E C O N O M I C S B U L L E T I N

# Spatial Discrimination with Quantity Competition and High Transportation Costs: a Note

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## *Abstract*

In this paper we extend the analysis of the standard model of spatial discrimination with quantity competition along the linear city to the case in which the unit transportation cost is greater than one. We show that in such a case the unique subgame perfect Nash equilibrium in locations is a dispersed symmetric equilibrium. Moreover, at this equilibrium firms' locations are not monotone in the transportation cost parameter.

We thank J. M. Chamorro Rivas for useful correspondence, the editor, Quan Wen, and an anonymous referee for valuable comments. The usual disclaimer applies.
**Citation:** Benassi, Corrado, Alessandra Chirco, and Marcella Scrimitore, (2007) "Spatial Discrimination with Quantity Competition and High Transportation Costs: a Note." *Economics Bulletin*, Vol. 12, No. 1 pp. 1-7
**Submitted:** August 18, 2006. **Accepted:** January 15, 2007.
**URL:** http://economicsbulletin.vanderbilt.edu/2007/volume12/EB-06L10022A.pdf

# 1 Introduction

The analysis of spatial price discrimination along the linear city with duopolistic Cournot competition and identical linear demand at all addresses has almost always be confined to situations with low transportation costs, so as to ensure full market coverage by both firms from any possible pair of locations (Anderson and Neven, 1991; Hamilton *et al*, 1989). Under the assumption that the ratio between the unit constant transportation cost $t$ and the maximum demand price (normalized to 1 in the sequel) is lower than 1/2, the only subgame perfect Nash equilibrium is agglomeration at the center of the linear city, which in turn implies a perfect symmetry of firms' behaviour at all market addresses. An extension of the analysis to the range $t \in [1/2, 1]$ has been provided by Chamorro Rivas (2000). He shows that the agglomerated equilibrium coexists with a dispersed equilibrium for $t > 2/3$: the latter entails duopolistic interaction at all addresses for $t \in (2/3, 10/11)$, while it allows for monopolistic areas (not including the firms' locations) at the extremes of the city for $t \in [10/11, 1]$. In this note we study the behaviour of the Cournot spatial markets for $t \in [1, 4)$. Our main result is that in this range a symmetric, dispersed equilibrium exists for all transportation costs, which entails duopolistic competition in a central area, and monopoly of each firm on a segment at the extreme of its market side – the monopolistic areas including the firms' location for $t \geq 1.3453$. The scope for duopolistic interaction at the center of the city vanishes when $t = 4$. For higher values of $t$ firms become fully separated monopolists.

# 2 The basic setup

Consider a linear city of unit length, along which consumers are uniformly distributed. Two firms, labelled 1 and 2, are engaged in a location-quantity game. They have to decide their respective locations and then compete in quantities at all market addresses $x \in [0, 1]$, bearing a unit transportation cost linear in distance, and facing the linear (inverse) market demand $P(Q(x)) = 1 - Q(x)$. We rule out arbitrage by assuming infinite transportation costs on the consumers' side. This two-stage game is solved by backward induction.

Denote with $a$ the location of firm 1 and with $1 - b$ the location of firm 2 (where $a \leq 1 - b$) The unit transportation costs incurred by the firms to serve an address $x$ can be written as $t|a - x| = td_1(x)$ and $t|1 - b - x| = td_2(x)$, where $t$ is a positive constant. Therefore, given that each point $x$ can be considered as a separate sub-market, the solution for the quantity stage is given by the following quantity schedules:

$$q_i(x) = \begin{cases} \max\left\{0, q_i^D\right\} & \text{if } q_j(x) > 0 \\ \max\left\{0, q_i^M\right\} & \text{if } q_j(x) = 0 \end{cases} \tag{1}$$

with $i, j = 1, 2$, $i \neq j$ and

$$q_i^D(x) = \tfrac{1}{3} - \tfrac{2}{3}t\ d_i(x) + \tfrac{1}{3}t\ d_j(x); \qquad q_i^M = \tfrac{1}{2} - \tfrac{1}{2}td_i(x) \tag{1'}$$

Clearly, $q_i^D > 0$ irrespective of $a$ and $b$, only if $t < 1/2$, while $q_i^M > 0$ irrespective of $a$ and $b$ only if $t < 1$. These assumptions highly simplify the solution of the location stage. The former ensures that for all $a$ and $b$ firms interact duopolistically at all addresses; the latter ensures that for all $a$ and $b$ all addresses are served by at least one firm. If these assumptions are relaxed, the pattern of market coverage (i.e., for any of the addresses full coverage, coverage by one firm, or no coverage) becomes contingent on the location pair. This is the issue we deal with in the next section.

## 3 Transportation costs, market coverage and equilibria

Assume $t \in [1/2, 4)$. Given that leapfrogging is ruled out by definition, the set of the firms' feasible choices in terms of distance from the right and left extremes of the line is $L = \{(a, b) \in \mathbb{R}^2_+ \mid b \leq 1 - a\}$. This set can be partitioned into several subsets, according to the possibility for firms to deliver positive quantities at the various market addresses. In particular, given that: (i) $q_1^D$ vanishes at the right endpoint $x = 1$ for $a = 1 - 1/2t - b/2$; (ii) $q_2^D$ vanishes at the left endpoint $x = 0$ for $b = 1 - 1/2t - a/2$; (iii) $q_1^D$ *and* $q_2^D$ vanish at the other firm's location for $b = 1 - 1/2t - a$; then for $t \in [0.5, 1)$ the set $L$ can be partitioned into the following subsets:

- $D \equiv \{(a, b) \in L \mid a > 1 - \frac{1}{2t} - \frac{1}{2}b \wedge b > 1 - \frac{1}{2t} - \frac{1}{2}a\}$, which includes all $(a, b)$ pairs such that $q_1^D(x)$ and $q_2^D(x) > 0$, $\forall x$;
- $A \equiv \{(a, b) \in L \mid a \leq 1 - \frac{1}{2t} - \frac{1}{2}b \wedge b > 1 - \frac{1}{2t} - \frac{1}{2}a\}$, where $(a, b)$ are such that $q_1^D(x) > 0$ for $x \in [0, \overline{x}_1)$, $q_1^D(x) = 0$ for $x \in [\overline{x}_1, 1]$, where $\overline{x}_1 > 1 - b$, while $q_2^D(x) > 0$ $\forall x$;
- $A' \equiv \{(a, b) \in L \mid a > 1 - \frac{1}{2t} - \frac{1}{2}b \wedge b \leq 1 - \frac{1}{2t} - \frac{1}{2}a\}$, which is the set symmetrical to $A$;
- $B \equiv \{(a, b) \in L \mid a \leq 1 - \frac{1}{2t} - \frac{1}{2}b \wedge b \leq 1 - \frac{1}{2t} - \frac{1}{2}a$ and $b > 1 - \frac{1}{2t} - a\}$, which comprises the $(a, b)$ pairs such that $q_1^D(x) > 0$ for $x \in [0, \overline{x}_1)$, $q_1^D(x) = 0$ for $x \in [\overline{x}_1, 1]$, where $\overline{x}_1 > 1 - b$, and $q_2^D(x) > 0$ for $x \in (\overline{x}_2, 1]$, $q_2^D(x) = 0$ for $x \in [0, \overline{x}_2]$, $\overline{x}_2 < a$;
- $C \equiv \{(a, b) \in L \mid b \leq 1 - \frac{1}{2t} - a\}$, whose elements are such that $q_1^D(x) > 0$ for $x \in [0, \overline{x}_1)$, $q_1^D(x) = 0$ for $x \in [\overline{x}_1, 1]$, where $\overline{x}_1 \leq 1 - b$, and $q_2^D(x) > 0$ for $x \in (\overline{x}_2, 1]$, $q_2^D(x) = 0$ for $x \in [0, \overline{x}_2]$, $\overline{x}_2 \geq a$.

This basic partition, already suggested by Chamorro Rivas (2000), is shown in Figure 1(a) for $t = 4/5$. When the analysis is extended to $t \geq 1$, the key problem arises that for some $(a, b)$ pairs, one or both firms may become unable to deliver positive quantities even to market sites not reached by the rival firm, i.e. market sites they could serve monopolistically, given the other firm's choice.

Given that $q_1^M$ vanishes in $x = 0$ for $a = 1/t$, and that $q_2^M$ vanishes in $x = 1$ for $b = 1/t$, for $t \geq 1$ the sets $A$, $A'$, $B$, and $C$ can in turn be partitioned according to the following scheme:

- For $t \in [1, 2)$, the set $A$ is partitioned into $\overline{A}$ and $\underline{A}$, with $\overline{A} \equiv \{(a, b) \in A | a < 1/t \wedge b \geq 1/t\}$ and $\underline{A} \equiv \{(a, b) \in A | a < 1/t \wedge b < 1/t\}$; similarly, $A'$ is partitioned into $\overline{A}'$ and $\underline{A}'$, with $\overline{A'} \equiv \{(a, b) \in A | a \geq 1/t \wedge b < 1/t\}$ and $\underline{A}' \equiv \{(a, b) \in A | a < 1/t \wedge b < 1/t\}$. Notice that $\underline{A}$ and $\underline{A}'$ become empty, so that $A = \overline{A}$ and $A' = \overline{A'}$, for $t \geq 2$;
- For $t \in (3/2, 2]$, $B$ is partitioned into three subsets: $\overline{B} \equiv \{(a, b) \in B | a < 1/t \wedge b \geq 1/t\}$, $\overline{B'} \equiv \{(a, b) \in B | a \geq 1/t \wedge b < 1/t\}$, and $\underline{B} \equiv \{(a, b) \in B | a \leq 1/t \wedge b \leq 1/t\}$; for $t \in (2, 5/2)$ a fourth subset can be defined, $\overline{\overline{B}} \equiv \{(a, b) \in B | a \geq 1/t \wedge b \geq 1/t\}$. Notice also that for $t \in (5/2, 4]$, $\underline{B}$ is empty, and hence $B$ turns out to be partitioned into $\overline{B}$, $\overline{B'}$ and $\overline{\overline{B}}$.
- For $t \in [3/2, 5/2)$, $C$ is partitioned into three subsets: $\overline{C} \equiv \{(a, b) \in C | a < 1/t \wedge b \geq 1/t\}$, $\overline{C'} \equiv \{(a, b) \in C | a \geq 1/t \wedge b < 1/t\}$, and $\underline{C} \equiv \{(a, b) \in C | a < 1/t \wedge b < 1/t\}$; for $t \in [5/2, 4]$ a fourth subset can be defined, $\overline{\overline{C}} \equiv \{(a, b) \in C | a \geq 1/t \wedge b \geq 1/t\}$, so that $C$ can be partitioned into the four subsets $\underline{C}$, $\overline{C}$, $\overline{C'}$, and $\overline{\overline{C}}$.

Figures 1(b) to 1(d) show the partition of the $L$ set for some relevant values of $t$.[1]

[1] To aid intuition, we remark that upper-barred sets include location pairs such that there exists at least one extreme portion of the market which is covered by no firm. In particular, this completely unserved portion of the market lies at the right extreme in $\overline{A}$, $\overline{B}$ and $\overline{C}$, at the left extreme in $\overline{A}'$, $\overline{B}'$ and $\overline{C}'$, and at both extremes in $\overline{\overline{B}}$ and $\overline{\overline{C}}$.

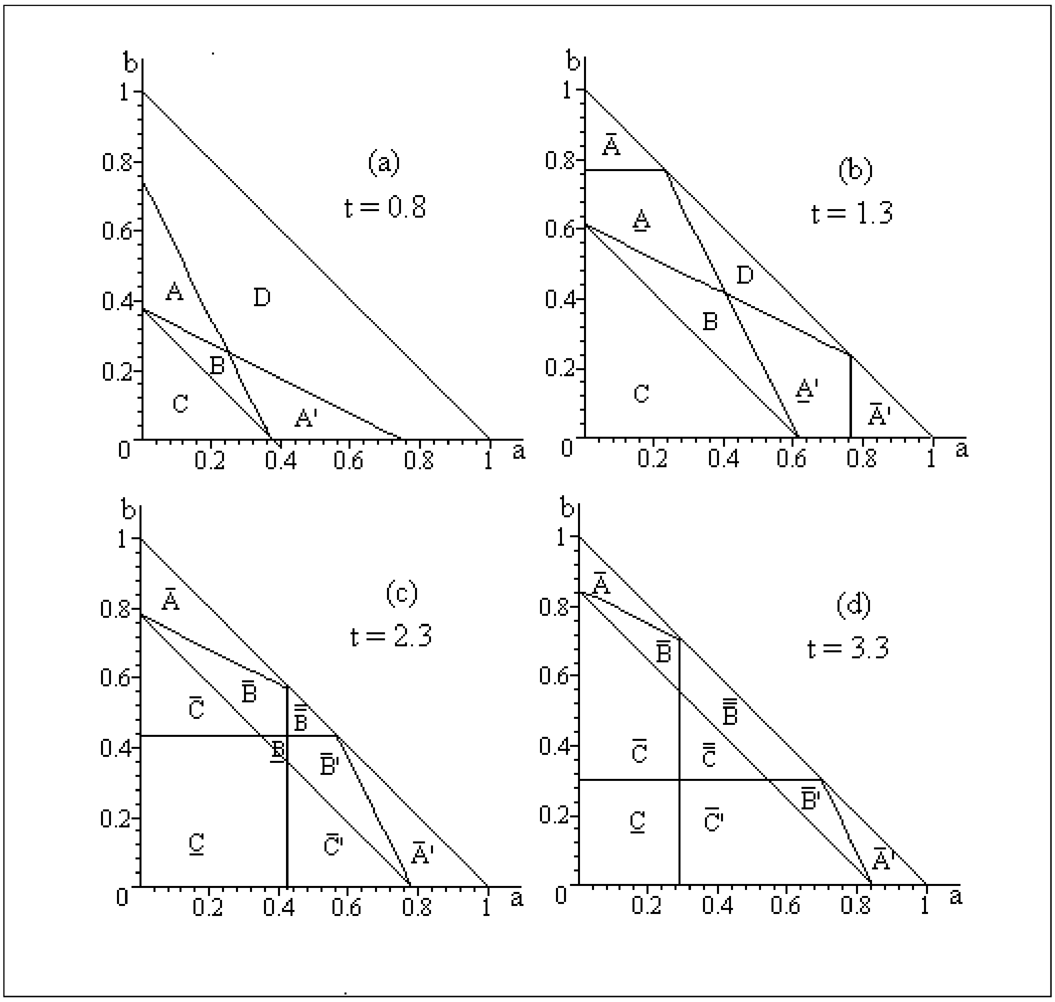


Figure 1. Partitions of the feasible locations set

We now turn to the solution of the model. We recall that the profits of firm $i$ at an address $x$ are $\pi_i^D(x) = \left[(1 + td_j(x) - 2td_i(x))/3\right]^2$, if at that address firms compete duopolistically. However, profits are $\pi_i^M(x) = \left[(1 - td_i(x))/2\right]^2$ and $\pi_j = 0$ if firm $i$ is monopolist at $x$. Moreover, given (1) and (1'), we can easily locate the boundaries between the segments of the market served and unserved by the two firms. In particular, $q_1^D$ vanishes at $x_1^r = 1/t - 1 + b + 2a$ when this occurs at the right of $1 - b$, while it vanishes at $x_1^l = (1/t + 1 - b + 2a)/3$, when this occurs at the left of $1 - b$; $q_1^M$ vanishes at $x_1^M = a - 1/t$. As for firm 2, $q_2^D$ vanishes at $x_2^l = 2 - 1/t - 2b - a$ when this occurs at the left of $a$, and at $x_2^r = (2 - 1/t - 2b + a)/3$ when it occurs at the right of $a$; finally, $q_2^M$ vanishes at $x_2^M = 1/t + 1 - b$.

By using the definitions of the duopolistic and monopolistic profits consistently with the boundaries of the regions where they apply, we prove in the Appendix the following Proposition.

**Proposition 1** *For* $t \in [1, 4)$ *no equilibria exist in which one or both firms leave uncovered an extreme segment of their market side.*

The intuition behind this result is quite straightforward. In the trade-off between serving regions of the market already covered by the rival firm and serving regions in which the firm may behave monopolistically, this second option is obviously preferred.

On the basis of Proposition 1, equilibria have to be looked for in the remaining sets, $D$, $A$ and $A'$, $\underline{A}$ and $\underline{A}'$, $B$, $\underline{B}$, $C$ and $\underline{C}$. In order to identify these equilibria, we proceed as follows: we first look for the maxima of the profit functions defined over each of the above sets, and identify the range of values of $t$ such that these maxima belong to the domains of those functions; secondly, we verify that these candidate equilibria are robust to unilateral deviations generating location pairs belonging to alternative sets.

Given this procedure, the analysis of the profit functions allows to establish the following proposition.

**Proposition 2** *(a) Let $t \in [1/2, 1]$; a unique agglomerated equilibrium, $a^D = b^D = 1/2$, exists for $t \leq 2/3$; for $t \in (2/3, 1]$ the agglomerated equilibrium coexists with a dispersed symmetric one. The latter is given by $a^{D'} = b^{D'} = (1/2t) - 1/4$, with full market coverage by both firms for $t < 10/11$; by $a^B = b^B = \left(104t - 23 - 2\sqrt{-117t^2 + 540t - 356}\right)/217t$ for $t \in [10/11, 1]$, a case which entails monopoly areas (not including the firms' locations) at the extremes of the market; (b) Let $t \in (1, 4)$; then there is a unique dispersed equilibrium at $a^B = b^B = \left(104t - 23 - 2\sqrt{-117t^2 + 540t - 356}\right)/217t$ (with the same characteristics as above) when $t < 1.3453$, and a unique dispersed equilibrium, at $a^C = b^C = \left(3\sqrt{3} - 4\right)\left(2t - 4 + 3\sqrt{3}\right)/11t$, when $t \geq 1.3453$.*

**Proof.** We first look for candidate equilibria. For part (a) of Proposition 2 see Chamorro Rivas (2000). In our setup, his proof amounts to showing that (i) given the definition of the profit functions in $D$, an intersection of the associated reaction functions consistently occurs in $D$ at $a^D = b^D = 1/2$ iff $t \in [1/2, 1]$, and at $a^{D'} = b^{D'} = (1/2t) - 1/4$ iff $t \in (2/3, 10/11)$; (ii) given the definition of the profit functions in $A$ ($A'$) an intersection of the associated reaction functions never occurs in $A$ ($A'$); (iii) given the definition of the profit functions in $B$, an intersection of the associated reaction functions consistently occurs in $B$ for $t \in [10/11, 1]$ at

$$a^B = b^B = \frac{1}{217t}\left(104t - 23 - 2\sqrt{540t - 356 - 117t^2}\right) \tag{2}$$

with $a^{D'} = b^{D'} = a^B = b^B$ for $t = 10/11$.[2]

As to part (b) of the Proposition, we first notice that part (a) rules out that an equilibrium might be in $D$, $A$ ($A'$), $\underline{A}$ ($\underline{A}'$), for $t > 1$. As to set $B$, we observe that in $B$ profits are:

$$\Pi_1^B(a, b; t) = \int_0^{x_2^l} \pi_1^M dx + \int_{x_2^l}^{x_1^r} \pi_1^D dx;\ \Pi_2^B(a, b; t) = \int_{x_2^l}^{x_1^r} \pi_2^D dx + \int_{x_1^r}^{1} \pi_2^M dx$$

The FOCs and the SOCs for their maximization are satisfied only by the pair given in (2), which lies in $B$ for all $t \in [10/11, 1.3453)$. Therefore, this symmetric dispersed solution, which coexists with the agglomerated one for $t \in [10/11, 1)$, is the unique candidate equilibrium in the interval $t \in [1, 1.3453)$. Moreover, since $\underline{B}$ coincides with $B$ for the above interval of values of $t$, while it is a subset of $B$ for $t \in [3/2, 5/2)$, we can exclude that in the latter interval there might be equilibria located in $\underline{B}$. In $C$ the profit functions are

$$\Pi_1^C(a, b; t) = \int_0^{x_2^r} \pi_1^M dx + \int_{x_2^r}^{x_1^l} \pi_1^D dx;\ \Pi_2^C(a, b; t) = \int_{x_2^r}^{x_1^l} \pi_2^D dx + \int_{x_1^l}^{1} \pi_2^M dx$$

The FOCs and the SOCs of the corresponding maximization problem are satisfied for

$$a^C = b^C = \left(3\sqrt{3} - 4\right)\frac{2t - 4 + 3\sqrt{3}}{11t} \tag{3}$$

[2] Notice that for $t = 10/11$ the intersection between the reaction functions associated to the sets $D$, $A$, $A'$, and $B$ occur at the same point, *viz.*, at the intersection of the boundaries of these sets, a point which we assumed to belong to $B$.

which is in $C$ for $t \in [1.3453, 4)$ and coincides with solution (2) for the boundary value $t = 1.3453$. In particular for $t \in [3/2, 4)$ the above solution lies in the $\underline{C}$ subset of $C$, while it is immediate to verify that for $t = 4$, $a = b = 1/4$, a pair which lies at the boundary between $\underline{C}$, $\overline{C}$, $\overline{C'}$, and $\overline{\overline{C}}$.

These candidate equilibria satisfy a consistency property: they are derived from profit functions which apply to specific subsets of the set of feasible locations, and actually belong to that subset, i.e. they are pairs of maxima over restricted domains. However, for them to be equilibrium strategies pairs over the whole space of strategies, the further condition is required that they be régime-change proof, i.e. be maxima over the unrestricted domain of the profit functions.

Consider for example firm 1's choice when firm 2's is set at the candidate equilibrium: (a) if $b = b^D$, then for $t \in [1/2, 1]$ the domain of $\Pi_1(a, b^D; t)$ lies in regions $A$ and $D$; (b) if $b = b^{D'}$, then for all $t \in (2/3, 4/5)$ the domain of $\Pi_1(a, b^{D'}; t)$ lies in regions $A$ and $D$, while for $t \in [4/5, 10/11)$ it lies in $C$, $B$, $A$, and $D$; (c) if $b = b^B$, then for $t \in [10/11, 1)$ the domain of $\Pi_1(a, b^B; t)$ lies in regions $C$, $B$, $A'$ and $D$, while for $t \in [1, 1.3453)$ it is in $C$, $B$, $\underline{A}'$ and $D$; (d) if $b = b^C$, then for $t \in [1.3453, 1.4444)$ the domain of $\Pi_1(a, b^C; t)$ lies in $C$, $B$, $\underline{A}'$ and $D$, for $t \in [1.4444, 1.5)$ it lies in $C$, $B$, $\underline{A}'$ and $\overline{A}'$, for $t \in [1.5, 1.756)$ in $\underline{C}$, $\underline{B}$, $\underline{A}'$ and $\overline{A}'$, for $t \in [1.756, 2.083)$ in $\underline{C}$, $\underline{B}$, $\overline{B}'$ and $\overline{A}'$, for $t \in [2.083, 4)$ in $\underline{C}$, $\overline{C}'$, $\overline{B}'$ and $\overline{A}'$. Straightforward but tedious calculations[3] confirm that all candidate equilibria are indeed maxima over the above unrestricted domains, given the intervals of values of $t$ for which they have been obtained: hence, they properly identify subgame perfect Nash equilibria. ■

The pattern of equilibria as $t$ increases from 0 to 4 is summarized in Figure 2, where we represent the agglomerated (dashed line) and the dispersed (solid line) equilibrium. The most relevant property of the pattern of the latter is its being non-monotone: the idea that an increase in transportation costs is associated to an incentive to relax competition through an increase of the distance from the rival firm is not confirmed, even for the dispersed equilibrium, over the entire domain of $t$. When the equilibrium pair lies in $B$, the distance from the endpoints is increasing in $t$ for $t \in (178/181, 1.3453)$, while it is always decreasing in $t$ when equilibrium is in $C$ ($\underline{C}$).

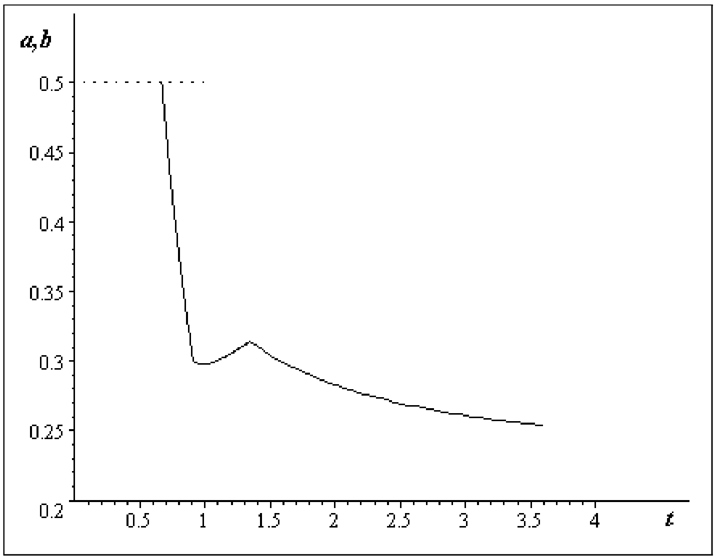

Figure 2: Equilibria as functions of $t$

[3]These calculations are available upon request.

In order to explain this behaviour, we notice the distinguishing feature of this model when transportation costs are high: the overall interaction between firms and the resulting location choices derive from different market configurations at different addresses. Indeed, each firm competes directly with the other in a central area, enjoys monopoly in its market side, and is prevented by high costs from covering distant locations. Accordingly, three basic incentives are at work in the dispersed equilibrium: (a) in the areas where firms compete in quantities, there is an incentive for both to locate farther apart form each other as $t$ increases; this is the 'strategic effect' due to the firms' incentive to serve at low costs markets where the rival firm delivers low quantities; (b) as the increase in $t$ makes it too costly to cover distant locations, firms perceive the incentive to move towards the centre: this is a 'market coverage effect', whereby firms try to preserve a wide market coverage in order to exploit all profit opportunities; (c) finally, each firm is subject to the cost minimizing incentive to locate at the centre of its monopoly area; this 'monopoly effect' leads firms to relocate towards the market endpoints, as the increase in $t$ widens the areas in which firms are monopolist. Clearly, the strategic effect dominates when the dispersed equilibrium lies in set $D$; the market coverage effect prevails when the equilibrium is in $B$ and $t > 178/181$; the monopoly effect is the strongest when the monopoly areas extend to the firms' own locations, so that equilibrium is in set $C$.

# 4 Final remarks

In this paper we have extended the analysis of spatial discrimination with quantity competition along the linear city to the whole range of relevant values of the transportation cost parameter, $t \in [1/2, 4)$. This extension looks particularly worthwhile – and not for the mere sake of completeness. It sheds some light on some fundamental properties of this model, which cannot emerge under the simplifying assumptions $t < 1/2$ or $t < 1$. First of all, for values of $t$ greater than 1 the dispersed equilibrium is the *unique* subgame perfect Nash equilibrium of the model. High values of $t$ lead unambiguously to dispersion, and dispersion is indeed the only outcome which makes the spatial dimension (be it a geographical space or a product space) relevant at equilibrium. Second, our extension shows that in a spatial market the strategic interaction between the firms at the location stage develops along several dimensions: it involves not only the conditions under which the firms coexist on some sub-markets in a central market area, but also each firm's attempt to enjoy and exploit at best full monopoly in a portion of the market, and to preserve the conditions for its own survival in distant market areas. In this sense high transportation costs widen the explanatory power of the Cournot approach to spatial discrimination. They generate endogenously those cost asymmetries and that multiplicity of sub-market configurations, which are fundamental ingredients of a realistic representation of markets with spatial properties.

# References


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[3] Hamilton, J.H., Thisse, J.-F., Weskamp, A., 1989. Spatial discrimination: Bertrand versus Cournot in a model of location choice, Regional Science and Urban Economics 19, 87–102.

## Appendix: Proof of Proposition 1

Proposition 1 amounts to ruling out the possibility that equilibria may arise in the sets $\overline{A}$, $\overline{A'}$, $\overline{B}$, $\overline{B'}$, $\overline{\overline{B}}$,$\overline{C}$, $\overline{C'}$ and $\overline{\overline{C}}$.

Let's start from set $\overline{A}$. Using the definitions of the boundaries, total profits in $\overline{A}$ can be written as

$$\Pi_1^{\overline{A}}(a,b;t) = \int_0^{x_1^r} \pi_1^D dx; \quad \Pi_2^{\overline{A}}(a,b;t) = \int_0^{x_1^r} \pi_2^D \, dx + \int_{x_1^r}^{x_2^M} \pi_2^M dx$$

The corresponding reaction functions intersect at $(a,b) = (9/17t, 1-25/17t)$, which however does not belong to $\overline{A}$ (including its boundaries) for $t > 0$. Given symmetry the same applies to $\overline{A'}$. Let's now consider set $\overline{B}$, where the total profits of the firms are:

$$\Pi_1^{\overline{B}}(a,b;t) = \int_0^{x_2^l} \pi_1^M dx + \int_{x_2^l}^{x_1^r} \pi_1^D dx; \quad \Pi_2^{\overline{B}}(a,b;t) = \int_{x_2^l}^{x_1^r} \pi_2^D dx + \int_{x_1^r}^{x_2^M} \pi_2^M(b,t,x)\, dx$$

The corresponding reaction functions intersect at $(a,b) = (1/t, 1-21/13t)$, a pair which does not belong to $\overline{B}$ (including its boundaries) for $t > 0$. Again, since $\overline{B'}$ is symmetrical to $\overline{B}$, this also rules out that an equilibrium might arise in $\overline{B'}$. As far as set $\overline{\overline{B}}$ is concerned, it is characterized by the following profit functions:

$$\Pi_1^{\overline{\overline{B}}}(a,b;t) = \int_{x_1^M}^{x_2^l} \pi_1^M dx + \int_{x_2^l}^{x_1^r} \pi_1^d dx; \quad \Pi_2^{\overline{\overline{B}}}(a,b;t) = \int_{x_2^l}^{x_1^r} \pi_2^D dx + \int_{x_1^r}^{x_2^M} \pi_2^M dx$$

In this case the intersection of the reaction functions is at $(a,b) = (a, 1-a-8/13t)$, a pair which does not belong to $\overline{\overline{B}}$ (including its boundaries) for $t > 0$. Consider now the set $\overline{C}$, where the total profits of the firms are:

$$\Pi_1^{\overline{C}}(a,b;t) = \int_0^{x_2^r} \pi_1^M dx + \int_{x_2^r}^{x_1^l} \pi_1^D dx; \quad \Pi_2^{\overline{C}}(a,b;t) = \int_{x_2^r}^{x_1^l} \pi_2^D dx + \int_{x_1^l}^{x_2^M} \pi_2^M dx$$

The reaction functions derived from $\Pi_1^{\overline{C}}$ and $\Pi_2^{\overline{C}}$ cross at $(a,b) = (1/t, 1-3/t)$, a pair which is not in $\overline{C}$ for all relevant $t \in [0,4)$. Once again, since $\overline{C'}$ is symmetrical to $\overline{C}$, this also applies to $\overline{C'}$. Finally, by considering the set $\overline{\overline{C}}$, we find that the profit functions are the following:

$$\Pi_1^{\overline{\overline{C}}}(a,b;t) = \int_{x_1^M}^{x_2^r} \pi_1^M dx + \int_{x_2^r}^{x_1^l} \pi_1^D dx; \quad \Pi_2^{\overline{\overline{C}}}(a,b;t) = \int_{x_2^r}^{x_1^l} \pi_2^D dx + \int_{x_1^l}^{x_2^M} \pi_2^M dx$$

Here the intersection occurs at $(a,b) = (1-b-2/t, b)$, which however does not belong to $\overline{\overline{C}}$ for for all relevant $t \in [0,4)$.[4]

[4]Notice that for $t = 4$ there exists a crossing of the reaction functions associated to the sets $\overline{C}$, $\overline{C'}$, $\overline{\overline{C}}$ at $a = b = 0.25$, which is at the intersection of the boundaries of those sets. However, according to our definitions, this point belongs to $\underline{C}$.