E C O N O M I C S B U L L E T I N

Income taxation, child-rearing policies, fertility and unemployment

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Abstract

We examine how subsidy policies to support child-rearing of households affect the fertility rate in a textbook OLG model extended to account for a labour market imperfection (e.g., a minimum wage or a monopolistic union's wage) as well as endogenous fertility. It is shown that increasing the child subsidy actually reduces population growth. The policy implications for countries with imperfect labour markets and low fertility (e.g., the most part of European Union countries) are noteworthy: if the government's objective is to increase the fertility rate, a child-subsidy support policy should not be introduced at all. Contrary to conventional wisdom, our findings in fact reveal that, for any given value of the minimum (or union's) wage, the introduction of a child subsidy reduces capital accumulation and increases unemployment in the long-run, and ultimately it negatively affects the demand for children in spite of a reduced cost of child-rearing.

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1 Introduction

The fertility rate in advanced countries has decreased dramatically in recent periods. Becker and Barro (1988) and Barro and Becker (1989) showed that increased child-rearing costs is an important factor contributing to low fertility rates, while Galor and Weil (1996) investigated the relation between fertility and growth, focusing on the gender gap. In particular, the latter authors analysed individuals' fertility choices in terms of men's and women's relative wages, and concluded that the two most important effects in the model are represented by a capital accumulation effect which acts positively on women's relative wages as well as by a women's relative wage effect which acts negatively on fertility.

Recently, Apps and Rees (2004) analysed a partial equilibrium model to account for endogenous fertility decisions of individuals and differences among male and female wages. After having described the historical evidence of an inverse relationship between female labour force participation rates and fertility behaviour, they assessed the effectiveness of family policies, in particular taxation and the system of child support. They found that countries that "support families through improved availability of alternatives to domestic child care, rather than through direct child payments, are likely to have both higher female labour supply and higher fertility", (Apps and Rees, 2004, p. 760).

Our paper is motivated by the recent and widespread debate around the effectiveness of publicly provided policies for the family, implemented mainly to reduce the drop in (or even to increase) population growth in a context where low fertility rates as well as population ageing are current stylised facts (e.g., Germany and Italy). Family policies may be adopted by the policymaker in different ways. For instance, child-care facilities (e.g., investments in day-care centres, schools and so on) or direct monetary transfers (a measure to increase directly the financial support to households with children). Some economists, have even suggested the introduction of tax penalties for households that choose low fertility, or, alternatively, the rise in monetary transfer payments or tax deductions for households that choose high fertility, as policy measures to reverse the decline in population growth. However, there seems to be a general scepticism in the literature concerning the issue of whether public policies for the family may have an impact on fertility choices, and, although such a policy debate is high on the political agenda (especially in the most part of European Union countries), a few number of theoretical contributions have tried to give an answer to the question of the effectiveness of child-subsidy support policies in a dynamic general equilibrium framework (see, for instance, Momota, 2000, who focused, differently from the present paper, on the gender gap by introducing a rather special form of public services to support child-rearing of households).

In this paper, we consider a standard dynamic general equilibrium OLG model (Diamond, 1965) extended to account for endogenous fertility decisions of individuals and a labour market imperfection, and where all household members allocate resources to raise children. Such an imperfection is represented, for simplicity, by the introduction of a regulated wage (which may be assumed to be either a minimum wage or a monopolistic union's wage) fixed over the prevailing market-clearing level. Further, by supposing that for some exogenous reasons (e.g., because of the population ageing in many European countries above mentioned) the government would like to increase fertility, we will examine how subsidy policies (direct monetary transfers) to support child-rearing of households affect the fertility rate.[1] It is shown that population growth is always lower than whether the child-subsidy policy is not implemented at all, regardless of the values of the key parameters of the model, even though the cost of child-rearing has been reduced.

The idea behind our results may be summarised in the following way: the regulation of wages determines a positive unemployment rate, which affects (negatively) individual's fertility decisions. In a partial equilibrium model, where unemployment is taken to be an exogenous variable, the introduction of child subsidies has straightforward effects, that is the higher the child subsidy the higher the demand for children owing to a decreased cost of child-rearing. However, by considering a general equilibrium model, where unemployment is an endogenously determined variable, things change dramatically. In particular, the final effect of the introduction of a publicly provided monetary child payment is the

[1] We note that we do not look for optimal taxes, but simply look at the effects of changes in tax rates in a given tax system which is almost certainly non-optimal.

results of two counterbalancing forces which act on fertility decisions of individuals: (*i*) a direct positive effect which tends to increase the fertility rate due to a reduced cost of child-rearing, and (*ii*) an indirect negative feedback effect which acts on fertility via the increased rate of unemployment due to a lower long-run stock of capital per-capita. In what follows, it is shown that the positive effect is permanently dominated by the negative effect regardless of the values of technology and preference parameters, and then the long-run rate of fertility is always lower than whether the subsidy policy is not implemented at all.

Our findings, which are at odds with conventional wisdom, have noteworthy policy implications, especially for European Union countries where labour markets are far from being competitive and the debate around the effectiveness of child-subsidy support policies is high on the political agenda. If the aim of the policymaker is to increase (reduce the drop in) population growth, child-subsidy support policies have the paradoxical result to depress further the long-run demand for children, and then they should not be introduced at all.

The paper is organised as follows: Section 2 presents the model and the main results. In Section 3 we extend the model presented in Section 2 to test for the robustness of the results. Finally, Section 4 bears the conclusions.

2 The model

Agents have identical preferences and are assumed to belong to an overlapping generations structure with finite lifetimes. Life is separated among three periods: childhood, young adulthood and old-age. During childhood individuals do not make economic decisions and thus they consume a fixed fraction of the time endowment from their parents. Adult individuals belonging to generation t have a homothetic and separable utility function defined over young-aged consumption ($c_t^{\,y}$), old-aged consumption ($c_{t+1}^{\,o}$) and the number of children (n_t),[2] as in Galor and Weil (1996).

Only young-adult individuals (N_t) join the workforce, and the labour supply is supposed to be constant and normalised to unity. As an adult, each young agent receives a constant binding regulated wage per hour worked ($\underline{w}$) higher than the prevailing market-clearing level:[3] thus, in each period, the labour market does not clear and involuntary unemployment occurs. This income is used to consume, to raise children, to pay taxes and to save. The aggregate unemployment rate (defined in terms of hours not worked) is $u_t = (N_t - L_t)/N_t$, where L_t is the labour demand. We assume that raising children requires a fixed amount of resources m per child (measured in units of market goods). Moreover, parents receive a lump-sum subsidy $\beta \in (0, m)$ – provided by the government at balanced budget – for each child to support child-rearing. During old-age agents are retired and live on the proceeds of their savings (s_t) plus the accrued interest at the rate r_{t+1} (which represents the rate of return from period t to period $t+1$). Therefore, the representative individual born at time t is faced with the following program:

$$\max_{\{c_t^{\,y}, c_{t+1}^{\,o}, n_t\}} U_t\left(c_t^{\,y}, c_{t+1}^{\,o}, n_t\right) = (1-\phi)\ln\left(c_t^{\,y}\right) + \gamma \ln\left(c_{t+1}^{\,o}\right) + \phi \ln(n_t), \qquad \text{(P)}$$

subject to

$$\begin{aligned} c_t^{\,y} + s_t &= \underline{w}(1-u_t) - (m-\beta)n_t - \tau_t \\ c_{t+1}^{\,o} &= (1+r_{t+1})s_t \end{aligned},$$

[2] Note that n_t represents the number of children with $n_t - 1$ being the population growth rate.

[3] It may be thought that such a wage is legally set (minimum wage) or fixed by a monopolistic union over the market-clearing level.

where $\tau_t > 0$ is a lump-sum tax levied on the young generation, $0 < \gamma < 1$ is the subjective discount factor and $0 < \phi < 1$ captures the importance in the welfare function of consuming while young relative to the utility of children.
The first order conditions for an interior solution are given by:

$$\frac{c_{t+1}^{o}}{c_t^{y}} \cdot \frac{1-\phi}{\gamma} = (1 + r_{t+1}), \tag{1}$$

$$\frac{\phi}{n_t} = \frac{1-\phi}{c_t^{y}} \cdot (m - \beta). \tag{2}$$

Eq. (1) equates the marginal rate of substitution between working period and retirement period consumption to their relative prices, whereas Eq. (2) equates the marginal utility of having a child with the involved marginal costs in terms of forgone utility of consumption. Note that a necessary and sufficient condition for the existence of a positive solution for n_t is $m - \beta > 0$, that is the net cost of raising children must be strictly positive.
Exploiting (1) and (2) together with the individual's intra-temporal budget constraints, the demand for children and the savings path are respectively given by:

$$n_t = \frac{\phi}{(1+\gamma)(m-\beta)}\left[\underline{w}(1-u_t) - \tau_t\right], \tag{3}$$

$$s_t = \frac{\gamma}{1+\gamma}\left[\underline{w}(1-u_t) - \tau_t\right]. \tag{4}$$

The government runs a balanced budget policy in every period. The fixed monetary child payment is assumed to be entirely financed by levying and adjusting over time lump-sum taxes upon young-aged individuals. Therefore, the per-capita time-t government constraint is simply:[4]

$$\beta n_t = \tau_t, \tag{5}$$

where the left-hand side represents the total child care expenditure and the right-hand side the tax receipt.
Inserting (5) into (3) to eliminate τ_t and rearranging terms yields:

$$n_t = \frac{\phi \underline{w}(1-u_t)}{(1+\gamma)(m-\beta) + \phi\beta}, \tag{6}$$

Now, combining (4), (5) and (6), the saving function is determined by:

$$s_t = \frac{\gamma \underline{w}(1-u_t)(m-\beta)}{(1+\gamma)(m-\beta) + \phi\beta}. \tag{7}$$

By looking at Eqs. (6) and (7) it can easily be seen that both the fertility rate and the saving function depend negatively on the unemployment rate.
As regards the production side, we assume firms are identical and act competitively. The (aggregate) constant returns to scale Cobb-Douglas technology of production is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$,[5] where Y_t, K_t and $L_t = (1-u_t)N_t$ are output, capital and the labour input respectively, $A > 0$ represents a scale parameter and $\alpha \in (0,1)$ is the capital's weight in technology. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per-capita respectively, the intensive form production function may be written as:

$$y_t = Ak_t^{\alpha}(1-u_t)^{1-\alpha}. \tag{8}$$

Assuming that physical capital totally depreciate at the end of each period and that final output is traded at unit price, profits maximisation leads to the following marginal conditions for capital and labour:

[4] We suppose individuals act atomistically and do not take into account the government budget when deciding on the desired number of children and the savings path.
[5] Adding exogenous growth in labour productivity does not alter any of the substantive conclusions of the model and, hence, it is not included here.

$$r_t = \alpha A \left[\frac{k_t}{1-u_t} \right]^{\alpha-1} - 1, \tag{9}$$

$$\underline{w} = (1-\alpha) A \left[\frac{k_t}{1-u_t} \right]^{\alpha}. \tag{10}$$

As far as labour is concerned, the marginal product of labour will adjust to meet the fixed real wage with unemployment being an endogenous variable.[6] Thus, exploiting Eq. (10), the current rate of unemployment is determined by:

$$u_t = 1 - \left[\frac{(1-\alpha)A}{\underline{w}} \right]^{\frac{1}{\alpha}} \cdot k_t, \tag{11}$$

which is positively related with the minimum wage and strictly decreasing in the per-capita stock of capital.

We now combine all the pieces of the model to characterise the long-run equilibrium. Given the government budget (5), and knowing also that $N_{t+1} = n_t N_t$, the market-clearing condition in goods as well as in capital markets is expressed by the equality $n_t k_{t+1} = s_t$, that is, the per-capita stock of capital in period $t+1$ equals the amount of resources saved in period t discounted by the number of individuals. Using (6) and (7) to substitute out for n_t and s_t respectively, such a condition boils down to the following (constant) long-run stock of capital per-capita (which is independent of the level of the regulated wage):

$$k^*(\beta) = \frac{\gamma(m-\beta)}{\phi}. \tag{12}$$

As it can easily be seen by looking at Eq. (12), an increase in the direct monetary transfer to support child rearing of households does always reduce the long-run stock of capital (the higher the child-subsidy the lower the net cost of raising children, $m-\beta$), that is $\frac{\partial k^*}{\partial \beta} = -\frac{\gamma}{\phi} < 0$ for any $0 < \beta < m$.

2.1 Comparative Static Analysis

We now suppose that for some exogenous reasons the government would like to increase fertility, and we will examine the impact of the child-subsidy on the long-run fertility decisions of individuals. Can a child-subsidy policy serve as an effective instrument to increase (reduce the drop in) population growth in a context where the wage rate is legally set by the government (or fixed by a monopolistic unions) over the prevailing market-clearing wage? This simple question gives rise to very interesting findings in our basic OLG model with equilibrium unemployment.

First of all, let us rewrite the long-run rate of fertility as a generic function of the child grant as follows:

$$n^* = n^*\{\beta, u^*[k^*(\beta)]\}. \tag{13}$$

with u^* being the steady-state unemployment rate. The total derivative of (13) with respect to β gives:[7]

$$\frac{dn^*}{d\beta} = \overbrace{\frac{\partial n^*}{\partial \beta}}^{+} + \underbrace{\overbrace{\frac{\partial n^*}{\partial u^*}}^{-} \cdot \overbrace{\frac{\partial u^*}{\partial k^*}}^{-} \cdot \overbrace{\frac{\partial k^*}{\partial \beta}}^{-}}_{-}. \tag{14}$$

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[6] Notice that firms hire workers according to the perceived labour demand curve at the wage rate legally set by the policymaker (minimum wage), or fixed by the monopolistic union, over the prevailing market-clearing level.

[7] Details are given in Appendix.

Eq. (14) says that the final effect of an increase in the direct monetary transfer to support child-rearing of households depends on two counterbalancing forces, and it appears to be ambiguous: (*i*) a positive (direct) effect of the child grant which tends to increase fertility by reducing the cost of raising children, and (*ii*) a negative (indirect) feedback effect which acts on fertility via the increased unemployment rate. In particular, a raise in the child-subsidy tends to lower the stock of capital per-capita (owing to a reduced cost of child-rearing) and this increases the long-run rate of unemployment. Given the negative relationship between unemployment and fertility, the higher u^* the lower the number of children desired by each individual, that is, the higher the child grant the more individuals substitute children for unemployed time.
To analyse ultimately which of the two forces dominates (either the direct positive effect of the child grant or the negative feedback effect which tends to reduce fertility via the increased unemployment) we combine Eqs. (6), (11) and (12) to obtain the long-run rate of fertility as a function of the child grant and the key parameters of the model:

$$n^*(\beta) = \frac{((1-\alpha)A)^{\frac{1}{\alpha}} \gamma(m-\beta)}{[(1+\gamma)(m-\beta)+\phi\beta]\underline{w}^{\frac{1-\alpha}{\alpha}}}, \tag{15}$$

From Eq. (15) thus the following proposition holds:

Proposition 1. *Introducing and/or raising a child-subsidy does always reduce the long-run rate of fertility.*

Proof. Differentiating (15) with respect to β gives

$$\frac{\partial n^*(\beta)}{\partial \beta} = \frac{-n^*(\beta) m \phi}{(m-\beta)[(1+\gamma)(m-\beta)+\phi\beta]} < 0, \tag{17}$$

for any $0 < \beta < m$. **Q.E.D.**

Proposition 1 reveals that the fertility rate is lower than whether child-subsidy policies are not implemented at all, and it stems directly from the role played by the long-run rate of unemployment on the rate of fertility. In particular, the negative feedback effect which acts on fertility decisions through the increased unemployment always prevails over the direct effect of the child payment which tends to increase the desired number of children via the reduced cost of child-rearing.
Therefore, the regulation of wages introduces a negative correlation among population growth and child-subsidy regardless of the values of preference and technology parameters, that is the higher the minimum (or union's) wage the higher the unemployment rate, and, in spite of a decreased cost of child-rearing, unemployment depresses individuals' fertility behaviour. An increasing child-subsidy therefore tends to overdraw such a negative effect on population growth, given the reduced stock of capital per-capita.
It is rather unusual to think about the possibility to increase fertility by introducing offspring taxes, but a straightforward corollary which emerges by Proposition 1 permits to conclude that countries with imperfect labour markets (such as the most part of European Union countries) should not introduce child-subsidy policies at all, but rather they should implement child-tax policies (lump-sum rebated to young-adult individuals at balanced budget).[8] In fact, for any given value of the regulated wage, the introduction of child-subsidies tends to reduce both the long-run stock of capital per-capita and the fertility rate, and the higher the child grant the higher the unemployment rate. Therefore, we may conclude that both unemployment and population ageing are worsened by the introduction of a (balanced budget) child-subsidy policy.

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[8] Notice that if we allow for the possibility that $\beta < 0$, in which case each child is taxed and the revenue of this tax is lump-sum rebated to young-aged individuals ($\tau_t < 0$), then the effect of such an offspring tax is to increase monotonically the demand for children in the long-run. Moreover, it is also important to note that – for any given value of the non-competitive wage – the higher the child-tax the higher the stock of capital per-capita and the lower the unemployment rate in the long-run. This is our current direction of research.

3 Endogenous fertility, time cost of child-rearing and unemployment benefit policies

In this section, in order to assess the robustness of the results stated in Section 2, we relax the assumption of a fixed child-rearing cost, by also introducing a proportional-to-wage cost of children, thus capturing the opportunity time-cost of raising each child.[9] We assume that u_t represents the time-t probability of being unemployed, and we allow for the possibility of an unemployment benefit mechanism financed at balanced budget with lump-sum taxes levied on young-aged individuals. In particular, following van Groezen et al. (2003), we now suppose that the government runs two distinct policies to balance out both unemployment benefit and child-care expenditure in every period. In the former case, the per-capita time-t budget constraint of the unemployment benefit system is given by:

$$z\,\underline{w}u_t = \theta_t, \tag{18}$$

where the left-hand side represents the unemployment benefit expenditure, paid with probability u_t to each young-adult agent ($0 < z < 1$ is the so-called replacement ratio), and the right-hand side the tax receipts ($\theta_t > 0$), whereas in the latter case (child-care expenditure) we suppose Eq. (5) still holds.[10]
Moreover, the total amount of resources needed to take care of one child is supposed to be divided among consumption and the time cost of children,[11] and it is therefore defined to be $m + q \cdot E_t(W_t)$,[12] where $0 < q < 1$ is the fraction of the time endowment that parents must be spent in order to raise one child, and $E_t(W_t) := \underline{w}(1 - u_t) + z\,\underline{w}u_t$ represents the expected income when young (which is simply sum of the labour income earned with probability $1 - u_t$, plus the unemployment benefit received with probability u_t).
Therefore, the problem faced by the representative agent born at time t is to maximise (P) subject to the following intra-temporal budget constraints:

$$\begin{aligned} c_t^{\,y} + s_t &= E_t(W_t)(1 - qn_t) - (m - \beta)n_t - \tau_t - \theta_t \\ c_{t+1}^{\,o} &= (1 + r_{t+1})s_t \end{aligned}.$$

The first order conditions for an interior solution are given by (1) and

$$\frac{\phi}{n_t} = \frac{1-\phi}{c_t^{\,y}} \cdot [qE_t(W_t) + m - \beta]. \tag{19}$$

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[9] The existence of a time cost of child-rearing might imply that people, anticipating that the minimum (or union's) wage generates unemployment (assumed to be distributed randomly across individuals when young), may take the opportunity to have a child while unemployed to capitalise on the lower opportunity cost. We thank the responsible editor Ian Walker for having pointed out this point.

[10] Since we assumed agents act atomistically, they do not take into account the government budget constraints when deciding on the desired number of children. Notice also that the policies specified by Eqs. (5) and (18) do not weight upon the old generation living in period t, as both the child allowance and the unemployment benefit schemes are financed exclusively with lump-sum taxes levied on the young-adult generation, that is the government redistribute resources within young-aged individuals living at the moment of the introduction of the policies.

[11] This child-cost structure is closely related to that one adopted by Boldrin and Jones (2002), who assumed agents do not derive utility from leisure. In particular, they examined an overlapping generations model of fertility choice where individuals live for three periods, young, middle age and old age, and assumed that only middle aged individuals are endowed with one unit of productive time supplied inelastically to the labour market, while they set the labour supply of young and old aged people to be equal to zero. Therefore, the percentage of child-rearing cost on working income (q) may be interpreted as an opportunity cost of the parents' home time which is increasing in their working income (see, among others, Cigno, 1991).

[12] It is worth noting that with this child cost structure a fraction $q \cdot n$ of the individual's time endowment is devoted to child-rearing activities. As a consequence, each young individual obtains an (expected) income $E_t(W_t)$ for each unit of supplied time.

Eq. (19) has the same economic interpretation than Eq. (2). However, the cost of raising children now depends also upon the time cost of children as well as the replacement ratio, q and z respectively. Note that, in this case, a necessary and sufficient condition for the existence of a positive solution for n_t is $qE_t(W_t)+m-\beta>0$, which implies that the child-subsidy can even be set at a higher level than the fixed cost of child-rearing.
Using (1), (19), the individuals' budget constraints as well as (5) and (18), the demand for children and the savings path are respectively determined by:

$$n_t = \frac{\phi \underline{w}(1-u_t)}{(1+\gamma)[qE_t(W_t)+m-\beta]+\phi\beta}, \tag{20}$$

$$s_t = \frac{\gamma \underline{w}(1-u_t)[qE_t(W_t)+m-\beta]}{(1+\gamma)[qE_t(W_t)+m-\beta]+\phi\beta}. \tag{21}$$

Note that if $q=0$, then Eqs. (20) and (21) collapse to (6) and (7), respectively.
The production side of the economy remains exactly the same than the one presented in Section 2, so that Eqs. (8)-(11) still hold. Therefore, using (20) and (21), equilibrium in goods as well as in capital markets implies:

$$k_{t+1} = \frac{\gamma}{\phi}\{q[\underline{w}(1-u_t)+z\,\underline{w}u_t]+m-\beta\}. \tag{22}$$

Using Eq. (11) to substitute out for u_t, the dynamic equilibrium sequence of capital is given by:

$$k_{t+1} = \frac{\gamma}{\phi}(m-\beta+q\,z\,\underline{w})+\frac{\gamma}{\phi}q(1-z)\underline{w}^{\frac{\alpha-1}{\alpha}}((1-\alpha)A)^{\frac{1}{\alpha}}k_t. \tag{23}$$

Steady-state implies $k_{t+1}=k_t=k^*$, so that the long-run per-capita stock of capital as a function of the child-subsidy may be expressed by the following equation (note that in this case the capital stock also depends on the level of the regulated wage):

$$k^*(\beta) = \frac{\underline{w}^{\frac{1-\alpha}{\alpha}}\frac{\gamma}{\phi}(m-\beta+q\,z\,\underline{w})}{\underline{w}^{\frac{1-\alpha}{\alpha}}-\frac{\gamma}{\phi}q(1-z)((1-\alpha)A)^{\frac{1}{\alpha}}}, \tag{24}$$

From Eq. (24) it can easily be ascertained that a necessary and sufficient condition for the existence of a positive steady-state equilibrium is $\underline{w}>w_D:=\left(\frac{\gamma q(1-z)}{\phi}\right)^{\frac{\alpha}{1-\alpha}}\cdot((1-\alpha)A)^{\frac{1}{1-\alpha}}$, which is always satisfied because w_D is smaller than the market-clearing wage.[13]
As in the previous section, the introduction of a child-subsidy clearly affects negatively the steady-state stock of capital per-capita.
Since in this paper we are mainly concerned with the analysis of the effects of the child-subsidy on the demand for children in the long-run, we now combine Eqs. (11), (20) and (24) to obtain the steady-state value of n as a function of β:

$$n^*(\beta) = \frac{\phi\underline{w}\cdot\{1-u^*[k^*(\beta)]\}}{(1+\gamma)\cdot\{q\underline{w}\cdot[1-u^*[k^*(\beta)]\cdot(1-z)]+m-\beta\}+\phi\beta}. \tag{25}$$

Differentiation of (25) with respect to β gives:

$$\operatorname{sgn}\left\{\frac{\partial n^*(\beta)}{\partial\beta}\right\} = \operatorname{sgn}\left\{-\underline{w}^{\frac{1-\alpha}{\alpha}}+\frac{\gamma}{\phi}q(1-z)((1-\alpha)A)^{\frac{1}{\alpha}}\right\}, \tag{26}$$

[13] Moreover, using (23) it is easy to verify that stability requires $\underline{w}>w_D$. For the sake of brevity, we do not report here the complete proofs both of: (1) w_D is smaller than the market-clearing wage, and (2) stability requirement, which are of course available on request.

which is negative if β is positive, meaning that the fertility rate is lower than whether child-subsidy policies are not introduced at all. Therefore, the opportunity to have a child while unemployed to capitalise on the lower opportunity cost does not affect the result stated in Proposition 1. This result reaffirms that one obtained in the previous section and reveals that it is a robust feature of minimum (or union's) wage economies.

We therefore suggest that countries with imperfect labour markets (e.g., the most part of European Unions countries) should not introduce child-subsidy policies at all, as not only capital accumulation is reduced and unemployment is increased in the long-run, but population growth is further depressed.

To sum up, in contrast with Becker and Barro (1988) and Barro and Becker (1989), who showed that increased child-rearing costs is an important factor contributing to low fertility rates, especially in developed countries, we found that if labour markets are far from being competitive, a reduced child-rearing cost may be an important factor contributing to low fertility rates, since a higher child-subsidy reduces both child-rearing costs and capital accumulation, and a lower capital accumulation ultimately implies a higher unemployment rate and a decreasing demand for children in the long-run. This negative (indirect) effect more than counterbalances the direct effect of the child-subsidy which acts positively on the long-run rate of fertility.

4 Conclusions

This paper examines how subsidy policies to support child-rearing of households affect the fertility rate in a textbook OLG model extended to account for a labour market imperfection (e.g., a minimum wage legally set by the government or a monopoly union's wage) as well as endogenous fertility decisions of individuals. We found that the fertility rate is lower than whether child-subsidy policies are not implemented at all. Our finding suggests that countries with imperfect labour markets and which, in order to increase population growth, support families through direct child payments, obtain the unexpected result to depress further fertility rates. Therefore, for the most part of European Union countries, where imperfect labour markets are the rule rather then exception, subsidy policies to support child-rearing of households should not be introduced at all, as they reduce both population growth and the employment rate.

Appendix

In this appendix we present details to clarify the role of the child grant on the long-run fertility rate. In particular, we have that:

$$\frac{\partial n^*}{\partial \beta} = \frac{(1+\gamma-\phi)n^* \phi \underline{w}}{(1+\gamma)(m-\beta)+\phi\beta} > 0, \tag{A1}$$

$$\frac{\partial n^*}{\partial u^*} = \frac{-\phi \underline{w}}{(1+\gamma)(m-\beta)+\phi\beta} < 0, \tag{A2}$$

$$\frac{\partial u^*}{\partial k^*} = -\left[\frac{(1-\alpha)A}{\underline{w}}\right]^{\frac{1}{\alpha}} < 0. \tag{A3}$$

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