E C O N O M I C S   B U L L E T I N

New Explanations for the Firm Size-Wage Premium

Nien-Pen Liu
PhD candidate, Graduate Institute of Industrial Economics, National Central University, Taiwan, R.O.C.

Dachrahn Wu
Department of Economics, National Central University, Taiwan, R.O.C.

Abstract

This paper contributes some new explanations for the firm size-wage premium. We find that if the difference between the two types¡¦ status quo utility levels is large enough, then the small firm will reject the good agent more often than the large firm. The reason behind this result is that the capital-like resource restriction interferes with the willingness of the small firm to mitigate the information rent caused by the countervailing incentive. Moreover, when the countervailing incentive exists and when both firms want to delegate the tasks to both types, the good agent produces more output in the large firm and therefore gets a higher payment. These findings support the labor quality explanation that larger firms employ higher-quality workers and the productivity hypothesis that workers are more productive in large firms and therefore ask for higher wages, but from a kind of markedly different reasoning.

We would like to thank Kevin Hallock and an anonymous referee for helpful comments.
Citation: Liu, Nien-Pen and Dachrahn Wu, (2007) "New Explanations for the Firm Size-Wage Premium." *Economics Bulletin*, Vol. 10, No. 2 pp. 1-7
Submitted: November 29, 2006. **Accepted:** March 19, 2007.
URL: http://economicsbulletin.vanderbilt.edu/2007/volume10/EB-06J30005A.pdf

1. Introduction:

In spite of the large and growing importance of the firm size-wage premium, previous attempts in the literature to account for this premium have had limited success. There still remains a large, significant, and unexplained premium paid to workers of large firms (e.g. Troske, 1999).

This paper proposes a contractual game to analyze the employment problems confronted by a large firm and a small firm. It is assumed that the only distinction between a large firm and a small firm is different sizes of capital-like resource endowments.[1] We find that if the difference between the two types' status quo utility levels is large enough,[2] then the small firm will reject the good agent more often than the large firm. The reason behind this result is not that the surplus generated by the small firm and the good agent cannot afford the good type's status quo utility, but rather the capital-like resource restriction interferes with the willingness of the small firm to mitigate the information rent caused by the countervailing incentive. Moreover, when the countervailing incentive exists and when both firms want to delegate the tasks to both types, the good agent produces more output in the large firm and therefore gets a higher payment.

The findings alluded to above contribute some new explanations for the firm size-wage premium. Under the results that small firms will reject good agents more often than large firms and good agents are more productive in large firms, this paper supports the labor quality explanation that larger firms employ higher-quality workers (e.g. Hamermesh, 1993; Brown and Medoff, 1989; Reilly, 1995) and the productivity hypothesis that workers are more productive in large firms and therefore ask for higher wages (e.g. Oi and Idson, 1999), but from a kind of markedly different reasoning.

The remainder of this paper is organized as follows: Section 2 introduces the asymmetric information model to delineate firms' employment problems. The explanations for the firm size-wage premium are provided in Section 3. Section 4 concludes.

2. The Model

Consider two firms that are endowed with different sizes of capital-like resources q_{cb} and q_{cs}, respectively, where $q_{cb} > q_{cs}$. Hence, the firm endowed with q_{cb} (q_{cs}) is called the large (small) firm. Each firm wants to delegate an agent to produce some units (denoted by q) of a certain product. Assume that the output q and the capital-like resource q_c are complements. Therefore, the revenue function (of both firms) can be characterized as follows.

$$R(q, q_c) = S(\min\{q, q_c\}) \quad \text{where} \quad S' > 0, S'' < 0 \quad \text{and} \quad S(0) = 0 .$$

The agent's production cost is $C(q, \theta) = \theta q$, where θ is a constant marginal cost that is observed by the agent, but unobservable to the firm. Nonetheless, it is

[1] Economies of scale and other financial advantages (e.g. lower interest rate) are often mentioned to explain why large firms might invest more in both human capital and physical capital. The point is that large firms can spread the fixed costs of their investments across more output and agents.

[2] Type-dependent utilities (or countervailing incentives) with interesting implications have appeared successively in Lewis and Sappington (1989) for a regulation model with fixed cost, and in Laffont and Tirole (1990) for the regulation of bypass, etc. Jullien (2000) provided a general theory of type-dependent reservation utility with a continuum of types.

common knowledge that the cost function is either:

$$C(q,\underline{\theta}) = \underline{\theta}q \quad \text{with probability } \nu \tag{1}$$

or

$$C(q,\overline{\theta}) = \overline{\theta}q \quad \text{with probability } 1-\nu, \tag{2}$$

where $\underline{\theta} < \overline{\theta}$. Let us call the low cost (high cost) agent the good (bad) agent. If the firm pays t for producing q units of the good, then the benefits of the good agent and the bad agent are $U = t - \underline{\theta}q$ and $U = t - \overline{\theta}q$, respectively, and the profit function (of both firms) is $\pi = S(\min\{q, q_c\}) - t$. For simplicity, we normalize the status quo utility of the bad agent to zero, and we assume that the status quo utility level of the good agent is $U_0 \geq 0$.

In the case of complete information, the production levels $\underline{q}^*$ and $\overline{q}^*$ are given by the following first-order conditions:

$$S'(\underline{q}^*) = \underline{\theta} \tag{3}$$

and

$$S'(\overline{q}^*) = \overline{\theta}. \tag{4}$$

In order to bring the influence of the capital-like resource restriction on firms' choices (of whether to delegate a task to an agent) under asymmetric information into focus, we make the following assumption:

$$q_{cb} \geq q^{CI} > q_{cs} = \underline{q}^*, \tag{5}$$

where q^{CI} will be defined later. Hence, both firms make the same decision on whether to reject a good agent under complete information. We further assume that ν is not too high so that it is always optimal for the firm to delegate the task to the bad agent.

3. Why does a small firm reject a good agent more often than a large firm?

We start with the analysis on the case where the large and small firms only delegate their tasks to the bad agents. The firm's optimization problem is to design a menu of contracts $\{(0,0);(t^R, q^R)\}$ to maximize ex ante expected profits, where (0,0) is the null contract and (t^R, q^R) is the non-zero contract that is accepted only by the bad agent. Define the information rent by $\overline{U}^R = t^R - \overline{\theta}q^R$. We can then replace the transfers in each firm's objective function as a function of information rents and outputs so that the new control variables are $\{(0,0),(\overline{U}^R, \overline{q}^R)\}$. Hence, the large and small firms' optimization problems (denoted by (P_b^R) and (P_s^R), respectively) can be written as follows:

(P_b^R): $$\max_{\{(0,0);(\overline{U}^R,\overline{q}^R)\}} (1-\nu)(R(\overline{q}^R, q_{cb}) - \overline{\theta}\overline{q}^R) - (1-\nu)\overline{U}^R$$

subject to

$$\overline{U}^R \geq 0 \qquad (\overline{IR}^R) \tag{6}$$

$$U_0 \geq \overline{U}^R + \Delta\theta\overline{q}^R. \qquad (\underline{NIC}^R) \tag{7}$$

(P_s^R): $$\max_{\{(0,0);(\overline{U}^R,\overline{q}^R)\}} (1-\nu)(R(\overline{q}^R, q_{cs}) - \overline{\theta}\overline{q}^R) - (1-\nu)\overline{U}^R$$

subject to (6) and (7).

One clearly sees that ($\overline{IR}^R$) must be binding irrespective of problems (P_b^R) or (P_s^R); otherwise, firms could decrease $\overline{U}^R$ by a sufficiently small positive amount ε that would slack ($\underline{NIC}^R$) and increase their profits. Furthermore, both firms will choose $\overline{q}^R = \overline{q}^*$ when $U_0 \geq \Delta\theta\overline{q}^*$ and $\overline{q}^R = \frac{U_0}{\Delta\theta}$ when $U_0 < \Delta\theta\overline{q}^*$. This leads to the following lemma.

Lemma 1 The profits of problems (P_b^R) and (P_s^R) are the same when both the large and small firms delegate the tasks only to the bad agents.

Next, when a firm decides to delegate the task to both types, its problem is in designing a menu of contracts $\{(\underline{t},\underline{q});(\bar{t},\overline{q})\}$ to maximize the expected profits, where $(\underline{t},\underline{q})$ and $(\bar{t},\overline{q})$ are designed to attract the good and the bad agents, respectively. Define the corresponding good agent and the bad agent's information rents by $\underline{U} = \underline{t} - \underline{\theta}\underline{q}$ and $\overline{U} = \bar{t} - \overline{\theta}\overline{q}$. We then replace the transfers in each firm's objective function as a function of information rents and outputs so that the new control variables are $\{(\underline{U},\underline{q}),(\overline{U},\overline{q})\}$. Hence, the optimization problems of the large and small firms (denoted by (P_b) and (P_s), respectively) are written as follows:

(P_b): $$\max_{\{(\underline{U},\underline{q});(\overline{U},\overline{q})\}} \nu(R(\underline{q},q_{cb}) - \underline{\theta}\underline{q}) + (1-\nu)(R(\overline{q},q_{cb}) - \overline{\theta}\overline{q}) - (\nu\underline{U} + (1-\nu)\overline{U})$$

subject to

$$\underline{U} \geq \overline{U} + \Delta\theta\overline{q} \qquad (\underline{IC}) \qquad (8)$$

$$\overline{U} \geq \underline{U} - \Delta\theta\underline{q} \qquad (\overline{IC}) \qquad (9)$$

$$\underline{U} \geq U_0 \qquad (\underline{IR}) \qquad (10)$$

$$\overline{U} \geq 0. \qquad (\overline{IR}) \qquad (11)$$

(P_s): $$\max_{\{(\underline{U},\underline{q});(\overline{U},\overline{q})\}} \nu(R(\underline{q},q_{cs}) - \underline{\theta}\underline{q}) + (1-\nu)(R(\overline{q},q_{cs}) - \overline{\theta}\overline{q}) - (\nu\underline{U} + (1-\nu)\overline{U})$$

subject to (8) to (11).

Let π_b and π_s denote the optimal values of problems (P_b) and (P_s), respectively. Note that because of $q_{cb} > q_{cs}$, the optimal value of problem (P_b) is weakly larger than that of problem (P_s) for any incentive feasible contract. However, is there any time that the optimal value π_b is strictly larger than the optimal value π_s? Straightforward arguments show that constraints (8) through (11) define five possible regimes, which are the following.

R1	($\underline{IC}$)($\overline{IR}$)	binding
R2	($\underline{IC}$)($\overline{IR}$)($\underline{IR}$)	binding
R3	($\overline{IR}$)($\underline{IR}$)	binding
R4	($\overline{IR}$)($\underline{IR}$)($\overline{IC}$)	binding

R5 $(\underline{IR})(\overline{IC})$ binding.

In what follows, we index each of the solutions to these problems with a superscript "SB" that represents second-best, and subscripts "b" and "s" that correspondingly represent problems (P_b) and (P_s). Depending on the value of U_0, the solutions (see Proposition A.1 and Proposition A.2 in Appendix) to these problems are classified into the following two cases.

Case 1: For regime 1, 2, or 3, both solutions fall into the same regime

This case occurs when $U_0 < \Delta\theta \underline{q}^*$, and the solutions to both problems are the same. The optimal (second best) output of the good (bad) agent is equal to (not larger than) the first best output, $\underline{q}^*$ ($\overline{q}^*$), irrespective of $\underline{q}_b^{SB}$ or $\underline{q}_s^{SB}$ ($\overline{q}_b^{SB}$ or $\overline{q}_s^{SB}$). This implies that π_b is identical to π_s. The combined effect of the capital-like resource restriction and the rent extraction-efficiency trade-off under asymmetric information does not make the small firm worse in this case.

Case 2: The solution to problem (P_b) falls into regime 4 or 5, and the solution to problem (P_s) falls into regime 5

This case occurs when $U_0 \geq \Delta\theta \underline{q}^*$, and here firms confront the situation of countervailing incentives. In order to attract the good agent who has handsome outside opportunities, a generous payment is necessary. Such a contract appears attractive to the bad agent (so that $\overline{IC}$ is binding). The bad agent's output yields no distortion, i.e. $\overline{q}^{SB} = \overline{q}^*$ for both firms. Without the capital-like resource restriction, the output of the good agent is distorted upwards to mitigate the bad agent's information rent, i.e., $\underline{q}_b^{SB} = \frac{U_0}{\Delta\theta}$ if $\Delta\theta \underline{q}^* \leq U_0 < \Delta\theta \underline{q}^{CI}$ and $\underline{q}_b^{SB} = \underline{q}^{CI}$ if $U_0 \geq \Delta\theta \underline{q}^{CI}$, where $\underline{q}^{CI}$ is given by:

$$S'(\underline{q}^{CI}) = \underline{\theta} - \frac{1-\nu}{\nu}\Delta\theta, \tag{12}$$

where the superscript "CI" represents countervailing incentives.

For the reason that the incremental value of $R(\underline{q}, \underline{q}^*)$ (over $\underline{q}$) is zero when $\underline{q}$ is larger than $\underline{q}^*$, it is now too costly for the small firm to mitigate the information rent of the bad agent by distorting the good agent's output upwards. The small firm, with the capital-like resource restriction, will leave the good agent's output at the first-best level, i.e. $\underline{q}_s^{SB} = \underline{q}^*$. As the small firm's second-best solution is incentive feasible to the large firm and the large firm takes another choice, π_b is therefore strictly larger than π_s when the good agent's outside opportunities are good enough (or the difference between the two types' status quo utility levels is large enough). The combined effect of the capital-like resource restriction and the rent extraction-efficiency trade-off under asymmetric information does make the small firm worse in this case. This leads to the following proposition.

Proposition 1 If the difference between the two types' status quo utility levels is large enough, then the small firm will reject the good agent more often than the large firm.

By Propositions A.1 and A.2 in Appendix, we obtain Proposition 2 below.

Proposition 2 When the countervailing incentive exists (i.e. the solutions fall into case 2) and when both firms want to delegate the tasks to both types, the good agent produces more output in the large firm and gets a higher payment.

The prevalence of the scenario that larger firms pay higher wages is well documented in the economic literature (Oi and Idson, 1999), yet the explanations for such significant premiums paid by large firms remain incomplete. The findings of Propositions 1 and 2 contribute some new explanations on why large firms pay higher wages. For the results that small firms will reject good agents more often than large firms and good agents are more productive in large firms, this paper supports the labor quality explanation that larger firms employ higher-quality workers (e.g. Hamermesh, 1993; Brown and Medoff, 1989; Reilly, 1995) and also supports the productivity hypothesis that workers are more productive in large firms and therefore ask for higher wages (e.g. Oi and Idson, 1999), but with a kind of markedly different reasoning.

4. Conclusion:

This paper proposes a contractual game to analyze the employment problems confronted by a large firm and a small firm. We find that if the difference between the two types' status quo utility levels is large enough, then the small firm will reject the good agent more often than the large firm. The reason behind this result is not that the surplus generated by the small firm and the good agent cannot afford the good type's status quo utility, but rather the capital-like resource restriction interferes with the willingness of the small firm to mitigate the information rent caused by the countervailing incentive. Moreover, when the countervailing incentive exists and when both firms want to delegate the tasks to both types, the good agent will produce more output in the large firm and therefore get a higher payment.

Appendix:

In what follows, define $\bar{q}^{SB*}$ by $S^{'}(\bar{q}^{SB*}) = \bar{\theta} + \frac{\nu}{1-\nu}\Delta\theta$. Hence, $\bar{q}^{SB*} < \bar{q}^{*}$. Also note that, depending on the value of U_0, the solution of problem (P_b) falls into one of five different regimes defined in section 3.

Proposition A.1: (For the proof, see Laffont and Martimort, 2002, pp101-104.)

1. If $U_0 < \Delta\theta\bar{q}^{SB*}$, then the solution falls into regime (1),

   $\underline{q}^{SB} = \underline{q}^{*}, \bar{q}^{SB} = \bar{q}^{SB*}, \underline{U}^{SB} = \Delta\theta\bar{q}^{SB*}$ and $\bar{U}^{SB} = 0$.

2. If $\Delta\theta\bar{q}^{*} \geq U_0 \geq \Delta\theta\bar{q}^{SB*}$, then the solution falls into regime (2),

$\underline{q}^{SB} = \underline{q}^*, \overline{q}^{SB} = \frac{U_0}{\Delta\theta}, \underline{U}^{SB} = U_0$ and $\overline{U}^{SB} = 0$.

3. If $\Delta\theta\underline{q}^* > U_0 > \Delta\theta\overline{q}^*$, then the solution falls into regime (3),
   $\underline{q}^{SB} = \underline{q}^*, \overline{q}^{SB} = \overline{q}^*, \underline{U}^{SB} = U_0$ and $\overline{U}^{SB} = 0$.
4. If $\Delta\theta\underline{q}^{CI} \geq U_0 \geq \Delta\theta\underline{q}^*$, then the solution falls into regime (4),
   $\underline{q}^{SB} = \frac{U_0}{\Delta\theta}, \overline{q}^{SB} = \overline{q}^*, \underline{U}^{SB} = U_0$ and $\overline{U}^{SB} = 0$.
5. If $U_0 > \Delta\theta\underline{q}^{CI}$, then the solution falls into regime (5),
   $\underline{q}^{SB} = \underline{q}^{CI}, \overline{q}^{SB} = \overline{q}^*, \underline{U}^{SB} = U_0$ and $\overline{U}^{SB} = U_0 - \Delta\theta\underline{q}^{CI} > 0$.

Similarly, depending on the value of U_0, the solution of problem (P_s) falls into one of five different regimes defined in section 3.

Proposition A.2:

1. If $U_0 < \Delta\theta\underline{q}^*$, then the outcomes are the same as outcomes 1-3 in Proposition A.1.
2. If $U_0 \geq \Delta\theta\underline{q}^*$, then the solution falls into regime (5), where:
   $\underline{q}^{SB} = \underline{q}^*, \overline{q}^{SB} = \overline{q}^*, \underline{U}^{SB} = U_0$ and $\overline{U}^{SB} = U_0 - \Delta\theta\underline{q}^* \geq 0$.

Proof: For the proof of result 1, see Laffont and Martimort, 2002. We focus on the case of $U_0 \geq \Delta\theta\underline{q}^*$. Note that the bad type will pretend to be the good type if the small firm adopts the first-best solution (i.e., proposing $(\underline{q}^*, U_0)$ and $(\overline{q}^*, 0)$ for the good type and the bad type, respectively). We presume that ($\underline{IC}$) and ($\overline{IR}$) are irrelevant so that both ($\overline{IC}$) and ($\underline{IR}$) are binding. The small firm's problem can be simplified to the following problem ($P_s^{'}$).

$$(P_s^{'}): \quad \max_{\{\underline{q},\overline{q}\}} \nu(R(\underline{q},\underline{q}^*) - \underline{\theta}\underline{q}) + (1-\nu)(R(\overline{q},\underline{q}^*) - \overline{\theta}\overline{q}) - (\nu U_0 + (1-\nu)(U_0 - \Delta\theta\underline{q})$$

As the expected reduction of information rent does not depend on the bad type's output $\overline{q}$, the maximization of ($P_s^{'}$) needs no distortion away from the bad type's first-best output - namely, $\overline{q}^{SB} = \overline{q}^*$. Because of the capital-like resource restriction (i.e. $q_{cs} = \underline{q}^*$), increasing the good type's output by an infinitesimal amount does not increase $R(\underline{q},\underline{q}^*)$ any more when $\underline{q}$ is larger than $\underline{q}^*$. It is therefore not worthwhile for the small firm to distort the good type's output level upwards in order to mitigate the information rent caused by the countervailing incentive, i.e. the binding ($\overline{IC}$) constraint. As a result, the maximization with respect to $\underline{q}$ yields $\underline{q}^{SB} = \underline{q}^*$. To validate this approach, one needs to check the presumption about ($\underline{IC}$) and ($\overline{IR}$), i.e. $U_0 \geq (U_0 - \Delta\theta\underline{q}^*) + \Delta\theta\overline{q}^*$ and $U_0 - \Delta\theta\underline{q}^* \geq 0$. The former inequality is correct obviously, while the latter one is automatically satisfied owing to the premise of result 2. ■

References

Brown, C., and J. Medoff (1989) "The Employer Size-Wage Effect" *Journal of Political Economy* 97, 1027-1059.

Hamermesh, D. S. (1993) *Labor Demand.* Princeton: Princeton University Press.

Jullien, B. (2000) "Participation Constraint in Adverse-Selection Models" *Journal of Economic Theory* 93, 1-47.

Laffont, J. J., and D. Martimort (2002) *The Theory of Incentives: The Principal-Agent Model*, Princeton: Princeton University Press.

Laffont, J. J., and J. Tirole (1990) "Bypass and Cream Skimming" *American Economic Review* 80, 1042-1061.

Lewis, T., and D. Sappington (1989) "Countervailing Incentives in Agency Problems" *Journal of Economic Theory* 49, 294-313.

Oi, W. Y., and T. L. Idson (1999) "Firm Size and Wages" In *Handbook of Labor Economics*, vol. 3 by Ashenfelter, O., and D. Cards, Eds., North-Holland: Amsterdam.

Reilly, K. T. (1995) "Human Capital and Information" *Journal of Human Resources* 30, 1-18.

Troske, K. R. (1999) "Evidence on The Employer Size-Wage Premium from Worker-Establishment Matched Data" *Review of Economics and Statistics* 81, 15-26.