E C O N O M I C S   B U L L E T I N

Mixed Motives of Simultaneous-move Games in a Mixed Duopoly: Comments and Erratum

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Abstract

If the public and private firm have mixed motives about payoff in a simultaneous-move game, Choi (2006) analyzes that the resulting equilibrium turns out to be an inefficient level with the monopoly of private firm even if there are Nash equilibria. However, we find that if we use equilibrium profit, we would have solved its unique Nash equilibrium that both firms aim to maximize the relative payoffs.

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1 Introduction

Although some theoretical works have already succeeded in explaining a mixed oligopoly, Choi (2006) investigates the simultaneous-move games in a mixed duopoly where a public firm and a private firm are maximizers of either profits or relative profits. Contrary to previous results (De Fraja and Delbono; 1989, 1990), if each firm has mixed motives about payoff in a simultaneous-move game, Choi (2006) analyzes that each firm's payoff motives is mixed, the resulting equilibrium turns out to be an inefficient level with the monopoly of private firm even if the public firm participates in the productive activity. The existence of the pubic firm never affects the equilibrium output in the simultaneous-move games discussed in mixed motives.

The result of endogenous simultaneous move in a mixed duopoly is a new one since so far the literature the absolute payoffs in a mixed oligopoly have found various robust results. In this sense, Choi (2006) makes a contribution to the literature. However, we show that comparing each firm's equilibrium levels of output and profits as defined Choi (2006) is only meaningful when his equilibrium levels of outputs is correct. We find that if we use equilibrium profit, we would have solved its unique Nash equilibrium that both firms aim to maximize the relative payoffs.

2 The Model

Consider that a public firm and a private firm, all producing a single homogenous product in a mixed duopoly model. Following Choi (2006), we use a linear inverse demand schedule for the industry $p = 1 - Q : Q = q_0 + q_1$ where p is a price for two firms ($i = 0, 1$), q_0 and q_1 denote the output of public firm and private firm, respectively. Assume that total cost to firm i of producing quantity q_i is $C(q_i) = cq_i$. Assume that $1 > c > 0$, so that there is some value for total output Q for which market price is greater than the firms' common marginal cost c.

The absolute payoff of each firm i is given by $\pi_i^a = pq_i - cq_i$ and the relative payoff of the firm i is defined in the evolutionary game theory (Samuelson(1997, pp. 66), Weibull (1995, pp. 71-74) and Vega-Redondo (1997)) as the difference between i's absolute payoff and the average absolute payoff of all firms. The average absolute payoff is given by $(1/2)(\pi_i^a + \pi_j^a), j \neq i$ in our framework. Thus, the relative payoff to firm i is as follows:

$$\pi_i^r = \pi_i^a - \frac{1}{2}(\pi_i^a + \pi_j^a) = \frac{1}{2}(q_i - q_j)(p - c). \tag{1}$$

To distinguish notations, the superscripts of lm are defined as when the private firm acting the l-payoff-maximizer and when the public firm acting as the m-payoff-maximizer where $l, m = a, r$. For simplicity, following Samuelson (1997, pp. 66) and Weibull(1995, pp. 71-74), we specify the public firm 0's objective function, W^{lm}, as

$$W^{lm} = \frac{Q^2}{2} + \pi_1^{lm} + \pi_0^{lm} \quad \text{where} \quad l = a, r; m = a, r, \tag{2}$$

where $Q^2/2$ is consumer surplus and each $\pi_i^{lm}, i = 0, 1$ is firm i's profits of both private and public firm.

3 New Equilibrium Outputs and Payoffs

To distinguish notations, each output is defined as q_i^{lm} when the public firm aims to maximize $l = a, r$ payoff and the private firm aims to maximize $m = a, r$ payoff.

First, consider simultaneous-move games in a mixed duopoly when both firms aim to maximize the absolute payoff. However, in Choi(2006), page 3, the equilibrium outputs of simultaneous-move games are given by $q_0^* = (1-c)/2$ and $q_1^* = (1-c)/4$. However, his calculation is not correct. That is, the public firm's objective is to maximize welfare defined as the sum of consumer surplus and absolute profits of all firms, and the private firm's objective is to maximize its own profit. So both firm's absolute-payoff maximization problems are as follows:

$$\max_{q_0} W^{aa} = \frac{Q^2}{2} + \pi_0^{aa} + \pi_1^{aa}, \quad \text{and} \quad \max_{q_1} \pi_1^{aa} = pq_1 - cq_1.$$

Straightforward computation yields each firm's reaction functions: $q_0 = 1 - c - q_1, q_1 = \frac{1-c-q_0}{2}$. Thus, the correct results should be $q_1^{aa} = 0$ and $q_0^{aa} = 1 - c$. Each firm's payoff is then $W^{aa} = \frac{(1-c)^2}{2}, \pi^{aa} = 0$.

Next, consider that the maximization problems for the relative-payoff-maximizer of public firm and private firm. Indeed, Choi (2006) computed that the public firm of relative-payoff-maximizer maximizes first equation (i.e., $\frac{\partial W^{rr}}{\partial q_0} < 0 \Leftrightarrow q_0 = -q_1$) on page 4 when the private firm aims to maximize relative payoff. Given the public firm of the relative-payoff-maximizer, the private firm's objective function is given by $\max_{q_1} \pi_1^{rr} = \frac{1}{2}(q_i - q_j)(p - c)$ which yields $q_1 = \frac{1-c}{2}$. Furthermore, when we calculate the maximization of W^{rr}, the relative-payoff-maximizer of public firm's maximization problem is given by $\max_{q_0} W^{rr} = \frac{1}{2}(q_0 + q_1)^2 =$ consumer surplus. When the public firm maximizes consumer surplus, it has to take into account the condition that consumer surplus is maximized when $Q = q_1 + q_0 = 1$. Thus, the best response of the public firm is $q_0 = 1 - q_1$. Hence, we obtain the equilibrium output levels as

$$q_0^{rr} = \frac{1+c}{2}, q_1^{rr} = \frac{1-c}{2}.$$

Each firm's payoff is then $W^{rr} = \frac{1}{2}, \pi^{rr} = \frac{c^2}{2}$.

Finally, other calculations of mixed motives of payoff in mixed duopoly are given by Choi (2006), page 4. However, his calculations are not correct. In these cases, the $W^{ar} = W^{ra} = \frac{1}{2}(q_0 + q_1)(1 - c)$ is obtained. Since each derivative of the public firm's payoff with respect to q_0 is $\partial W^{ra}/\partial q_0 = \partial W^{ar}/\partial q_0 = \frac{(1-c)}{2} > 0$, we have similar response functions as in the W^{rr} case. That is, the relative (respectively, absolute)-payoff maximizing private firm's reaction function is $q_1 = \frac{1-c}{2}$ (respectively, $q_1 = \frac{1-c-q_0}{2}$). Thus, the correct results of the equilibrium output levels should be

$$q_0^{ar} = 1, q_0^{ra} = 1 - c, q_1^{ar} = q_1^{ra} = 0$$

because the public firm has to take into account the condition that the consumer surplus is maximized when $Q = 1$. Each firm's payoff is then $W^{ar} = W^{ra} = \frac{1-c}{2}, \pi_1^{ar} = 0$ and $\pi_1^{ra} = \frac{c^2}{2}$. As a result, Choi's (2006) main result might be changed by using pure Nash equilibrium defintion[1].

3.1 Simultaneous-Move Games with Mixed Motives

Choi (2006) summarized each equilibrium output from mixed motives where different firms coexist that value the relative payoff and absolute payoff, respectively. He analyzes that each firm's payoff motives is mixed, the resulting equilibrium turns out to be an inefficient level with the monopoly of private firm even if the public firm participates in the productive activity. Instead, we discuss four simultaneous-move games with equilibrium payoffs and investigate the unique Nash equilibrium of the extended quantity setting mixed duopoly game.

[1] I am grateful to Noriaki Matsushima for drawing my attention to the issue of optimal solutions.

Thus, we have some doubts on the result: (i) how can quantities be used as payoffs to solve a game in Choi (2006)? we need to justify this, (ii) our work do not deal with the application of the evolutionary game theory; rather, it might assume that the profit motives of public and private firms participating in mixed duopoly could be predicted. Accordingly, choosing the production quantity endogenously should be taken to mean strategically choosing the profit motive as an independent variable.

In the table, "$a_i, i = 0, 1$" and "$r_i, i = 0, 1$" represent absolute-payoff-maximizer and relative-payoff-maximizer with regard to payoff motive choice respectively. Straightforward calculation yields the payoff table below.

Table 1: Firms' payoffs in the simultaneous-move games

public firm

		a_0	r_0
private firm	a_1	$0, \frac{(1-c)^2}{2}$	$0, \frac{1-c}{2}$
	r_1	$\frac{c^2}{2}, \frac{1-c}{2}$	$\frac{c^2}{2}, \frac{1}{2}$

As shown in Table 1, we can solve its Nash equilibrium that satisfies uniqueness: the Nash equilibrium outcome in Table 1 is (r_1, r_0). From this result, the proposition is derived as follows.

Proposition 1: Suppose that there is a mixed duopoly with mixed motives. Then there is a unique pure Nash equilibrium outcome: both firms always act as the relative-payoff-maximizers.

4 Concluding Remarks

In this paper, we investigate the simultaneous-move game in a mixed duopoly where firms are maximizers of either profits or relative profits. Thus, we find that if we use equilibrium profit, we solved its unique Nash equilibrium that both firms aim to maximize the relative payoffs. As a next step, investigating the sequential-move game in the mixed oligopoly is needed for future research[2].

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[2]For the sequential-move games in the mixed duopoly, it is available from author upon request.