E C O N O M I C S   B U L L E T I N

Samuelson machines and the optimal public-private mix

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Abstract

Standard economic analysis assumes the sets of public and private goods to be exogenously given. Yet societies very often choose the public-private mix, using resources to convert seemingly private goods into ones with public goods characteristics and vice versa. In practice, we see a bewilderingly large variety of public-private mixes across societies. This papers advances an analysis of the choice of the public-private mix in the framework of voluntary contributions to public goods provision, by envisaging that, starting from a situation where all goods have private characteristics, some goods can be changed to have public goods characteristics at a cost (by purchasing a "Samuelson machine"). It characterizes the jointly optimal choice of the public-private mix and the efficient supply or not of the public goods in the mix. This characterization generates a number of testable predictions on the public-private mix, and on the prevalence of free riding

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1. Introduction

In standard public economics, the division between private goods and public goods is taken as immutable. Following Samuelson (1954), the assumption that some goods have the technological feature that their consumption is non-rival and non-excludable is part of the specification of the model. Given this division, the analysis then asks about such matters as efficient supply and free riding. Yet in the real world such a neat exogenous division is not observed. Whether the consumption of a good is in fact non-rival and non-excludable is often a social or historical construct rather than given by technology.

For example, defence expenditure is often cited as a typical example of a public good: to defend a county, or to deter an attack on it, is to protect all of its inhabitants. But national defence is a public good because of the artefact of the nation state, one of whose functions is to maintain the integrity of its territory. Before the re-unification of Germany in 1991 an attack on West Germany was not an attack on East Germany. Now it is, and the inhabitants of the former East Germany benefit from defence expenditure by the inhabitants of what was West Germany, and vice versa. Who benefits from whose defence expenditure, i.e. the degree of 'publicness' of defence, depends on the political constitution of the territory. At the international level, signing defence treaties or setting up organisations such as NATO whose central principal is that 'an attack on one is an attack on all' further extends the benefits of defence expenditure.[1] When individuals and countries agree to collective defence arrangements, they convert a previously private good to one which now has the properties of non-rivalry and non-excludability.

A second example concerns the dissemination of information and signals.[2] Television programmes, once produced, are in principle available to everyone: there is no need to produce another programme or to hire additional actors if more people are to view the programme. But for viewers to watch the programme they must be able to receive the TV signal. This requires some form of network, such as transmitters that can relay a signal across long distances, or a cable network. The network acts as a platform, without which the TV programme is a private good, rather like a private theatre performance. With the network in place, the marginal cost of including further viewers is minimal. Thus the network has the potential to transform a private good into a public good. Of course, some forms of network do allow viewers to be excluded: the cable company can cut off your signal if you do not pay. But 'free-to-air' signals via conventional transmitters do not allow this. What we observe, then, is that every society that has access to a dissemination technology has the opportunity to build a platform and to make TV signals a non-rivalrous and non-excludable public good. Some societies choose to avail themselves of this opportunity; others do not. Whether the good is private or public is not exogenously given, but socially determined.

More generally, the "public-private mix" varies considerably across societies. By this we mean that the composition of goods that are supplied through collective action varies greatly, even allowing for the share of the government in national income. For example, local services such as garbage collection demonstrate a broad spectrum between collective and private provision; in some areas it is individualized, in other areas it is a collective enterprise to which the whole community contributes. In poor village economies, there is a

[1] For a review of the economics of alliances and transnational institutions, see Sandler and Hartley (2001).
[2] For an early analysis, see Samuelson (1958).

wide variation in the degree to which irrigation is individualized or collectivized. Even when a good is supplied through collective action, the free rider problem is solved to different degrees in different situations, leading to yet more variations in observed patterns for the supply of the same good (Ostrom, 1990).

The general point is that while in some cases technology is destiny, in others societies can choose whether a good is private or public. In this paper, we develop a framework to analyse such choices and to delineate the costs and benefits of one form of social organization over another. This will allow us begin to identify the factors that determine the public-private mix in the sense intended here, and thus to explain the variations we observe in practice.

To fix ideas, consider the standard model of voluntary contributions to public goods; for example see Bergstrom, Blume and Varian (1986). In this model, an individual's expenditure on the private good benefits only that individual, whereas for public goods each individual's expenditure benefits all individuals. In the most common formulation, each individual benefits from the sum of all individuals' contributions. If there are two individuals and they both contribute \$10, each will enjoy \$20 worth of the public good. In other words, a machine for turning \$10 bills into \$20 bills! Such "Samuelson machines" clearly convey a benefit relative to the alternative of autarchy. If goods can be costlessly converted from private to public, then a society will want all its good to be public, whether or not there is free riding in the supply of the public good. If it can be done costlessly, it is better to eliminate free riding than not; but society is better off in either case. What is important is that the conversion provides a platform for positive externalities, which are beneficial relative to autarchy regardless of whether they are ultimately internalized through further collective action.

In the context of the voluntary contributions to public goods model we thus envisage three scenarios for a typical good. One is that the good in question is private. The second is that the good is public but that the contributions are the Nash equilibrium of a non-cooperative game. The third scenario is that the good is public but the contributions maximize joint welfare - the efficient outcome.[3] The value of converting from private to public thus depends on what sort of collective action takes place after conversion.

Suppose now that the arrangements that need to be established to generate positive externalities are costly: defence treaties need to be agreed on and signed, TV networks have to be put in place. In other words, Samuelson machines are costly. How much would society be willing to pay to get one? If it already had m such machines, how much would it be willing to pay for one more? The answer depends in part on whether society will be able to enforce an efficient outcome once the machine is bought. But suppose that achieving efficiency is itself costly: the institutional conditions for monitoring and penalising free riding do not come free. However, the value of curbing free riding will itself not be independent of the number of Samuelson machines in play. Our society must then solve two interdependent problems—whether or not to curb free riding, and how many machines to buy. It is this joint solution that the paper develops and interprets in the sections to come.

[3] This three way classification leaves out some interesting institutional environments, for example in which the outcome is determined via a political process such as by majority rule. Also, because our Samuelson machines convert pure private goods into pure public goods, we do not consider scenarios involving local public goods (which are non-excludable but with rivalry across localities), or club goods (which are excludable and partially rival); see for example Cornes and Sandler (1986) and Sandler and Tschirhart (1997).

Section 2 presents the basic model: we develop the simplest possible setting in which our basic points can be made cleanly and tractably. Section 3 considers the value of Samuelson machines with and without free riding, and the optimal public-private mix in each setting. Section 4 analyses the joint decision on public-private mix and whether to control free riding. Section 5 concludes the paper with a discussion and interpretation of the key results.

2. Samuelson Machines

Our basic model is essentially the log-linear version of the standard model of private contributions to public goods (Bergstrom, Blume and Varian, 1986). There are n goods in all, indexed $j = 1, 2, \ldots, n$. Let m of these goods be public and $n - m$ goods be private. Consumption of a public good is non-excludable and non-rival. Denote the set of public goods by P. There are two individuals, each with identical lump sum income y. All prices are normalized at unity. Individual i spends x_{ij} on good j, so the budget constraint is

$$\sum_j x_{ij} = \ y \ ; i = 1, 2 \tag{1}$$

Let the consumption of good j by individual i be denoted c_{ij}. Then

$$c_{ij} = x_{ij} \text{ for } j \notin P \tag{2}$$
$$c_{ij} \ = c_{\sim ij} = c_j \ = x_{ij} \ + x_{\sim ij} \text{ for } j \in P \tag{3}$$

where $\sim i$ denotes the individual other than i. Individual i has a utility function

$$U_i = \sum_j \alpha_j \log c_{ij} \ ; \alpha_j > 0 \ \forall_j \ ; \ \sum_j \ \alpha_j = 1 \tag{4}$$

where *log* denotes the natural logarithm. Using (2) and (3), we have

$$U_i = \sum_{j \notin P} \alpha_j \log c_{ij} + \sum_{j \in P} \alpha_j \log c_j \ ; \ \ i = 1, \ 2 \tag{5}$$
$$\sum_{j \notin P} c_{ij} + \sum_{j \in P} c_j = \text{y} + \sum_{j \in P} x_{\sim ij} \ ; \ \ i = 1, \ 2. \tag{6}$$

We start by considering the Nash equilibrium. Individual i maximizes (5) subject to (6). Using (2) and (3) the first order conditions are

$$x_{ij} \ = \ \alpha_j \left[y + \sum_{k \in P} x_{\sim ik} \right], \ j \notin P \tag{7}$$

$$x_{ij} + x_{\sim ij} \ = \ \alpha_j \left[y + \sum_{k \in P} x_{\sim ik} \right], \ j \in P \tag{8}$$

With identical individuals, $x_{ij} = x_{\sim ij} = x_j$, so (8) can be rewritten as

$$2x_j^N = \alpha_j \left[y + \sum_{k \in P} x_k^N \right], \ \ j \in P \tag{9}$$

where the superscript N denotes the Nash equilibrium outcome. Solving (9) yields:

$$\sum_{k\in P} x_k^N = \left[\frac{\beta}{2-\beta}\right] y, \quad j \in P \tag{10}$$

where $\beta = \sum_{k\in P} \alpha_k$. Then (2), (3), (7), (8), and (10) give consumption levels:

$$c_j^N = \alpha_j \left[\frac{2}{2-\beta}\right] y \text{ for all } j. \tag{11}$$

Thus utility for each individual in the Nash equilibrium is

$$V^N = K + \log y + \log \frac{2}{(2-\beta)} \tag{12}$$

where $K = \sum_{j=1}^{n} \alpha_j \log \alpha_j$. $V^0 = K + \log y$ is the utility level that can be achieved if an individual is unable to take advantage of the other's provision of public goods.

The efficient allocation is derived as the solution to

$$\max \sum_{j=1}^{n} \alpha_j \log c_{1j} + \sum_{j=1}^{n} \alpha_j \log c_{2j} \ s.t. \ \sum_{j\notin P} (c_{1j} + c_{2j}) + \sum_{j\in P} c_j = 2y \tag{13}$$

The common consumption levels are thus given by

$$c_j^E = \alpha_j y \quad j \notin P \tag{14}$$
$$c_j^E = 2\alpha_j y \quad j \in P \tag{15}$$

and the achieved level of utility by

$$V^E = K + \log y + \beta \log 2, \tag{16}$$

where the superscript E denotes the socially efficient outcome.

It can be seen directly from (12) and (16) that utility in both regimes N and E depends on the set of goods that are public. In particular, given the parameters $\alpha_1, ..., \alpha_n$, both V^N and V^E increase with β. This is shown in Figure 1, where V^0 is set equal to 0. As expected, when $\beta = 0$ (i.e. when all goods are private), there can be no free riding in the Nash Equilibrium and $V^N = V^E = V^0$. The possibility of free riding arises when $\beta > 0$, and for $0 < \beta < 1, V^0 < V^N < V^E$. But surprisingly, $V^N = V^E$ when $\beta = 1$. At first, this seems counter-intuitive; one might expect that the more public goods there are, the greater the welfare loss from free riding. But free riding operates when agents under-supply public goods and consume more private goods; in other words, free riding needs a public good/private good margin at which to operate. If $\beta = 1$ this margin does not exist; and even if β is close to 1, then private goods carry little weight in agents' utility functions, so the incentive to free ride is very low. The efficiency loss $V^E - V^N$ thus depends both on the opportunity for free riding, which requires a high level of β, and the incentive to free ride, which requires a low level of β. For extreme values of β, one of the requirements is absent. In our model, it is easily checked that $V^E - V^N$ is greatest when $\beta = 2 - (\log 2)^{-1}$. When consumers have identical preferences, the result that the efficiency loss is zero if there are only public goods is quite general, since all consumers will choose their private supply of public goods to maximise the same utility function $u(c_1, c_2, ...c_n)$.

This completes the setting up of the basic model. The next section analyses outcomes when the set of public goods, P, is itself a choice variable.

3. The Public-Private Mix With and Without Efficient Supply

If this society had a costless choice, it would always opt for increasing β. It does not matter through what combinations of public goods β is increased - all sets P for which β is the same are identical from the point of view of social welfare. We therefore focus on β as the key choice variable, and characterize the cost of public goods technology as the cost of increasing β. Moreover, for analytical ease we work with the approximation that there are a large number of goods and treat β as a continuous variable.

Let us suppose that society could "purchase" β at a unit cost (in terms of endowment) of 2θ, so that the *per capita* marginal cost of converting a private goods technology into a public goods technology is θ. Then for a given level of β utility in regimes N and E is

$$V^N = K + \log(y - \theta\beta) + \log\frac{2}{(2-\beta)} \tag{17}$$

$$V^E = K + \log(y - \theta\beta) + \beta\log 2 \tag{18}$$

As in the previous section $V^N = V^E$ if β equals zero or one and $V^N < V^E$ for intermediate values. Maximising (17) and (18) yields optimal choices of β in the two cases, $\widehat{\beta}^N$ and $\widehat{\beta}^E$, and the achieved utility levels, $\widehat{V}^N$and $\widehat{V}^E$, as functions of θ and y:

$$\left.\begin{array}{lll} \widehat{\beta}^N = 1; & \widehat{V}^N = K + \log(y-\theta) + \log 2 & \text{if } \frac{\theta}{y} \leq \frac{1}{2} \\ \widehat{\beta}^N \in (0,1); & \widehat{V}^N = K + \log y & \text{if } \frac{\theta}{y} = \frac{1}{2} \\ \widehat{\beta}^N = 0; & \widehat{V}^N = K + \log y & \text{if } \frac{\theta}{y} \geq \frac{1}{2}. \end{array}\right\} \tag{19}$$

$$\left.\begin{array}{lll} \widehat{\beta}^E = 1; & \widehat{V}^E = K + \log(y-\theta) + \log 2 & \text{if } \frac{\theta}{y} \leq \frac{\log 2}{1+\log 2} \\ \widehat{\beta}^E = \frac{y}{\theta} - \frac{1}{\log 2}; & \widehat{V}^E = K + \log\theta + \frac{y}{\theta}\log 2 - \log\log 2 - 1 & \text{if } \frac{\log 2}{1+\log 2} \leq \frac{\theta}{y} \leq \log 2 \\ \widehat{\beta}^E = 0; & \widehat{V}^E = K + \log y & \text{if } \frac{\theta}{y} \geq \log 2 \end{array}\right\} \tag{20}$$

The optimal choices of "the degree of publicness" for regimes N and E are shown in Figure 2 as a function of $\frac{\theta}{y}$, the unit cost of β as a fraction of the endowment. For sufficiently low costs, ($\frac{\theta}{y} < \frac{\log 2}{1+\log 2}$), all goods are chosen to be public in both N and E. For sufficiently high costs ($\frac{\theta}{y} > \log 2$), all goods are private in N and E. For intermediate values, interesting differences appear. As $\frac{\theta}{y}$ falls below $\log 2$, E is the first to start acquiring public goods, but for $\frac{\theta}{y}$ above 0.5 N stays completely private. At $\frac{\theta}{y} = 0.5$ N switches to completely public. As $\frac{\theta}{y}$ continues to fall, E becomes increasingly public until $\widehat{\beta}^E = 1$ at $\frac{\theta}{y} = \frac{\log 2}{1+\log 2}$.

4. The Joint Choice of Regime and the Public-Private Mix

If obtaining the efficient equilibrium were costless, then society would always be in regime E, with the choice of β depending on $\frac{\theta}{y}$. However, efficiency itself is not free. It requires the setting up and operation of costly institutions. In what follows we relate these costs to the choice of E versus N, taking into account the optimal choice of β in each case.

We adopt the simple assumption that there is a fixed *per capita* cost π of achieving efficiency. $\widehat{\beta}^N$ and $\widehat{V}^N$ are still given by (19) but in regime E the maximand is now

$$V^E = K + \log(y - \pi - \theta\beta) + \beta \log 2$$

so that in the expressions for $\widehat{\beta}^E$ and $\widehat{V}^E$ in (20), y is now replaced by $y - \pi$, i.e.

$$\left.\begin{array}{lll} \widehat{\beta}^E = 1; & \widehat{V}^E = K + \log(y-\pi-\theta) + \log 2 & \text{if } \frac{\theta}{y-\pi} \le \frac{\log 2}{1+\log 2} \\ \widehat{\beta}^E = \frac{y-\pi}{\theta} - \frac{1}{\log 2}; & \widehat{V}^E = K + \log\theta + \frac{y-\pi}{\theta}\log 2 - \log\log 2 - 1 & \text{if } \frac{\log 2}{1+\log 2} \le \frac{\theta}{y-\pi} \le \log 2 \\ \widehat{\beta}^E = 0; & \widehat{V}^E = K + \log(y-\pi) & \text{if } \frac{\theta}{y-\pi} \ge \log 2 \end{array}\right\}$$

Thus, conditional on society choosing E rather than N, higher values of π result in lower values of $\widehat{\beta}^E$. In regime E, an increase in π is equivalent to a reduction in y, and given the diminishing marginal utility of income embodied in the term $\log(y - \pi - \theta\beta)$ the appropriate response is to economise by choosing a lower level of publicness.

It is now straightforward to analyse the choice of regime. For any given values of π, θ, and y, the higher of $\widehat{V}^E$ and $\widehat{V}^N$ indicates whether it is worth putting in place those institutions which enable society to overcome free-rider problems and achieve efficiency. An equiproportional increase in π, θ, and y has no effect on $\widehat{\beta}^E$ or $\widehat{\beta}^N$, or on the difference $\widehat{V}^E - \widehat{V}^N$, and Figure 3 shows the optimal regime, E or N, as a function of $\frac{\pi}{y}$ and $\frac{\theta}{y}$

To interpret Figure 3, recall a result of Section 3 that in regime E (with π implicitly equal to 0) it is optimal to choose an intermediate degree of publicness if $\frac{\log 2}{1+\log 2} < \frac{\theta}{y} < \log 2$, whereas in regime N β^N switches from one to zero at $\frac{\theta}{y} = 0.5$. But since $V^E \ge V^N$, with equality if and only if β equals zero or one, it follows that $V^E > V^N$ if $\frac{\log 2}{1+\log 2} < \frac{\theta}{y} < \log 2$. For positive values of $\frac{\pi}{y}$, the range of $\frac{\theta}{y}$ for which it is optimal to choose E over N becomes narrower. If $\frac{\pi}{y}$ is high enough (greater than $\frac{1}{2} - \frac{1+\log\log 2}{2\log 2}$) then the costs of efficiency are too great, whatever the cost of using the Samuelson machine to convert private to public goods.

Figure 3 thus divides $(\frac{\pi}{y}, \frac{\theta}{y})$ space into three regions: (i) an area where it is optimal to choose N and set $\beta^N = 1$. Here either the costs of efficiency are so great or the costs of publicness so low that a Nash regime can avoid any loss of welfare from free-riding by making all goods public; (ii) an area where it is optimal to choose N and set $\beta^N = 0$. In this region either the costs of publicness are so high that both regimes would set $\beta = 0$ (so that for $\pi > 0$ choosing E would only incur additional costs), or for $\frac{\theta}{y}$ just above 0.5 the gain from choosing positive levels of publicness under E would not offset the additional cost of efficiency; (iii) an area where it is optimal to choose E, with $\widehat{\beta}^E = \frac{(y-\pi)}{\theta} - \frac{1}{\log 2}$.

5. Discussion and Conclusion

Across societies there seems to be a wide variety of public-private mixes. Yet the standard framework in public economics is one where the division of goods into public and private is given exogenously as a technological datum. This is unsatisfactory theoretically, and implausible as a depiction of reality, since we know that societies can and do choose to make the consumption characteristics of some goods as public or as private as they wish.

The central question of inefficient supply of the public good does not of course disappear in the broader framework of choice of the set of public goods. Rather, the two aspects interact in interesting ways. Figure 3 helps us to begin to understand the different decisions that societies make when faced with the joint choice of the degree of publicness and the degree of efficiency in the supply of public goods. Not surprisingly, when the cost of turning private goods into public goods is high, a society will go for a very low degree of publicness and when this cost is low, it will go for a very high degree of publicness. However, both for very high and very low costs of publicness, it is not worthwhile to pay the costs of solving collective action problems. This is because when the degree of publicness is very low or very high, the costs of free riding are small relative to the costs of enforcing collective action. Paradoxically, therefore, observing free riding in the supply of public goods does not necessarily indicate social inefficiency, once the costs of the "efficient" outcome are taken into account. Thus a specific hypothesis that emerges from our analysis is that we will observe a Nash equilibrium in the supply of public goods in societies with both very high and very low degrees of publicness.

For intermediate values of the cost of publicness, and for low enough costs of efficiency, we predict that society will choose efficient supply of those public goods it chooses to produce. In this range the degree of publicness will decline with the cost of efficiency and the cost of publicness. This scenario perhaps comes closest to economists' basic intuition about variation in the degree of publicness: as the cost of free riding increases, the value of public goods declines, it is not worth paying the price of the marginal Samuelson machine, and the degree of publicness falls.

Finally, consider what happens as societies become richer. If the costs of efficiency and of publicness remain constant, then in Figure 3 the outcome moves along a ray towards the origin. The most variegated pattern occurs when the ray crosses the efficient region. The prediction then is that as income increases the degree of publicness rises, but that the likelihood of Nash equilibrium has an inverse-U shape—it is highest in both very rich and very poor societies.

We hope that the line of enquiry begun here will prove fruitful in opening up the black box of an assumed given set of public goods. A direct consideration of how and why societies choose to convert some private goods into public ones, and vice versa, generates both interesting theory, and interesting predictions and hypotheses for empirical work.

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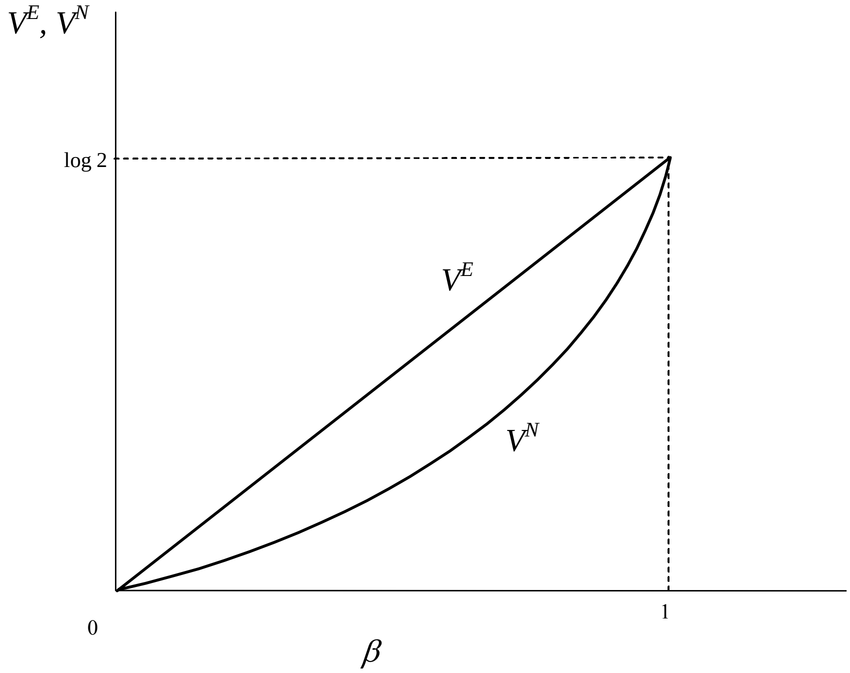

Figure 1: The effect on V^N and V^E of changes in β

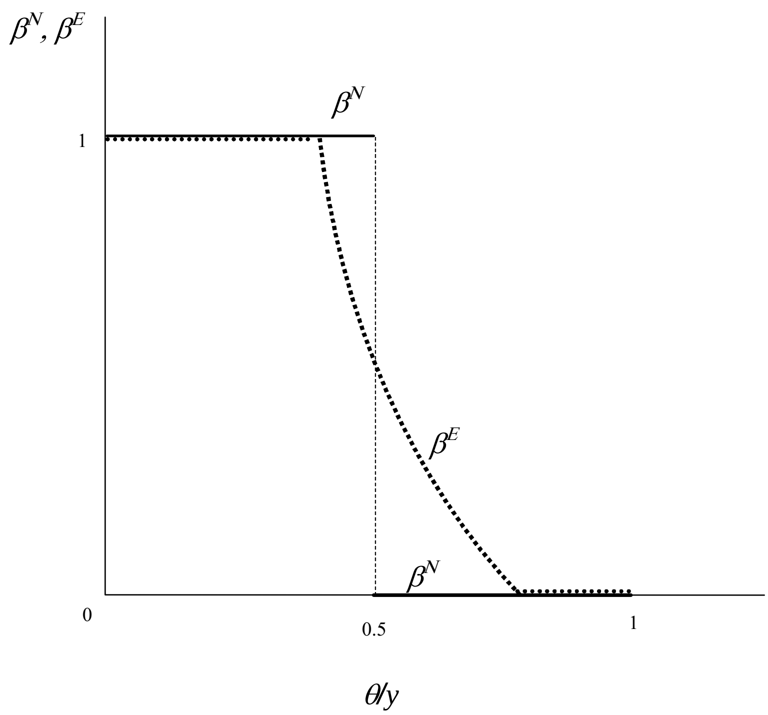

Figure 2: The effect on β^N and β^E of changes in θ/y

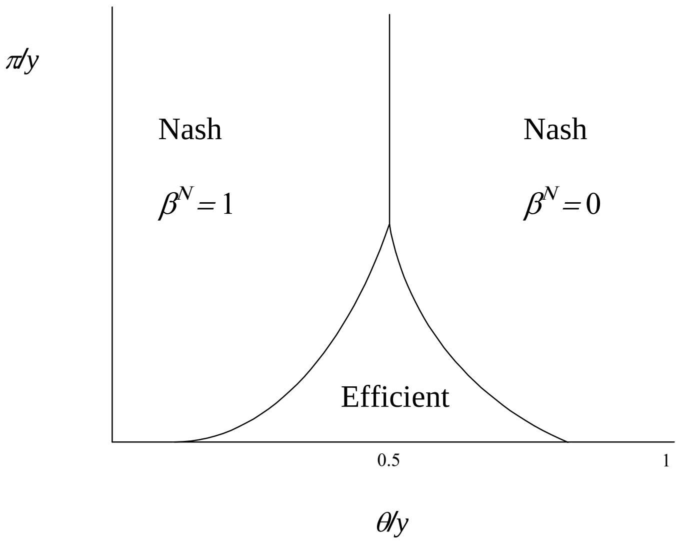

Figure 3: The effect of π/y and θ/y on choice of regime E or regime N