E C O N O M I C S   B U L L E T I N

Labour income taxation, child rearing policies and fertility

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Abstract

We examine how subsidy policies to support child-rearing affect the fertility rate in a textbook general equilibrium overlapping generations model extended to account for endogenous fertility decisions of individuals. It is shown the counter-intuitive result that increasing the child grant may actually reduce the long-run fertility rate.

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1 Introduction

A recent and widespread decrease in fertility rates occurred in advanced countries. As a consequence, an issue of major policy concern is the problem of supporting social security systems, such as pay-as-you-go pension schemes, in the face of declining ratios of economically active to retired households. Here, we assume that for some exogenous reason, such as the above mentioned one, the government would like to increase fertility.[1] So far the literature has ignored a fully general equilibrium dynamic context in child policies-fertility studies: for instance, Apps and Rees (2004) is limited to a static context, van Groezen et al. (2003) is limited to a partial equilibrium dynamic context. An exception is Momota (2000) which however builds a model based on very specific features.[2]

This paper studies the effect of child-subsidy support policies on the long-run rate of fertility using the simplest possible OLG model (Diamond, 1965) extended to account for endogenous fertility decisions of individuals and a simple balanced budget policy with only two instruments, a wage tax and a lump-sum child payment.[3] We found that, if capital is important enough in production (or, alternatively, the preference for children is sufficiently high), the fertility rate may be lower than whether the subsidy policy is not implemented at all. Otherwise, if the capital's weight in technology is sufficiently low (or, alternatively, the preference for children is sufficiently low), a value of the child grant for which population growth is maximised does exist. Beyond such a level the relation between long-run fertility rate and childcare policy is negative.

The paper is organised as follows. In Section 2 we develop the model and the main steady-state results are analysed and discussed. Section 3 bears the conclusions.

2 The Model

Agents have identical preferences and are assumed to belong to an overlapping generations structure with finite lifetimes. Life is separated among three periods: childhood, young adulthood and old-age. During childhood individuals do not make economic decisions and thus they consume a fixed fraction of the time endowment from their parents. Adult individuals belonging to generation t (N_t) have a homothetic and separable utility function defined over young-aged consumption ($c_t^{\ y}$), old-aged consumption ($c_{t+1}^{\ o}$) and from the number of children they have (n_t),[4] as in Galor and Weil (1996). As an adult each young agent is supposed to have an endowment of one unit of time which is supplied inelastically to the labour market. The perceived market-clearing wage (w_t) is used to consume, to pay taxes, to raise children and to save. We assume that raising children requires a fixed cost m per child (measured in units of market goods). Moreover, parents receive a lump-sum subsidy – provided by the government at balanced budget – for each child to support child-rearing, $\beta \in (0, m)$. During old-age agents are retired and live on the proceeds of their savings (s_t) plus the accrued interest at the rate r_{t+1}. The representative individual born at time t is faced with the problem of maximising the following logarithmic utility function:

$$\max_{\{c_t^{\ y}, c_{t+1}^{\ o}, n_t\}} U_t\left(c_t^{\ y}, c_{t+1}^{\ o}, n_t\right) = (1-\phi)\ln\left(c_t^{\ y}\right) + \gamma \ln\left(c_{t+1}^{\ o}\right) + \phi \ln(n_t),$$

[1] Notice that we do not look for optimal child subsidy policy, but simply look at the effects of changes in the child grant in a context which is almost certainly non-optimal.

[2] The model used in this paper is, among the related papers, relatively close to, though simpler than, the Momota's model (2000). Differently from this paper, Momota (2000) assumed two kinds of individuals, a gender wage gap, a time cost for childrearing and a rather special form of the subsidy policy and showed that the male and female labour shares, the productivity of child-rearing and the level of the tax rate are key variables for explaining the child-subsidy policy effects.

[3] It is important to stress that we have sought to clarify these theoretical findings using as parsimonious a model as possible.

[4] Note that n_t represents the number of children with $n_t - 1$ being the population growth rate.

subject to the inter-temporal budget constraint

$$c_t^{\,y} + \frac{c_{t+1}^{\,o}}{1+r_{t+1}} = w_t(1-\tau_t) - (m-\beta)n_t \,,$$

where $0<\tau_t<1$ is a tax rate on labour income, $0<\gamma<1$ is the subjective discount factor and $0<\phi<1$ captures the importance in the welfare function of consuming while young relative to the utility of children.

The first order conditions for an interior solutions are therefore given by:

$$\frac{c_{t+1}^{\,o}}{c_t^{\,y}} \cdot \frac{1-\phi}{\gamma} = 1 + r_{t+1} \,, \qquad \text{(FOC1)}$$

$$\frac{\phi}{n_t} = \frac{1-\phi}{c_t^{\,y}}(m-\beta). \qquad \text{(FOC2)}$$

Eq. (FOC1) equates the marginal rate of substitution between working period and retirement period consumption to their relative prices, whereas Eq. (FOC2) equates the marginal utility of having a child with the involved marginal costs in terms of forgone utility of consumption. Note that a necessary and sufficient condition for the existence of a positive solution for n_t is $m-\beta>0$, that is the net cost of raising children must be strictly positive.

Using (FOC1) and (FOC2) along with the inter-temporal budget constraint, the demand for children and the savings path are respectively given by:

$$n_t = \frac{\phi}{1+\gamma} \cdot \frac{w_t(1-\tau_t)}{m-\beta} \,, \qquad (1)$$

$$s_t = \frac{\gamma}{1+\gamma} \cdot w_t(1-\tau_t). \qquad (2)$$

The government runs a balanced budget policy in every period. The child-rearing subsidy is supposed to be entirely financed by levying and adjusting over time taxes on labour income. Therefore, the per-capita time-t government constraint is simply:

$$\beta n_t = w_t \tau_t \,, \qquad (3)$$

where the left-hand side represents the total child care expenditure and the right-hand side the tax receipt. We assume agents act atomistically and do not take the government budget constraint into account when deciding on the desired number of children.

Inserting (3) into (1) to eliminate τ_t and rearranging terms yields:

$$n_t = \frac{\phi w_t}{(1+\gamma)(m-\beta)+\phi\beta} \,, \qquad (4)$$

Now, combining (2), (3) and (4), the savings function is determined by:

$$s_t = \frac{\gamma(m-\beta)w_t}{(1+\gamma)(m-\beta)+\phi\beta}. \qquad (5)$$

As regards the production sector, firms are identical and act competitively. The (aggregate) constant returns to scale Cobb-Douglas technology of production is $Y_t = AK_t^{\,\alpha} L_t^{\,1-\alpha}$,[5] where Y_t, K_t and $L_t = N_t$ are the time-t output, capital and labour input respectively, $A>0$ represents a scale parameter and $\alpha \in (0,1)$ is the capital's weight in technology. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per-capita respectively, the intensive form production function may be written as $y_t = Ak_t^{\,\alpha}$. Assuming total depreciation of capital at the end of each period and knowing that final output is treated at unit price, profits maximisation leads to the following marginal conditions for capital and labour, respectively:

[5] Adding exogenous growth in labour productivity does not alter any of the substantive conclusions of the model and, hence, it is not included here.

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{6}$$

$$w_t = (1-\alpha) A k_t^{\alpha}. \tag{7}$$

Knowing that $N_{t+1} = n_t N_t$, the market-clearing condition in goods as well as in capital markets is expressed by the condition $n_t k_{t+1} = s_t$. Substituting out for n and s from Eqs. (4) and (5) respectively, such a condition boils down to the following long-run capital per-capita:

$$k^*(\beta) = \frac{\gamma}{\phi}(m - \beta). \tag{8}$$

As it can easily be seen by looking at Eq. (8), an increase in the child grant does always reduce the long-run stock of capital per-capita (the higher the child grant the lower the net cost of raising children, $m - \beta$), that is $\frac{\partial k^*(\beta)}{\partial \beta} = -\frac{\gamma}{\phi} > 0$.

2.1 Comparative Static Analysis

Which are the effects of the child-rearing subsidy on the long-run rate of fertility? This simple question gives rise to very interesting findings in our basic OLG model.

The long-run fertility rate may be written as a generic unction of the child grant in the following way:

$$n^* = n^*\{\beta, w^*[k^*(\beta)]\}, \tag{9}$$

so that the total derivative of Eq. (9) with respect to β gives:[6]

$$\frac{dn^*}{d\beta} = \overbrace{\frac{\partial n^*}{\partial \beta}}^{+} + \underbrace{\overbrace{\frac{\partial n^*}{\partial w^*}}^{+} \cdot \overbrace{\frac{\partial w^*}{\partial k^*}}^{+} \cdot \overbrace{\frac{\partial k^*}{\partial \beta}}^{-}}_{-}. \tag{10}$$

Eq. (10) says that the final effect of an increase in the child grant on the long-run rate of fertility depends on two counterbalancing forces, and it appears to be ambiguous: (1) a positive (direct) effect which tends to increase fertility by decreasing the cost of child-rearing, and (2) a negative (indirect) feedback effect which acts on fertility through the decreased wage rate. In particular, a raise in the child grant tends to decrease the stock of capital per-capita (owing to an increased cost of child-rearing) and this reduces the wage rate perceived by young-adult individuals for each hour worked. Given the positive relationship between fertility and wages (Malthusian Fertility Effect), the lower w^* the lower the desired number of children.

To analyse ultimately which of the two forces dominates, we now combine Eqs. (4), (7) and (8) to obtain the long-run rate of fertility as a function of β and the key parameters of the model:

$$n^*(\beta) = \frac{\phi(1-\alpha) A \left[\frac{\gamma}{\phi}(m-\beta)\right]^{\alpha}}{(1+\gamma)(m-\beta) + \phi\beta}, \tag{11}$$

and the following propositions hold:

Proposition 1. *The introduction of a child grant increases the long-run rate of fertility if and only if* $\alpha < \overline{\alpha}$.

Proof. The proof straightforwardly derives by differentiating (11) with respect to the child grant and evaluating it at $\beta = 0$:

[6] Details are given in Appendix A.

$$\frac{\partial n^*(\beta)}{\partial \beta}\Big|_{\beta=0} = \frac{n^*(0)\cdot[1+\gamma-\phi-\alpha(1+\gamma)]}{m(1+\gamma)}, \tag{12}$$

so that

$$\begin{cases} \frac{\partial n^*(\beta)}{\partial \beta}\Big|_{\beta=0} > 0 \quad iff \quad \alpha < \overline{\alpha} \\ \frac{\partial n^*(\beta)}{\partial \beta}\Big|_{\beta=0} < 0 \quad iff \quad \alpha > \overline{\alpha} \end{cases}. \tag{13}$$

where $\overline{\alpha} \equiv \frac{1+\gamma-\phi}{1+\gamma}$ **Q.E.D.**

Proposition 2. *Let* $\alpha < \overline{\alpha}$ *hold. Then* $n^*(\beta)$ *is an inverted U-shaped function with* $\beta = \beta^*$ *being an interior global maximum. Let* $\alpha > \overline{\alpha}$ *hold. Then an increase in the child grant does always reduce the long-run fertility rate.*

Proof. Differentiation of Eq. (11) with respect to β gives:

$$\frac{\partial n^*(\beta)}{\partial \beta} = \frac{n^*(\beta)\cdot\{m[1+\gamma-\phi-\alpha(1+\gamma)]-\beta(1-\alpha)(1+\gamma-\phi)\}}{(m-\beta)[(m-\beta)(1+\gamma)+\phi\beta]}. \tag{14}$$

If $\alpha < \overline{\alpha}$ then

$$\frac{\partial n^*(\beta)}{\partial \beta} \gtrless 0 \Leftrightarrow \beta \lessgtr \beta_n. \tag{15}$$

where

$$\beta_n \equiv \frac{m[1+\gamma-\phi-\alpha(1+\gamma)]}{(1-\alpha)(1+\gamma-\phi)}.[7] \tag{16}$$

If $\alpha > \overline{\alpha}$ then $\frac{\partial n^*(\beta)}{\partial \beta} < 0$ for any $0 < \beta < m$. **Q.E.D.**

Proposition 1 shows that if the capital's weight in technology is high enough (or, alternatively, if the subjective discount factor is low enough and/or the intra-temporal relative weight between consuming while young relative to the utility of children is sufficiently high), then the introduction of a fixed child grant brings upon a reduction in the rate of fertility, which is always lower then whether the subsidy policy is not introduced at all.

Proposition 2 says that if labour is important enough in the production process (alternatively, if individuals prefer to postpone consumption in the future and/or the parents' taste for children is sufficiently low), then raising the child grant increases the long-run rate of fertility if and only if $\beta \le \beta_n$; otherwise, if the child subsidy is fixed at too high a level, the fertility rate is reduced as the child grant is increased.

3 Conclusions

This paper examines how subsidy policies to support child-rearing of households affect the fertility rate in the standard OLG model (Diamond, 1965) extended to account for endogenous fertility and a balanced budget policy. The main results are: (1) when the capital's weight in production is sufficiently high, the fertility rate is lower than whether the subsidy policy is not implemented at all; (2) when the capital's weight in production is low enough, on the technological side, and the subjective discount

[7] It is worth to note that $\beta_n < m$ holds true whatever the values of α, ϕ and γ.

factor is high enough and/or the intra-temporal relative weight between consuming while young relative to the utility of children is sufficiently low, on the preference side, raising the child grant increases the long-run fertility rate but only up a certain level, where population growth is maximised: beyond such a level, the higher the child subsidy the lower the long-run fertility rate.

The policy implications are straightforward: in an otherwise identical economy, family policies may have at all opposite effects depending on the weight of capital in production relative to preference parameters.

The interest of these results lies in: (1) the relevance of their messages showing a new perspective for family policies, (2) the simplicity with which are obtained, that is within a standard dynamic general equilibrium overlapping generations model extended to account for endogenous fertility and a balanced budget child-subsidy support policy, and (3) having analytically picked up the relevant "threshold" value of the child grant beyond which population growth is reduced.

Appendix

In this appendix we present details to clarify the role of the child grant on the long-run fertility rate. In particular, we have that:

$$\frac{\partial n^*}{\partial \beta} = \frac{(1+\gamma-\phi)n^*}{(1+\gamma)(m-\beta)+\phi\beta} > 0, \tag{A1}$$

$$\frac{\partial n^*}{\partial w^*} = \frac{\phi}{(1+\gamma)(m-\beta)+\phi\beta} > 0, \tag{A2}$$

$$\frac{\partial w^*}{\partial k^*} = \alpha(1-\alpha)A(k^*)^{\alpha-1} > 0, \tag{A3}$$

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