E C O N O M I C S B U L L E T I N

Employment and Fertility Choice: A Differential Game Approach

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Abstract

For OECD countries there is an intriguing variety of combinations between total fertility rate (TFR) and female labor force participation rate (FPR) suggesting the existence of multiple equilibria. This paper provides a differential game framework where the employment choices by husband and wife affect a family's fertility. The model has multiple open-loop equilibria characterized by different combinations of FPR and TFR that are consistent with the empirical cross-country evidence. The dynamic trajectory from one equilibrium point to another also sheds lights on possible demographic transition of individual countries as displayed in their time series data. The model stresses that the husband's employment decisions are as important as wife's in determining family size.

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1 Introduction

One of the striking features regarding actual fertility in some industrialized countries such as Italy, Spain and Greece is the existence of low levels of fertility rate and low levels female labor force participation [Ahn and Mira, 2002]. This evidence [see Figure 1] is interesting and somewhat surprising since traditional economic theory [e.g., Becker, 1965; Mincer, 1985] predicts a negative relationship between total fertility rates [TFR] and female labor force participation rates [FPR]. The idea lies on the opportunity costs of women's time. Increasing female wages increase female labor supply; the more time individual women spend at work means less time dedicated to child rearing. As child rearing is intensive in mother's time there is a trade-off between time spent at work and time spent in child rearing. In this framework individual women are unable to combine work with child rearing being constrained to make either-work-or-children choices. As a consequence it is expected an inverse relationship between FPR and TFR. This is why the recent cross-country evidence from southern European countries and Japan of neither-work nor-children decision is puzzling.

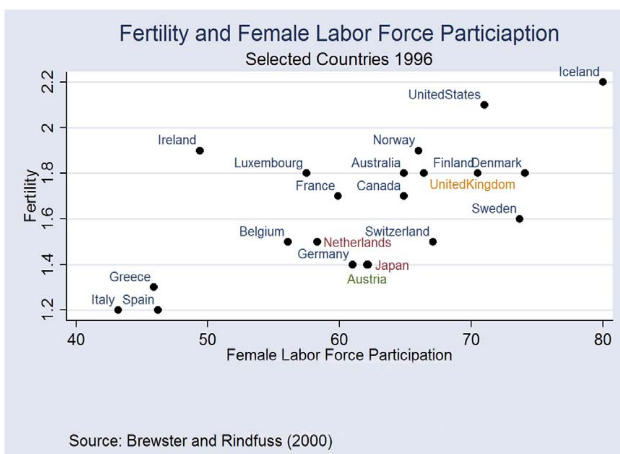

Figure 1: Fertility and Female Labor Force Participation

The sign of the cross-country correlation between TFR and FPR changed over the decades, in the 1970 the correlation was negative and it became positive in the mid eighties. In spite of the change in the sign of the cross-country correlation, TFR and FPR are negatively correlated within countries, i.e., the mean number of children is always lower for working woman than non-working woman. This is consistent with the micro economic theories of the family. Therefore the macro and micro evidence run in opposite directions [De Laat and Sanz, 2004].

Another interesting feature concerning the relationship between TFR and FPR is that when we analyze the time series for individual countries, we have a wide variety of dynamic behavior. There are countries, such as Sweden and Norway that went from low FPR and TFR to high FPR and TFR, while there are other countries, such as Italy and Spain that went from high fertility and low FPR to low fertility and low FPR.

Such diversity of combinations between TFR and FPR, not surprisingly, has triggered a large literature. Several papers that focus on the compatibility of work and children advanced some explanations for the observed combinations between TFR and FPR. The explanations can be divided into two broad categories [Rindfuss and Brewster, 1996]: 1) social-structural factors and 2) ideational factors.

The social-structural factors emphasize family policies and cash benefits, on the one hand, and labor market issues and the availability of childcare, on the other hand. The family policies studied involve housing allowances, tax relief, parental leave and child benefits [e.g., Hantrais, 1997; Brewster and Rindfuss, 2000]. Some of the labor market issues studied are the flexibility of employment and work hours and the unemployment rate [e.g., Ahn and Mira, 2002]. Regarding child care, the literature discusses its provision, whether it is private or public, and its cost, whether it is private, state-funded or a mixed system [e.g., Di Tomasso, 1999; Del Boca, 2002] and the transmission of human capital [Ortega and Tanaka, 2004].

The ideational factors emphasize the role of cultural factors in shaping the division of domestic work. The key here is the understanding of social norms regarding the role of women and men in domestic labor [e.g., Bettio and Villa, 1998; Akerlof and Kranton, 2000;Lagerlof, 2003]. In more traditional societies [e.g.,Munshi and Myaux, 2002], women devote significantly more time to unpaid work than men. In addition, there is evidence that in high-fertility countries men are increasingly involved in childcare. For instance, the average weekly hours devoted to housework by Japanese men is 3.5, whereas in the USA it is 13.8 and in Sweden 18.1 [De Laat and Sanz, 2004].

The evidence regarding cross-country data presented above raises an interesting question: Is it possible to explain such different cross-country outcomes within a unified framework? The empirical evidence suggests multiple equilibria. This paper provides a dynamic employment choice and fertility model that has multiple steady state equilibria consistent with the observed current cross-country variation of FPR and TFR.

The dynamic model can also address, at least partially, the time series properties of individual countries. That is, it can tackle the question of why some countries went from low FPR and TFR to high FPR and TFR, like Sweden and Norway. This question can be addressed through the analysis of the dynamic trajectories between different steady state equilibrium points.

Therefore, the theoretical model can be used to help understand the available data. By focusing in the present time, the cross-section data can be addressed through the analysis of the steady state equilibria of the model, while the time series data can be illuminated by the transitional dynamics from one equilibrium point to another.

The model's departure point is that a couple makes choices regarding their work and home time and that the number of children is affected by these decisions. We model the husband and wife's employment decisions in a differential game framework since they may have different preferences concerning how much time to spend at work or at home. The model has multiple open-loop equilibria characterized by different combinations of FPR and TFR that resemble the empirical cross-country evidence. The dynamic trajectory from one equilibrium point to another also sheds lights on possible demographic transition of individual countries as displayed in their time series data. Moreover, the model stresses that the husband's employment decisions are as important as the wife's in determining the family size.

2 Model

The model adapts the framework put forward by Feichtinger and Wirl (1993). The husband and the wife decide how much time to spend at home and outside home [working and/or in leisure]. The husband's working hours plus the leisure time spent outside home are denoted by $x(t)$; thus $1 - x(t)$ stands for the husband's home time, where the total available time is normalized to 1. The wife's domestic time is denoted by $y(t)$; so $1-y(t)$ is the wife's number of hours spent outside home. Therefore, x and y are the control variables. An important feature of this model is that the family size, or the number of children,$z(t)$, is a result of the couple's interaction, which is a function [quite complex, probably non-linear] of the time they spend together. We postulate that the number of children increases with the couple's time spent at home and decreases with the existing number of children. This leads to the following differential equation:

$$\dot{z(t)} = f(1 - x(t), y(t)) - \delta z(t) \tag{2.1}$$

By emphasizing the role of cultural factors in shaping the division of domestic work, we assume that the preferences of women and men regarding home and outside home activities are socially determined, including mating decisions [e.g., Dawkins, 2004][1]. This means that for the wife, given that child rearing is intensive in mother's time, it is assumed that her objective is to maximize the present value of instantaneous utility from children, $B(z)$, and domestic time, $n(y)$. Similarly, assuming the husband's role is shaped culturally as the

[1] See Bisin and Topa (2002) for empirical models of cultural transmission.

family's main economic provider, the husband's objective is to maximize the present value of instantaneous utility from outside home time, $m(x)$ and children, $A(z)$. The couple discount the future benefits at the same interest rate, $r > 0$.[2] [3]

$$\text{Wife:} \qquad \max_{y(t)} \qquad J_1 = \int_0^\infty e^{-rt}[B(z(t)) + n(y(t))]dt \tag{2.2}$$

$$\text{Husband:} \qquad \max_{x(t)} \qquad J_2 = \int_0^\infty e^{-rt}[A(z(t)) + m(x(t))]dt \tag{2.3}$$

Equations (2.1)-(2.3) describe a differential game. This paper focuses only on precommitment (open-loop) strategies. An open-loop Nash equilibrium is the pair of time dependent strategies $\{x^*(t), y^*(t), t \in [0, \infty)\}$such that the wife maximizes her objective J_1 and the husband his objective J_2. This means that the necessary optimality conditions for the control problems of both players must hold simultaneously; these conditions are listed below using the following notation: H_1 for the wife's and H_2 for the husband's current value Hamiltonian,λ and μ for the associated costate variables [time arguments are suppressed]:

Wife	Husband	
$H_1 = B(z) + n(y) + \mu[f(1-x, y) - \delta z]$	$H_2 = A(z) + m(x) + \lambda[f(1-x, y) - \delta z]$	(2.4)
$H_y^1 = n'(y) + \mu f_y(1-x, y) = 0$	$H_x^2 = m'(x) + \lambda f_x(1-x, y) = 0$	(2.5)

[2] The assumptions about household decision making on labor supply and child-care are implicit in our formulation (2.1)-(2.3). It is noted that parental net income [gross income, wages plus government subsidies, less child-care costs] and child-care costs affect labor supply decisions and family fertility (for example, Mincer (1962) and Ashenfelter and Heckman (1974)). They also affect our model's functions: $f(1-x, y); A(z); B(z); m(x); n(y)$. For simplicity, in equation (2.1), we rather work with parents' allocation of time, x and y, and the number of children, z, than with income and prices. We can see below the simplification is sufficient to generate some interesting results that are consistent with empirical observations. In addition to the important roles of market prices and income in intra-family decision-making, we acknowledge the fact that the intra-family decisions are also regulated by several other non-market variables, such as cultural factors, extended family and other social networks, and tradition.

[3] Our formulation is also inspired by the recent literature on non-cooperative models. There are several approaches to model household decision making process (see Bergstrom (1996, 1997) and Lundberg and Pollak (1996) for the survey of this literature). While the conventional unitary models are simple and powerful, empirical evidence supporting such models is limited. One of important features of the non-cooperative models is that the equilibrium is self-enforcing (see Chen and Woolley (2007) for related discussions). Following this literature, we thus assume that each household member maximizes his/her own objective function.

$$\dot{\mu} - r\mu = -[B'(z) - \mu\delta] \qquad \dot{\lambda} - r\lambda = -[A'(z) - \lambda\delta] \tag{2.6}$$

In order to solve the model, we must assume explicit functional forms:

$$f(1-x, y) = h_0 + h_1(1-x) + h_2(1-x)^2 + w_0 + w_1 y - w_2 y^2$$
$$A(z) = a_0 + a_1 z \qquad B(z) = b_0 + b_1 z$$
$$m(x) = m_0 + m_1 x \qquad n(y) = n_0 + n_1 y$$

where all parameters are positive.

Differentiation of equations (2.5) with respect to time and substitution of the costate variables and their time derivatives into the Euler equations (2.6) yields the following system of differential equations:

$$\dot{x} = \frac{[h_1 - 2h_2(1-x)][a_1 h_1 - 2a_1 h_2(1-x) + m_1(\delta + r)]}{2m_1 h_2} \tag{2.7}$$

$$\dot{y} = \frac{[2w_2 y - w_1][2b_1 w_2 y - b_1 w_1 - n_1(\delta + r)]}{2n_1 w_2} \tag{2.8}$$

$$\dot{z} = h_0 + h_1(1-x) - h_2(1-x)^2 + w_0 + w_1 y - w_2 y^2 - \delta z \tag{2.9}$$

This dynamic system shows how the husband's working and leisure time, wife's home time and the number of children varies along time. An interesting property of this dynamic system is the existence of multiple steady state equilibria.

Notice that by setting $\dot{x} = 0$ in equation (2.7) it follows that we have two steady state values for x [denoted x_1, x_2]:

$$x_1 = 1 - \frac{h_1}{2h_2} \qquad x_2 = 1 - \frac{h_1}{2h_2} + \frac{m_1(r+\delta)}{2a_1 h_2} \tag{2.10}$$

Non-negativity of the solutions implies that $h_1 < 2h_2$. Thus, it is sufficient to assume that: $h_1 < h_2$, implying that: $0 < x_1 < x_2$. Therefore, in x_1 the husband spends more time at home than in x_2.

In the same vein, by setting $\dot{y} = 0$ in equation (2.8) it follows that we have two steady state values for y [denoted y_1, y_2]:

$$y_1 = \frac{w_1}{2w_2} \qquad y_2 = \frac{b_1 w_1 + n_1(r+\delta)}{2b_1 w_2} \tag{2.11}$$

It's clear that: $0 < y_1 < y_2$, as a result in y_1 the wife spends more time at work and leisure than in y_2.

Given that we have multiple equilibrium values for x and y, it follows that we have multiple equilibrium values for z, since setting $\dot{z} = 0$ in equation (2.9) [where the asterisk denotes possible steady state equilibrium] yields:

$$z^* = [h_0 + h_1(1 - x^*) - h_2(1 - x^*)^2 + w_0 + w_1 y^* - w_2 {y^*}^2]\delta^{-1}$$

In total this model provides four steady state equilibrium points:

1) $x_1, y_1, z_1 = [h_0 + h_1(1 - x_1) - h_2(1 - x_1)^2 + w_0 + w_1 y_1 - w_2 y_1^2]\delta^{-1}$
2) $x_2, y_2, z_2 = [h_0 + h_1(1 - x_2) - h_2(1 - x_2)^2 + w_0 + w_1 y_2 - w_2 y_2^2]\delta^{-1}$
3) $x_1, y_2, z_3 = [h_0 + h_1(1 - x_1) - h_2(1 - x_1)^2 + w_0 + w_1 y_2 - w_2 y_2^2]\delta^{-1}$
4) $x_2, y_1, z_4 = [h_0 + h_1(1 - x_2) - h_2(1 - x_2)^2 + w_0 + w_1 y_1 - w_2 y_1^2]\delta^{-1}$

The points (x_1, y_1, z_1) and (x_2, y_1, z_4) are stable focus equilibrium points, while (x_2, y_2, z_2) and (x_1, y_2, z_3) are saddle point equilibriums.

Noticing that $h_2(x_2 + x_1) > 2h_2 - h_1$ and $w_2(y_2 + y_1) > w_1$, it is easy to see that $z_4 < z_1$ and $z_2 < z_3$. In order to simplify the comparison among the equilibrium points assume that $\frac{y_2 - y_1}{x_2 - x_1} > \frac{b_1 m_1}{a_1 n_1}$. This assumption makes $z_4 > z_3$; therefore, we have $z_1 > z_4 > z_3 > z_2$.

As claimed in the introduction, this model can be used to understand the actual cross-country evidence. Given the resemblance with actual country cases, we can associate each equilibrium point to actual country characteristics. Using Figure 2, a 3-Dimensional graph, as reference, the point (x_1, y_1, z_1) can be associated with the USA where FPR and TFR are high [which means high $(1 - y)$ and z, such as $(1 - y_1)$ and z_1], but the husband's time spent at home is larger than other OECD countries [low x, such as x_1]. The equilibrium (x_1, y_2, z_3) resembles the Spanish example because of the low TFR and FPR and high unemployment rate which may affect positively the amount of time husband's spent at home [4]. In the same fashion, one can associate the point (x_2, y_1, z_4) with Norway, and (x_2, y_2, z_2) with Japan. This mapping leads us to another important contribution of our model. Instead of focusing on a two dimensional relationship between FPR and TFR we must introduce a third dimension to capture the husband's employment time, which is of fundamental importance in determining and understanding the relationship between FPR and TFR [see also De Laat and Sanz, 2004].

Another way of using the model is to focus on the dynamic trajectories from one equilib-

[4] Here an important distinction is necessary since the total amount of time the husband spends at home can be spent as leisure time and/or domestic work. In the USA the time the husband devotes to household activities is higher than in Spain, where men enjoy more leisure time.

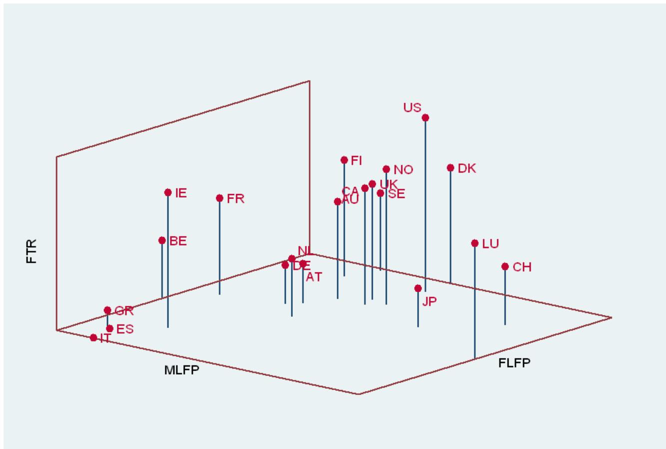

Figure 2: Total Fertility, Male and Female Labor Force Participation

rium point to another.[5] For instance, the dynamic trajectory of countries such as Sweden and Norway that went from low FPR and TFR to high TFR and FPR, can be seen in our model as the transition from equilibrium point (x_2, y_2, z_2), to equilibrium point (x_2, y_1, z_4). In order to understand this, think of Sweden or Norway in 1970 as initially in point (x_2, y_2, z_2), and in 2000 in point (x_2, y_1, z_4). The dynamic transition they experienced, through the increase in FPR and TFR is described by the trajectory that departs from equilibrium point (x_2, y_2, z_2) and ends in equilibrium point (x_2, y_1, z_4). Notice that in this case the number of hours the husband spends at home did not change and only FPR and TFR increased.

3 Concluding Remarks

For OECD countries, a variety of combinations between total fertility rate (TFR) and female participation rate (FPR) in the labor market have been documented empirically. There are countries that present low TFR and low FPR, countries that have high TFR and high FPR and other countries that have low FPR and high TFR. The empirical evidence suggests multiple equilibria. This paper provides a differential game framework where employment

[5] Mathematically the transition can be studied by analyzing the dynamic system formed by equations (2.1), (2.7), (2.8), and (2.9). Given any initial condition for z, we can study the paths towards the four identified equilibria. We use Scandinavia as an example, but our paper is able to fit other cases as well. However, the causes of such a transition can be different across countries and identifying these causes is interesting in its own right but beyond the scope of this paper.

choices by husband and wife affect family's fertility. The model has multiple open-loop equilibria characterized by different combinations of FPR and TFR consistent with the cross-country empirical evidence. The dynamic trajectory from one equilibrium point to another also sheds lights on possible demographic transition of individual countries as displayed in their time series data. In addition, the model stresses that the husband's employment decisions are as important as wife's in determining family size.

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