E C O N O M I C S B U L L E T I N

Endogenous timing in a mixed oligopoly consisting of a single public firm and foreign competitors

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Abstract

We investigate endogenous timing in a mixed oligopoly consisting of a single public firm and foreign competitors and compare the results with those in Pal (1998) to see the effect of the nationality of private firms on the endogenous role of the public firm. We find that the results are the same in two cases: (i) there are only two time periods for quantity choice, and (ii) there are more than two time periods for quantity choice and there are more than two private firms; but quite different when there are more than two time periods for quantity choice and there are only one or two private firms.

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1. Introduction

Endogenous order of moves is an important issue in a pure private oligopoly and in a mixed oligopoly as well. In the literature on mixed oligopoly, Pal (1998) analyzed endogenous order of moves in quantity choice in a mixed oligopoly consisting of a single public firm and *N* domestic private firms. Matsumura (2003) considered endogenous roles of firms in a mixed duopoly consisting of a state-owned public firm and a foreign private firm. Lu (2006) discussed endogenous timing in a mixed oligopoly with both domestic and foreign private firms in the linear demand case.

Given the results in Pal (1998), Jacques (2004) and Lu (2007), the last two of which slightly correct Proposition 4.1 in the first paper, it is interesting to investigate endogenous timing in a mixed oligopoly consisting of a single public firm and foreign competitors. What is the effect of the nationality of private firms on the endogenous role of the single public firm? This is exactly what we do in this paper by adopting the observable delay game of Hamilton and Slutsky (1990) in the context of a quantity setting mixed oligopoly where the firms first choose the timing of choosing their quantities.

Using a general demand function, Matsumura (2003) discussed a mixed duopoly case in which there are only two possible time periods for quantity choice. The differences between this paper and Matsumura (2003) are: (1) the number of foreign private firms can be more than one; (2) the number of possible time periods can be more than two; (3) we use a linear demand function in order to compare the results with those in Pal (1998), Jacques (2004) and Lu (2007). We find that the results are the same in two cases: (i) there are only two time periods for quantity choice, and (ii) there are more than two time periods for quantity choice and there are more than two private firms; but quite different when there are more than two time periods for quantity choice and there are only one or two private firms.

The organization of the paper is as follows. In Section 2, we describe the model. Section 3 presents the results when there are only two possible time periods for quantity choice. The SPNEs are presented in Section 4 when there are more than two possible time periods to be chosen. Section 5 closes the paper.

2. The model

Consider a mixed oligopoly model consisting of one single public firm and N (≥ 1) foreign private firms, all producing a single homogenous product. Let q_0 and q_i (i=1, 2,…, N) be the quantities of the public firm and the foreign private firms, respectively. Let $Q = q_0 + \sum_{i=1}^{N} q_i$ denote the aggregate quantity. The market price is determined by the inverse demand function $p = a - Q$.

To make the results in this paper directly comparable to those of Pal (1998), Jacques (2004) and Lu (2007), we make the same assumptions except that the nationality of the private firms is different. Specifically, the following assumptions are made: (1) a is sufficiently large; (2) All foreign private firms have constant and identical marginal costs of production, which are normalized to 0; (3) The public firm has a positive, constant marginal cost of production, $c > 0$; (4) Fixed costs are zero for all firms; (5) The public firm's objective is to maximize domestic social surplus defined as the sum of consumer surplus and its profit, whereas each foreign private firm's objective is to maximize its own profit.

We consider the observable delay game of Hamilton and Slutsky (1990) in the context of a quantity setting mixed oligopoly where firms first announce at which time they will choose their quantities and are committed to this choice before they actually choose their quantities. There are $M \geq 2$ possible time periods for quantity choice and each firm may choose its quantity in only one of those M periods.

The objective functions of the public firm and foreign private firm i are respectively given by

$$SS = a(q_0 + \sum_{i=1}^{N} q_i) - \frac{1}{2}(q_0 + \sum_{i=1}^{N} q_i)^2 - (a - q_0 - \sum_{i=1}^{N} q_i)\sum_{i=1}^{N} q_i - cq_0$$

and

$$\pi_i = pq_i = (a - q_0 - \sum_{i=1}^{N} q_i)q_i .$$

Our objective is to solve the SPNEs of this extended quantity setting mixed oligopoly game. We restrict our attention to symmetric equilibria in which all firms of the same type choose to produce in the same period. First, we derive the results for two time periods (M=2). Next, we present the results for more than two time periods.

3. Results for two time periods ($M = 2$)

First, we prove that the public firm will not produce simultaneously with all foreign private firms. This is stated in the following proposition.

Proposition 3.1: All firms producing simultaneously in the same time period cannot be sustained as a SPNE outcome.[1]

This proposition is the same as Proposition 3.1 in Pal (1998) except that the private firms in Pal's model are domestic and also the same as Lemma 3.1 in Lu (2006) except that there is no domestic private firm in our work. It implies that this result is robust regardless of the type of private firms in the market.

Given Proposition 3.1 and that we restrict our attention to symmetric equilibria, there are two possible equilibria when M=2: one involves all private firms producing simultaneously in period 1 and the public firm producing in period 2, while in the other possible equilibrium, the public firm produces in period 1 and all private firms produce simultaneously in period 2. We show that the former possible equilibrium is really a SPNE for any N while the latter one is a SPNE only when $N \leq 2$.

Proposition 3.2: If $N \geq 3$, there is a unique SPNE, at which the private firms produce in period 1 and the public firm produces in period 2. If $N \leq 2$, then there is a second SPNE in which the public firm produces in period 1 and all private firms produce in period 2.

One might wonder why Proposition 3 in Matsumura (2003) states there exists a unique SPNE in which the public firm produces in period 1 and all private firms produce in period 2 while we identify two SPNEs for the same mixed duopoly. The reason is that Matsumura restricts his attention to the equilibria which are not supported by weakly

[1] All proofs are in the appendix.

dominated strategies. We can check that for a mixed duopoly case (N=1), the additional SPNE identified in Proposition 3.2 is indeed supported by a weakly dominated strategy.

Comparing the results of this section with those in Pal (1998), we find that the endogenous order of moves is actually the same. It seems that the nationality of private firms does not affect the endogenous timing. However, this is not completely true when there are more than two time periods for quantity choice.

4. Main Results for more than two periods ($M > 2$)

Proposition 4.1: If $M > 2$, then

(1) when $N \geq 3$, there is a unique SPNE, at which all private firms produce simultaneously in period 1 and the public firm produces in a subsequent period.

(2) when $N = 2$, there is a second SPNE, at which the public firm produces in any period except the last one and the two private firms produce in the subsequent period.

(3) when $N = 1$, there are two SPNEs. In one SPNE, the private firm produces in period 1 and the public firm produces in the last period; in the other SPNE, the public firm produces in any period except the last one and the private firm produces in a subsequent period.

Comparing the results of this section with those in Pal (1998), Jacques (2004) and Lu (2007), we find that the endogenous order of moves is actually the same when $N \geq 3$ but quite different when $N \leq 2$. When $N = 2$, we still have the same SPNE as in Pal (1998), but we also have a second SPNE at which the public firm produces in any period except the last one and the two private firms produce in the subsequent period. When $N = 1$, we still have two SPNEs but they are totally different from Jacques (2004) and Lu (2007). The reason is simple. That is because the public firm prefers to be a leader when private firms are foreign while it prefers to be a follower when competing with domestic private firms.

5. Concluding Remarks

In this paper, we investigate endogenous timing in a mixed oligopoly consisting of one single public firm and N (≥ 1) foreign private firms by considering the observable delay game of Hamilton and Slutsky (1990) in the context of a quantity setting mixed oligopoly. We find that the results are the same when there are only two time periods for quantity choice and when there are more than two time periods for quantity choice and there are more than two private firms but quite different when there are more than two time periods for quantity choice and there are one or two private firms. This difference is the result of the public firm's different desired role when competing with private firms of different nationality.

References

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Appendix

In the following proofs, we let q_0^*, Q^* and p^* respectively denote the public firm’s quantity, the total quantity and the price in equilibrium for any given timing, and q_f^* denote a foreign private firm’s quantity for any given timing in which all foreign private firms produce in the same period. When we consider whether a foreign private firm has the incentive to deviate from any given timing, we always choose foreign private firm 1 to be the defector. If foreign private firm 1 deviates, we let q_1^* denote the defector’s quantity, and q_i^* (i=2, 3,…, N) denote the quantity of those foreign private firms who do not defect.

If all firms produce simultaneously in period t(=1, 2), then every firm’s payoff maximization problem gives us the following first-order conditions:

$$\frac{\partial SS}{\partial q_0} = a-(q_0+\sum_{i=1}^{N} q_i)+\sum_{i=1}^{N} q_i - c = a - q_0 - c = 0, \tag{A1}$$

$$\frac{\partial \pi_i}{\partial q_i} = a - q_0 - \sum_{k=1,k\neq i}^{N} q_k - 2q_i = 0 \text{, for } i = 1,2,...,N. \tag{A2}$$

Solving these equations gives us $q_0^* = a-c$ and $q_f^* = c/(N+1)$. It follows that $Q^* = a - c/(N+1)$, $p^* = c/(N+1)$, $SS^* = a^2/2 - ac + \left(2N^2+2N+1\right)c^2/\left[2(N+1)^2\right]$, and $\pi_f^* = c^2/(N+1)^2$.

Proof of Proposition 3.1

We can show that either the public firm or a foreign private firm has the incentive to deviate if all firms produce simultaneously in the same period, that is, deviate from the following two cases.

Case 1.1: All firms produce simultaneously in period 1.

Consider foreign private firm 1 deviating to be a follower. Then in period 2, it will choose q_1 to maximize $\pi_1 = \left(a - q_0 - q_1 - \sum_{i=2}^{N} q_i\right) q_1$ and the first order condition (A.2) ($i=1$) implies $q_1 = \left(a - q_0 - \sum_{i=2}^{N} q_i\right)\Big/2$. It follows that $p = \left(a - q_0 - \sum_{i=2}^{N} q_i\right)\Big/2$ and thus in period 1, foreign private firm i’s ($i = 2,...,N$) profit function is $\pi_i = q_i\left(a - q_0 - \sum_{i=2}^{N} q_i\right)\Big/2$

and the public firm's objective function is

$$SS = a\left(a + q_0 + \sum_{i=2}^{N} q_i\right)\Big/2 - \frac{1}{2}\left[\left(a + q_0 + \sum_{i=2}^{N} q_i\right)\Big/2\right]^2 - \left(a - q_0 - \sum_{i=2}^{N} q_i\right)\Big/2 * \left(a - q_0 + \sum_{i=2}^{N} q_i\right)\Big/2 - cq_0 \ .$$

The first order conditions imply $q_0^* = a - 4Nc/(2N+1)$ and $q_i^* = 4c/(2N+1)$ ($i = 2,...,N$). It follows that $q_1^* = 2c/(2N+1)$ $Q^* = a - 2c/(2N+1)$, $p^* = 2c/(2N+1)$, and $\pi_1^* = 4c^2/(2N+1)^2 > c^2/(N+1)^2$. Therefore, foreign private firm 1 has the incentive to deviate.

Case 1.2: All firms produce simultaneously in period 2.

Consider the public firm deviating to be a leader. Then in period 2, (A.2) implies $q_i = (a - q_0)/(N+1)$. It follows that in period 1, the public firms' objective function is

$$SS = a(Na + q_0)/(N+1) - \frac{1}{2}\left[(Na + q_0)/(N+1)\right]^2 - N\left[(a - q_0)/(N+1)\right]^2 - cq_0 .$$

The first order condition implies $q_0^* = a - (N+1)^2 c/(2N+1)$. It follows that $q_f^* = (N+1)c/(2N+1)$, $\pi_f^* = (N+1)^2 c^2/(2N+1)^2$,
$SS^* = a^2/2 - ac + (N+1)^2 c^2/[2(2N+1)] > a^2/2 - ac + (2N^2 + 2N + 1)c^2/\left[2(N+1)^2\right]$. Therefore, the public firm has the incentive to deviate. ■

Proof of Proposition 3.2

(1) We prove that the possible equilibrium in which all private firms produce simultaneously in period 1 and the public firm produces in period 2 is really a SPNE for any *N* by showing that no firm has the incentive to deviate.

First we obtain the equilibrium quantities, price and each firm's payoff in this possible equilibrium. In period 2, (A.1) implies $q_0^* = a - c$. It follows that $p = c - \sum_{i=1}^{N} q_i$ and in period 1, foreign private firm *i*'s profit function is $\pi_i = \left(c - \sum_{i=1}^{N} q_i\right) q_i$. The first order conditions imply $q_f^* = c/(N+1)$. It follows that $SS^* = a^2/2 - ac + (2N^2 + 2N + 1)c^2/\left[2(N+1)^2\right]$, and $\pi_f^* = c^2/(N+1)^2$. Clearly the public firm has no incentive to deviate since the social surplus would be the same if it deviated to produce simultaneously with all the foreign private firms in period 1.

Now consider foreign private firm 1 deviating to produce in period 2. (A.1) and (A.2) ($i = 1$) imply $q_0^* = a - c$ and $q_1 = \left(c - \sum_{i=2}^{N} q_i\right)\Big/2$. In period 1, foreign private firm *i*'s ($i = 2,...,N$) profit function is $\pi_i = q_i\left(c - \sum_{i=2}^{N} q_i\right)\Big/2$ and the first order conditions imply $q_i^* = c/N$. It follows that $q_1^* = c/(2N)$, $p^* = c/(2N)$, and $\pi_1^* = c^2/4N^2 \le c^2/(N+1)^2$ (equal if and only $N = 1$). So no foreign private firm wants to deviate.

(2) We prove that the possible equilibrium in which the public firm produces in period 1 and all private firms produce simultaneously in period 2 is a SPNE only when $N \le 2$.
The equilibrium quantities, price and each firm's payoff in this possible equilibrium have been obtained in the proof of proposition 3.1 (case 1.2), $q_0^* = a - (N+1)^2 c/(2N+1)$, $q_f^* = (N+1)c/(2N+1)$, $\pi_f^* = (N+1)^2 c^2/(2N+1)^2$, $SS^* = a^2/2 - ac + (N+1)^2 c^2/[2(2N+1)]$.
Clearly, the public firm has no incentive to deviate.
Now consider foreign private firm 1 deviates to produce in period 1. (A.2) ($i = 2,...,N$) imply $q_i = (a - q_0 - q_1)/N$. In period 1, foreign private firm 1's profit function is $\pi_1 = q_1(a - q_0 - q_1)/N$ and the public firm's objective function is

$$SS = a\frac{(N-1)a + q_0 + q_1}{N} - \frac{1}{2}\left[\frac{(N-1)a + q_0 + q_1}{N}\right]^2 - \frac{a - q_0 - q_1}{N}\left[q_1 + \frac{N-1}{N}(a - q_0 - q_1)\right] - cq_0 .$$

The first order conditions imply $q_0^* = a - 2N^2c/(3N-1)$ and $q_1^* = N^2c/(3N-1)$. It follows that $q_i^* = Nc/(3N-1)$ ($i = 2,...,N$), $p^* = Nc/(3N-1)$, and $\pi_1^* = N^3c^2/(3N-1)^2$ which is lower than $(N+1)^2 c^2/(2N+1)^2$ when $N = 1$ or 2 but higher when $N \ge 3$. ■

Proof of Proposition 4.1:
Firstly, clearly, simultaneous play cannot be sustained as a SPNE outcome.

Secondly, private firms producing in period *t*(>1) while the public firm producing as a follower cannot be sustained as a SPNE outcome. To prove this, we list domestic social surplus in three different cases: (1) when the public firm produces simultaneously with all private firms, $SS = a^2/2 - ac + (2N^2 + 2N + 1)c^2/\left[2(N+1)^2\right]$; (2) when the public firm produces as a leader of all private firms, $SS = a^2/2 - ac + (N+1)^2 c^2/\left[2(2N+1)\right]$; (3) when the public firm produces as a follower of all private firms, $SS = a^2/2 - ac + (2N^2 + 2N + 1)c^2/\left[2(N+1)^2\right]$. So the public firm prefers to be a leader. If private firms produce in period *t*(>1), the public firm will choose to produce in period 1.

Thirdly, if private firms produce in period 1 and the public firm produces in period *t* ($2 \le t < T$), then we can show a foreign private firm has the incentive to deviate to be a follower of the public firm when $N = 1$ but no incentive to do so when $N \ge 2$.[2] We can also show a foreign private firm has no incentive to deviate to produce in period *s* ($2 \le s < t$) when $t \ge 3$.[3] If private firms produce in period 1 and the public firm produces

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[2] Straightforward calculation yields the defector's profit is $\pi_1^* = 4c^2/(9N^2)$, which is lower than $\pi_f^* = c^2/(N+1)^2$ when $N \ge 3$, equal when $N = 2$, but higher when $N = 1$.

[3] Straightforward calculation yields the defector's profit is $\pi_1^* = c^2/(4N^2)$, which is lower than $\pi_f^* = c^2/(N+1)^2$ when $N \ge 2$, equal when $N = 1$.

in period T, then clearly no firm has the incentive to deviate.

So far we have proved that if private firms want to be leaders of the public firm, they have to produce in period 1. When they do so, the public firm producing in any subsequent period when $N \geq 2$ can be sustained as SPNE, while the public firm has to choose to produce in the last period when $N = 1$.

Fourthly, by Proposition 3.2, the public firm producing as a leader of all private firms cannot be sustained as SPNE when $N \geq 3$.

Fifthly, if the public firm produces in period $t(<T)$ and $N \leq 2$ private firms produce in a subsequent period, then clearly the public firm has no incentive to deviate, and we can show that a private firm has no incentive to deviate to be a leader of the public firm when $t>1$, that a private firm has the incentive to deviate to be a leader of the other private firm when $N = 2$ except that private firms produce in the subsequent period, and that a private firm has no incentive to deviate if $N = 1$.■