)\alpha}{pA^{n} + (1-p)}\mu$$
(2)

Given an MSC structure with *n* controlling shareholders, let the ex ante return of Type *i* controlling shareholder be  $E_{t=0}^{Ti}(R/n)$ .

## Proposition 1

- (i) the market believes that each of the controlling shareholders can be T1 with probability q and T2 with probability (1-q),
- (ii) each controlling shareholder knows her type and believes that each of the (n-1) controlling shareholders can be T1 with probability q and T2 with probability (1-q),

then

If

 $E_{t=0}^{T_1}(R/n)$  is increasing in *n* with

$$\lim_{n \to \infty} E_{t=0}^{T_1}(R/n) = E^0 = p\mu + (1-p)\alpha\mu$$
(3-a)

and

 $E_{t=0}^{T2}(R/n)$  is decreasing in *n* with

$$\lim_{n \to \infty} E_{t=0}^{T_2}(R/n) = E^0 = p\mu + (1-p)\alpha\mu$$
(3-b)

Proof

<sup>7</sup> Prob(All Sell/G) = 
$$\sum_{k=0}^{n} {n \choose k} q^{k} u^{k} (1-q)^{n-k} v^{n-k} = (qu+(1-q)v)^{n}$$
.

<sup>&</sup>lt;sup>6</sup> Selling is a dominant strategy for a controlling shareholder when the state is bad. If she does not sell in the bad state, she gets  $\alpha\mu$ . If she sells, her return is a linear combination of  $\alpha\mu$  and  $\mu$ .

Type 1

$$E_{t=0}^{T1}(R/n) = \operatorname{Prob}(\operatorname{All sell at date 1}) E_{t=1}(R^M) + \operatorname{Prob}(i \text{ sells \& at least one of the}$$
(3)  
n-1 doesn't sell)  $\mu$  + Prob(i doesn't sell at date 1)  $\mu$ 

where Prob(All sell at date 1) =  $pu(qu+(1-q)v)^{n-1}+(1-p)$ , Prob(*i* sells & at least one of the *n*-1 doesn't sell)=  $pu(1-(qu+(1-q)v)^{n-1})$ , Prob(*i* doesn't sell at date 1) = p(1-u).

Appendix 1 shows that the first derivative of  $E_{t=0}^{T1}(R/n)$  with respect to *n* is positive.  $E_{t=0}^{T1}(R/n)$  is quasiconcave or concave. It can also be verified that  $\lim_{n \to \infty} E_{t=0}^{T1}(R/n) = E^0 = p\mu + (1-p)\alpha\mu$ .

#### Type 2

The derivation of  $E_{t=0}^{T2}(R/n)$  is similar to the previous case. In Appendix 2 we show that  $E_{t=0}^{T2}(R/n)$  is quasiconvex or convex. It can also be verified that  $\lim_{n\to\infty} E_{t=0}^{T2}(R/n) = E^0 = p\mu + (1-p)\alpha\mu$ .

As *n* increases, the ex ante return of T1 increases and the ex ante return of T2 decreases. The inability of the market to distinguish T1 from T2 benefits T2 as the later has a lower risk of liquidity shock. T2 is happier with smaller *n*. As *n* increases, the ex ante return for both types approaches the true ex ante return (i.e. when types are known). If investors' opportunity cost is  $E^{O}$ -g, g>0, then T2 is always willing to invest and T1's incentive to invest is increasing with *n*.

<u>Proposition 2</u> If types are public knowledge, then  $E_{t=0}^{T1}(R/n) = E_{t=0}^{T2}(R/n) = E^{O} = p \mu + (1-p) \alpha \mu$ 

<u>Proof</u> To show that  $E_{t=0}^{T1}(R/n) = E^{O}$ , simply replace q=1 in equation A1.1 in Appendix 1 (the equation for the ex ante return of T1). To show that  $E_{t=0}^{T2}(R/n) = E^{O}$ , simply replace q=0 in equation A2.2 in Appendix 2 (the equation for the ex ante return of T2).

According to proposition 2, if types are public information, the ex ante return of any type is independent of the investor's type and depends only on the characteristics of the investment. This result is similar to Garleanu and Pedersen  $(2003)^8$ . Investor characteristics affect ex ante returns only when there is asymmetric information about types.

Figures 1 and 2 below plot  $E_{t=0}^{T1}(R/n)$  and  $E_{t=0}^{T2}(R/n)$  as functions of *n* for different parameter values.

<sup>&</sup>lt;sup>8</sup> Garleanu and Pedersen (2003) showed that if agents are symmetric ex ante, present value does not depend on future adverse selection – in expectation, the future losses an agent will incur when trading due to liquidity reasons will be offset by gains when trading based on information.



**Fig.1**  $E_{t=0}^{Ti}(R)$  as a function of *n* for  $\mu=1, p=0.51, v=0, \alpha=0$  and different *u* and *q*.

**Fig. 2**  $E_{t=0}^{Ti}(R)$  as a function of *n* for  $\mu=1, p=0.51, v=0, \alpha=0$  and different *u* and *q*.



For both types, as *n* increases, the date 0 expected return converges to  $p \mu = 0.51$ , which is the expected return when there is full information about the types. The speed of the convergence depends on the parameters. Everything else constant, the smaller *q*, the faster is the convergence. Similarly, for given *p* and *q*, the smaller *u*, the faster is the convergence.

In addition to benefits, increasing the number of controlling shareholders can add to costs. For example, MSC can give raise to bargaining problems among controlling shareholders and result in corporate paralysis (Gomes and Novaes, 2005). With positive costs, the number of controlling shareholders will be finite.

# 3. Allowing for Costless Collusion

The above results were unconditional – for any positive p, q, u and v. The possibility of collusion between the controlling shareholders was assumed away. However, if the market believes that the good state is realized if at least one controlling shareholder does not sell, then in the bad state they can collude so that n-1 sell and one does not sell, with the sellers making side payments to the non-seller. Then market's assignment of Prob(G/ at least 1 controlling shareholder doesn't sell)=1 would not be optimal.

Next we allow for the possibility of costless collusion, i.e. there are no bargaining and enforcement costs. Adding transaction costs of collusion will only reinforce our results. We show that even with costless collusion, there is a range of parameter values for which the above model is valid. Let  $\theta = [\theta_1, ..., \theta_n]$  be the share distribution for *n* controlling shareholders, where  $\sum_{i=1}^{n} \theta_i$  is normalized to 1. Let  $\theta_k = min[\theta_1, ..., \theta_n]$ . For notational simplicity, let  $\beta = Prob(G/All)$ 

$$\text{Sell}) = \frac{pA^n}{pA^n + (1-p)}.^9$$

<u>Proposition 3</u> If  $\theta_k > 1 - \beta$ , then

- (i) the market assigns Prob(G/ At least 1 controlling shareholder doesn't sell)=1,
- (ii) In the bad state everyone sells.

<u>Proof</u> Fix Prob(G/ At least 1 controlling shareholder doesn't sell)=1. When at least 1 controlling shareholder does not sell, the market offers  $E_{t=1}(R^M) = \mu$ . If everyone sells, then  $E_{t=1}(R^M) = \beta\mu + (1-\beta)\alpha\mu$ . For these market beliefs, if the control group members collude in the bad state, then the maximum size of the pie to be shared is  $(1-\theta_k)\mu + \theta_k\alpha\mu$ .<sup>10</sup> If they do not collude in the bad state, then the total wealth is  $E_{t=1}(R^M) = \beta\mu + (1-\beta)\alpha\mu$ . The controlling shareholders will not collude if  $(1-\theta_k)\mu + \theta_k\alpha\mu < \beta\mu + (1-\beta)\alpha\mu$  or  $\theta_k > 1-\beta$ .

The intuition for Proposition 3 is the following. Even if collusion is costless, for a range of parameter values the controlling shareholders can be better off if all sell in the bad state instead of colluding and making side payments to each other. These parameter values are publicly observable, so the market has enough information to calculate when collusion is unlikely. For example, if n=2, p=0.8, q=0.6, u=0.6, v=0.4, then  $min[\theta, 1-\theta]$  should be greater than 0.48 – the supporting share distributions of controlling shareholders could be [0.5, 0.5], [0.51, 0.49], etc. (recall that  $\sum_{i=1}^{n} \theta_i$  was normalized to 1). Or, if p=0.8, q=0.9, u=0.8, v=0.7, then  $min[\theta, 1-\theta]$  should be greater than 0.29 – the supporting share distributions could be [0.7, 0.3], [0.6, 0.4], etc.

### 4. Conclusion

Multiple shareholder control (MSC) can arise as a signaling mechanism aimed at reducing the degree of information asymmetry between the firm and the market. A controlling shareholder knows more about the state of the firm than the market does. In addition, investors have privately observed and ex ante varying liquidity risks. If the only controlling

<sup>10</sup> If a shareholder has liquidity needs at date 1, then date 2 consumption has no value for her. Thus, if all need liquidity at date1 and they collude (only 1 sells), they have to share (1- $\theta_k$ )  $\mu$ . If at least 1 does not need liquidity at date 1, then the size of the pie to be shared is (1- $\theta_k$ )  $\mu + \theta_k \alpha \mu$ .

<sup>&</sup>lt;sup>9</sup> Recall that A=qu+(1-q)v.

shareholder sells her shares, the market is unable to distinguish the motivation for sale. The sales price is affected. Moreover, the ex ante return for investors with high liquidity risk is adversely affected, discouraging them from acquiring controlling shares. However, MSC structure makes the market's inference problem easier. With MSC, it becomes easier for the market to disentangle shocks to the fundamentals of the firm from liquidity shocks of controlling shareholders. The ex ante returns come closer to true returns and increase incentives of investors with high liquidity risk to acquire controlling shares.

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# Appendix 1 Deriving the Date 0 Return for T1

Recall A = (qu+(1-q)v). After substituting  $E_{t=1}(\mathbb{R}^M)$  and respective probabilities into (3), we get

$$E_{t=0}^{T_1} (R/n) = \frac{A^n (p(1-p)uA^{-1}(\alpha-1)+p) + (1-p)^2 \alpha + p(1-p)}{pA^n + (1-p)} \mu$$
(A1.1)

It can be verified that as  $n \to \infty$ ,  $E_{t=0}^{T1}(R/n) \to E^{O} = p\mu + (1-p)\alpha\mu$ . Assuming *n* is continuous, the first derivative of (A.1) with respect to *n* will be

$$\frac{\partial E_{t=0}^{T1}(R/n)}{\partial n} = \frac{pA^n(\ln A)(1-p)^2(1-\alpha)(1-uA^{-1})}{(pA^n+(1-p))^2}\mu$$
(A1.2)

where  $(1 - uA^{-1}) = \frac{(1 - q)(v - u)}{qu + (1 - q)v}$ .

Since qu + (1-q)v < 1 and u > v, then  $\frac{\partial E_{t=0}^{T1}(R/n)}{\partial n} > 0$ .

The second derivative of  $E_{t=0}^{T_1}$  (*R*/*n*) with respect to *n* is

$$\frac{\partial^2 E_{t=0}^{T1}(R/n)}{\partial n^2} = \frac{pA^n (\ln A)^2 (1-p)^2 (1-\alpha)(1-uA^{-1})(pA^n+1-p)(1-p-pA^n)}{(pA^n+(1-p))^4}\mu$$
(A1.3)

For small *n* the sign of the above equation depends on the sign of  $(1 - p - pA^n)$ . For large *p*, *q*, *u*, *v* and small *n*,  $(1 - p - pA^n)$  can be negative and since  $(1 - uA^{-1})$  is negative too, (A.3) can be positive. But as *n* increases,  $(1 - p - pA^n)$  becomes positive and (A.3) becomes negative. Thus  $\frac{\partial^2 E_{t=0}^{T1}(R/n)}{\partial n^2}$  is quasiconcave or concave.

# Appendix 2 Deriving the Date 0 Return for T2

 $E_{t=0}^{T_2} (R/n) = \operatorname{Prob}(\operatorname{All sell at date } 1) E_{t=I}(R^M)$   $+ \operatorname{Prob}(i \text{ sells \& at least one of the } n-1 \operatorname{doesn't sell}) \mu$   $+ \operatorname{Prob}(i \operatorname{doesn't sell at date } 1) \mu$ (A2.1)

where Prob(All sell at date 1) =  $pv(qu+(1-q)v)^{n-1}+(1-p)$ , Prob(*i* sells and at least one of n-1 doesn't sell)=  $pv(1-(qu+(1-q)v)^{n-1})$ , Prob(*i* doesn't sell at date 1) = p(1-v).

Denoting A = (qu + (1-q)v), after substitutions, we get

$$E_{t=0}^{T_2} (R/n) = \frac{A^n (p(1-p)vA^{-1}(\alpha-1)+p) + (1-p)^2 \alpha + p(1-p)}{pA^n + (1-p)} \mu$$
(A2.2)

It can be verified that as  $n \to \infty$ ,  $E_{r=0}^{T_2} (R/n) \to E^O = p\mu + (1-p)\alpha\mu$ . Assuming *n* is continuous, the first derivative of (A2.2) with respect to *n* will be

$$\frac{\partial E_{t=0}^{T2}(R/n)}{\partial n} = \frac{pA^n(\ln A)(1-p)^2(1-\alpha)(1-vA^{-1})}{(pA^n+(1-p))^2}\mu$$
(A2.3)
where  $(1-vA^{-1}) = \frac{q(u-v)}{q(u-v)}$ 

where  $(1 - vA^{-1}) = \frac{q(u - v)}{qu + (1 - q)v}$ 

Since qu + (1-q)v < 1 and u > v, then  $\frac{\partial E_{t=0}^{T2}(R/n)}{\partial n} < 0$ .

The second derivative of  $E_{t=0}^{T_2}$  (*R*/*n*) with respect to *n* is

$$\frac{\partial^2 E_{t=0}^{T2}(R/n)}{\partial n^2} = \frac{pA^n (\ln A)^2 (1-p)^2 (1-\alpha)(1-vA^{-1})(pA^n+1-p)(1-p-pA^n)}{(pA^n+(1-p))^4}\mu$$
(A2.4)

For small *n* the sign of the above equation depends on the sign of  $(1 - p - pA^n)$ . For large *p*, *q*, *u*, *v* and small *n*,  $(1 - p - pA^n)$  can be negative and since  $(1 - vA^{-1})$  is positive, (A2.4) can be negative. But as *n* increases,  $(1 - p - pA^n)$  becomes positive and (A2.4) becomes positive too. Thus  $\frac{\partial^2 E_{t=0}^{T2}(R/n)}{\partial n^2}$  is quasiconvex or convex.