E C O N O M I C S   B U L L E T I N

Forward Vertical Integration: The Fixed-Proportion Case Revisited

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Abstract

Assuming a fixed-proportion downstream production technology, partial forward integration by an upstream monopolist may be observed whether the monopolist is advantaged or disadvantaged cost-wise relative to fringe firms in the downstream market. Integration need not induce cost-predation and the profits of the fringe may increase. The output price falls and welfare unambiguously rises.

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1. Introduction

Many textbooks (e.g., Church and Ware, 2000) begins their treatment of forward integration by showing that it is pointless when downstream fringe firms and an upstream monopolist have access to the same downstream fixed-proportions constant-return technology. It is then shown that if the upstream monopolist was able to secure a better downstream technology, it would eject the fringe. Given this “razor’s edge” effect, also found in Quirmbach’s (1992) model, one must appeal to regulations to rationalize the existence of a partial integration outcome. We analyze a fixed-proportion case of vertical integration that supports partial integration over a range of downstream cost advantages and disadvantages on the part of the upstream monopolist.

A concern with forward integration is what Salop and Scheffman (1987) call cost-predation. This arises when a dominant firm supplying inputs to its competitors purposely raises the input price to reinforce its dominant position on the downstream market. Our results suggest that forward integration need not induce cost-predation and may even increase both the margin of fringe firms and the fringe’s output.

The model is presented in the next section. The third section begins by showing that the upstream monopolist may be partially integrated over a range of cost disadvantages and advantages in downstream production relative to fringe firms. Then, the implications of partial forward integration on output and input prices, the margin of fringe firms, profits and welfare are analyzed.

2. The model

A dominant/predator firm, referred to as *P*, is a monopolist in an upstream market and a dominant firm in a downstream market. It is assumed that the integration is a forward one, with firm *P* being the sole input supplier for the downstream firms with which it competes in the final good’s market. Thus, firm *P* and a competitive fringe, denoted by *f*, face a demand curve $D(p)$ for a homogenous product sold at price p and $D_p < 0$.[1] Each firm relies on a fixed-proportion technology that requires one unit of input to produce one unit of output. The cost functions for firm P and the firms in the fringe are respectively $C^P(q) = F(q) - \lambda q + R(q+x)$ and $C^f(x) = F(x) + rx$, where outputs are given by q and x, such that $D \equiv q + x$, r is the price of the input sold by firm *P* to the fringe firms and $R(.)$ is the cost to produce that input for firm *P*. It is assumed that $F_i > 0$, $R_D > 0$, $C_i^j > 0$ with $i = x, q$ and $j = f, P$. We refer to the portion $F(q) - \lambda q$ as the output manufacturing cost of the predator firm and to $R(q+x)$ as its input manufacturing cost. The predator firm can be more cost-efficient (inefficient) than the fringe firms by setting $\lambda > 0$ ($\lambda < 0$).[2]

For simplicity, there is no fixed cost on the downstream market. The fringe’s supply curve is denoted by $S(p,r)$. It is derived from the fringe’s cost function:

[1] The demand must not be “too convex” for the second order conditions to hold. To simplify the exposition, it is assumed in most instances that $D_{pp} = 0$.

[2] Riordan (1998) relies on the same assumption in a backward vertical integration model.

$$S(p,r) = Arg\max_x \pi^f \tag{1}$$

where $\pi^f \equiv [px - F(x) - rx]$.[3] The fringe's supply depends on *p-r*. As such, $S_p \equiv \partial S(.)/\partial p = 1/F_{xx}$, $S_r \equiv \partial S(.)/\partial r = -1/F_{xx}$ and $S_p = -S_r$. We assume that $F(x) = \frac{\gamma}{\gamma+1} x^{\frac{\gamma+1}{\gamma}}$, which implies that $S(.) = (p-r)^\gamma$ and $S_{pp} = S_{rr} = -S_{pr} < 0$ when $0 < \gamma < 1$. The second order condition requires that $\partial^2 \pi^f / \partial x^2 \equiv -F_{xx} < 0$ and it is satisfied when $\gamma > 0$. The predator firm maximizes its profit π^P:

$$\max_{p,r} \pi^P = [pq - F(q) + \lambda q + rx - R(q+x)] \tag{2}$$

subject to $q \equiv D(p) - x$ and $x \equiv S(p,r)$.[4]

The residual demand facing firm *P* is such that $\partial q / \partial p = D_p - S_p$, $\partial q / \partial r = -S_r$, where $D_p \equiv \partial D(.)/\partial p$. The first order conditions of firm *P* with respect to *p* and *r* can be expressed in terms of the elasticity of its residual demand on the downstream market, $\varepsilon^{D-S} \equiv -(D_p - S_p)\, p/q$, and the elasticity of demand on the upstream market, $\varepsilon^S \equiv -S_r\, r/S$:

$$\frac{p - F_q + \lambda + r\dfrac{S_p}{D_p - S_p} - R_D \dfrac{D_p}{(D_p - S_P)}}{p} = \frac{1}{\varepsilon^{D-S}} \tag{3}$$

$$\frac{(p - r - F_q + \lambda)}{r} = \frac{-1}{\varepsilon^S} \tag{4}$$

From (3) and (4), we can deduce that: $R_D < p - F_q + \lambda < r$. Provided that $F_{qq} = F_{xx} > 0 \ \forall q = x$ and $\lambda > 0$, the last inequality and the fact that $F_x = p - r$ jointly imply that in equilibrium $F_q > F_x$, which in turn implies $q > x$. This clearly shows how the dominant position of the predator firm on the downstream market is related to its cost advantage. The second order condition is respected if:

$$\pi^P_{pp}\, \pi^P_{rr} - \pi^P_{pr}\, \pi^P_{rp} > 0 \tag{5}$$

with $\pi^P_{pp}, \pi^P_{rr} < 0$. If $D_{pp} = 0$, (5) holds when economies of scale in input manufacturing are not too strong: $R_{DD} > 2/D_p$ (see Appendix 1).[5]

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[3] As in Salop and Scheffman (1987) and Riordan (1998), we assume that $F_{xx} > 0$ which implies $S_p > 0$.

[4] We refer to $q \equiv D(p) - S(p,r)$ as the dominant firm's residual demand in the downstream market.

[5] It is important to note that $R_{DD} > 2/D_p$ is sufficient, but not necessary, for the second order condition to hold.

3. Forward Integration

The predator firm finds it profitable to partially integrate the downstream market as long as (3) holds. When it is active in the input and output markets, it chooses equilibrium input and output prices $r^*(\lambda)$ and $p^*(\lambda)$. However, equilibria for which either the fringe firms or the predator firm are not active in the downstream market are possible. Defining p^c as the competitive output price when the predator is not integrated and $p^{\min}(\lambda)$ as the output price when the fringe is just about to be ejected, then $p^*(\lambda) \in \left(p^{\min}(\lambda), p^c\right)$. Because there are no fixed costs, the fringe does not produce any output when its average cost is minimized and $p^{\min}(\lambda) = r^*(\lambda)$. If λ was sufficiently low to permit $p^*(\lambda) \geq p^c$, the fringe would supply the whole market demand. Conversely, if the predator's cost structure was such that $p^*(\lambda) \leq r^*(\lambda)$, then it would eject the fringe. We implicitly define a lower bound $\underline{\lambda}$ such that $p^*(\underline{\lambda}) = p^c$. In this case the equilibrium prices $r^m \equiv r^*(\underline{\lambda})$ and $p^c \equiv p^*(\underline{\lambda})$ are consistent with (3) and (4), but also with the first order condition of the profit maximizing predator firm when it is only an upstream monopolist:

$$\max_r [rS(p,r) - R(S(p,r))] \tag{6}$$
$$\text{subject to } D(p*) = S(p*, r*).$$

Thus, for $\lambda = \underline{\lambda}$, then $r^*(\underline{\lambda}) = r^m$, $p^*(\underline{\lambda}) = p^c$, $q^* = 0$ and $x^* = D(p^c)$. As long as $\lambda < \underline{\lambda}$, increases in λ have no effect because it is more profitable for the predator firm to restrict its activities to the upstream market.

By the same token, we may implicitly define an upper bound $\overline{\lambda}$ such that $p^*(\overline{\lambda}) = r^*(\overline{\lambda})$. When $\lambda \geq \overline{\lambda}$, it is profitable for the predator firm to monopolize the output market. Consequently, at $\lambda = \overline{\lambda}$, the equilibrium prices are consistent with (3) and (4) as well as the first order condition of the profit maximization problem of the predator firm when it is a downstream monopolist:

$$\max_p [pD(p) - F(D(p)) + \lambda D(p) - R(D(p))] \tag{7}$$

When the input price is equal to its chosen output price, the fringe is ejected and the output price is the monopoly price, $p^m(\overline{\lambda})$. For $\lambda = \overline{\lambda}$, then $r^*(\overline{\lambda}) = p^*(\overline{\lambda})$, $p^*(\overline{\lambda}) = p^m(\overline{\lambda})$, $q^* = D(p^m)$ and $x^* = 0$.

The condition under which partial integration takes place is:[6,7]

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[6] Implicitly defining $\lambda^{\max}$ such that $C^P(q) = 0$, the domain for partial integration is $\lambda \in \left(\underline{\lambda}, \lambda^{\max}\right)$ as full integration equilibrium cannot be observed if $\lambda^{\max} < \overline{\lambda}$.

[7] A different motivation for entry can be found in Blair, Cooper and Kaserman (1985).

Lemma 1: $\forall \lambda \in \left(\underline{\lambda}, \overline{\lambda}\right)$ *then* $r(\overline{\lambda}) < p^*(\lambda) < p^c$ *and the predator firm is partially integrated on the downstream market.*

Unlike in Quirmbach's (1992) model in which the dominant firm would choose full integration unless it is arbitrarily restricted from doing so, the dominant firm does not necessarily want to monopolize the downstream market when it is free to choose its degree of integration. Figure 1 portrays the predator's profit as a function of λ when $D(.)$ is linear, both $R(.)$ and $F(.)$ are cubic functions and $R_{DD} > 0$ in the neighborhood of the equilibria. On the left of $\underline{\lambda} = -0.226$, the upstream monopolist chooses not to be integrated, but it is partially integrated between $\underline{\lambda}$ and $\overline{\lambda} = 0.111$. For $\lambda > \overline{\lambda}$, the fringe is ejected.[8] Thus, an upstream monopolist with a cost disadvantage in the downstream market may profitably integrate. Likewise, a predator that enjoys a technological advantage over the fringe firms may prefer a partial integration scenario to a monopoly scenario. The intuition behind these results is simple. Going back to the textbook case involving downstream competitive firms with constant unit costs (i.e. $F_{xx} = 0$ and $F_{qq} = 0$), the predator is indifferent between integrating or not when $\lambda = 0$, but would (not) integrate when $\lambda > 0$ ($\lambda < 0$). In our model, the downstream market is an increasing cost one from the perspective of the firms in the competitive fringe as well as the predator firm (i.e., $F_{xx} > 0$ and $F_{qq} > 0$). As such, it is profitable for the predator to enter the market even when it has a cost disadvantage and not to force the exit of the fringe when it has a cost advantage.

As mentioned before, an increase in λ can cause a non integrated upstream monopolist to partial integrated the downstream market Hence, we may use static comparative to compare an equilibrium without integration ($\lambda = \underline{\lambda}$) to one characterized by partial integration ($\lambda > \underline{\lambda}$). Before dwelling on the welfare implications, we analyze the impact of partial integration upon output and input prices.

Proposition 1: *A)* $dp/d\lambda < 0$. *B)* $\frac{dr}{d\lambda} \gtrless 0$. *A necessary, but not sufficient, condition for a cost-predation effect,* $\frac{dr}{d\lambda} > 0$, *is* $R_{DD} > 0$. *C)* $\frac{dp}{d\lambda} - \frac{dr}{d\lambda} > 0$ *if and only if* $R_{DD} < 2/D_p < 0$.

Proof: See Appendix 2.

Integration unambiguously induces a lower output price to the benefit of consumers, but it has ambiguous effects on the input price and the fringe's margin. If the upstream technology is characterized by increasing returns such that $R_{DD} < 2/D_p$, then the integration makes the input price fall enough to increase the fringe's margin! To make sense of this result, note that the condition on the upstream technology is derived under the assumption of decreasing returns in the downstream market. As a result, the predator firm may lower the input price to avoid moving up too high on its output manufacturing marginal cost curve. Finally, the fringe's output, like its margin, may decrease or increase with λ.[9]

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[8] For $\lambda > \lambda^{\max} = 0.157$, the predator's cost is non-positive. Naturally, such cases are dismissed as non-pertinent.

[9] We obtain $sign\left(dS(.)/d\lambda\right) > 0$ if and only if $R_{DD} < 2/D_p$.

Proposition 2: *A)* $\partial \pi^{p} / \partial \lambda > 0$, *B)* $\partial \pi^{f} / \partial \lambda > 0$ *if and only if* $R_{DD} < 2 / D_{p} < 0$, *C)* $\partial CS / \partial \lambda > 0$, *and D)* $\partial W / \partial \lambda > 0$.[10]
Proof: See Appendix 3.

Proposition 2 states that integration increases the predator's profit, consumer surplus and welfare. The fringe's surplus can increase provided that there are sufficient economies of scale in upstream production.

4. Conclusions

In this paper, we show that relaxing constant return to scale in producing the upstream input and the downstream output modifies the classic result about forward integration with fixed proportion technology discussed in most textbooks. Partial forward integration by an upstream monopolist may be observed in equilibrium as long as the integrated firm is not too advantaged or disadvantaged relative to a downstream competitive fringe. Interestingly, economies of scale in the production of the upstream input imply that the forward integration is never implemented by a cost-predation strategy and the profits of the fringe can even increase.

5. References

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[10] *C*S and *W* represent respectively the consumer surplus and total welfare.

Figure 1

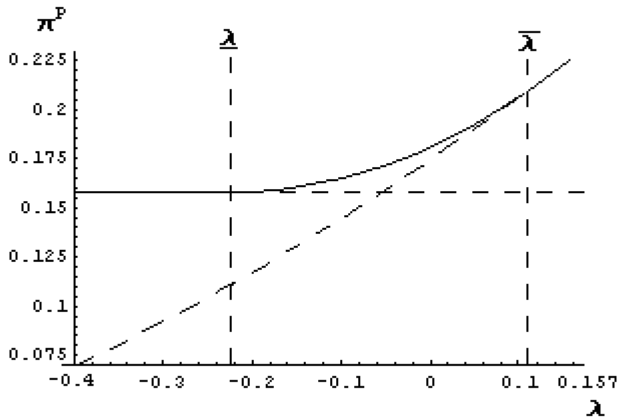

Figure 1. The impact of λ on the predator's profit when $D(p)=1-p$, $R(D)=D^3$ and $\gamma=\frac{1}{2}$.

Appendix

Appendix 1: Proof of the second order condition.

The derivatives of the dominant firm's profit are:

$$\pi_p^P = (p - F_q + \lambda)(D_p - S_p) + (D - S) + r\,S_P - R_D\,D_p = 0$$
$$\pi_r^P = (p - F_q + \lambda - r)(-S_r) + S = 0$$
$$\pi_{pp}^P = (p - F_q + \lambda)(D_{pp} - S_{pp}) + \left(2 - F_{qq}(D_P - S_P)\right)(D_p - S_P) + r\,S_{pp} - R_{DD}D_p^2 - R_D\,D_{pp} < 0$$
$$\pi_{rr}^P = -(p - F_q + \lambda - r)S_{rr} + \left(2 - F_{qq}\,S_r\right)S_r < 0$$
$$\pi_{pr}^P = -(p - F_q + \lambda - r)S_{pr} + S_P - S_r\left(1 - F_{qq}\,(D_p - S_p)\right) > 0$$
$$\pi_{rp}^P = \pi_{pr}^P > 0$$
$$\pi_{p\lambda}^P = (D_p - S_p) < 0$$
$$\pi_{r\lambda}^P = -S_r > 0$$

Assuming $D_{pp} = 0$, the second order condition for the dominant firm's optimization problem ($\pi_{pp}^P\pi_{rr}^P - \pi_{pr}^P\pi_{rp}^P > 0$) is reduces to:

$$D_p\left(2\left(-2 + D_p\left(F_{qq} + R_{DD}\right)\right)S_p + F_{qq}\left(-2 + D_pR_{DD}\right)S_p^2 + \left(p - r + \lambda - F_q\right)\left(-2 + D_p\left(F_{qq} + R_{DD}\right)\right)S_{rr}\right)$$

Given that $F_{qq} > 0,\ S_{rr} < 0,\ S_p > 0,\ D_p < 0$, it can readily be seen that this expression is unambiguously positive when $R_{DD} \geq 0$. However, economies of scale in input manufacturing are allowed, as $0 > R_{DD} > \left(2/D_p\right) - F_{qq}$. Hence, $0 > R_{DD} > 2/D_p$ is sufficient, but not necessary for the second order condition to hold.

Appendix 2: Proof of Proposition 1.

The impact of λ on input and output prices can be determined by totally differentiating the first order conditions, $\pi_p^p = \pi_r^p = 0$, and by applying Cramer's rule to the resulting equations. Then, it can be shown that:

$$\frac{dr}{d\lambda} = \frac{\pi_{rp}^P\ \pi_{p\lambda}^P - \pi_{r\lambda}^P\ \pi_{pp}^P}{\pi_{pp}^P\ \pi_{rr}^P - \pi_{pr}^P\ \pi_{rp}^P} \qquad \text{(A8)}$$

where $\pi_{ij}^P = \partial^2\pi^P / \partial i \partial j$ with $i, j = p, r$. From (A8), the denominator is positive and hence :

$$sign\frac{dr}{d\lambda} = sign\left(\pi_{rp}^P\ \pi_{p\lambda}^P - \pi_{r\lambda}^P\ \pi_{pp}^P\right) \qquad \text{(A9)}$$

Applying the same steps for the output price, we obtain:

$$sign\frac{dp}{d\lambda} = sign\left(\pi_{r\lambda}^P\ \pi_{pr}^P - \pi_{p\lambda}^P\ \pi_{rr}^P\right) \qquad \text{(A10)}$$

Part A): The proof directly follows from (A10), the definitions provided in the appendix 1 and the restrictions on the derivatives of the fringe's supply curve arising from the fixed proportion technology, specifically ($S_p = -S_r > 0, S_{pp} = S_{rr} = -S_{pr} < 0$). From this, it can be shown that: $sign\left(\frac{\partial p}{\partial \lambda}\right) = sign\left(D_p\left[2S_p + \left(p - r + \lambda - F_q\right)S_{rr}\right]\right)$. The bracketed expression is clearly positive and given that the demand for the final good is negatively sloped, it turns out that the output price is unambiguously decreasing in λ.

Part B): The proof follows from (A9)**.** It can be shown that the sign of $\pi^P_{rp}\,\pi^P_{p\lambda} - \pi^P_{r\lambda}\,\pi^P_{pp}$ = sign of $D_p\left[D_p R_{DD} S_p + \left(p - r + \lambda - F_q\right)S_{rr}\right] - D_{pp} S_p\left(p - F_q + \lambda - R_D\right)$. Obviously, setting $D_{pp} = 0$ does not resolve the ambiguity about $\frac{dr}{d\lambda}$ and the cost predation result hinges on the bracketed expression being negative. This requires that $R_{DD} > 0$.

Part C): It can be shown that: $sign\left(\frac{dp}{d\lambda} - \frac{dr}{d\lambda}\right) = sign\left(\left(\pi^P_{r\lambda}\,\pi^P_{pr} - \pi^P_{p\lambda}\,\pi^P_{rr}\right) - \left(\pi^P_{rp}\,\pi^P_{p\lambda} - \pi^P_{r\lambda}\,\pi^P_{pp}\right)\right) =$

$sign\left(S_r\left(D_p(-2 + D_p R_{DD}) - D_{pp}(p - F_q + \lambda - R_D)\right)\right)$ as $D_{pp} = 0$ we have $\frac{dp}{d\lambda} - \frac{dr}{d\lambda} > 0$ if and only if $R_{DD} < 2/D_p < 0$. We have demonstrated in appendix 1 that $R_{DD} > 2/D_p$ is only a sufficient condition for the second order condition to hold. The second order condition holds if $R_{DD} = 2/D_p - \varepsilon$ as long as ε is not too high. **QED**

Appendix 3: Proof of Proposition 2.

Part A): By the envelope theorem $\frac{\partial \pi^P\left(p(\lambda), r(\lambda), \lambda\right)}{\partial \lambda} = q\left(p(\lambda), r(\lambda), \lambda\right) > 0$.

Part B): The decrease or increase in the fringe's surplus comes from the facts that the fringe's surplus and output are conditioned by $p(\lambda) - r(\lambda)$ and that the output may shrink or increase as λ increases, $sign\left(\frac{dS(.)}{d\lambda}\right) = sign\left(S_p^2\left(2D_p - D_p^2 R_{DD}\right)\right) \underset{>}{\overset{<}{=}} 0$. Thence, for $R_{DD} < 2/D_p$ then $\frac{dp}{d\lambda} - \frac{dr}{d\lambda} > 0$ and $sign\left(dS(.)/d\lambda\right) > 0$, the fringe's surplus is increasing in λ. Inversely, for $R_{DD} > 2/D_p$ then $\frac{dp}{d\lambda} - \frac{dr}{d\lambda} < 0$ and $sign\left(dS(.)/d\lambda\right) < 0$, the fringe' surplus is decreasing in λ.

Part C): As $\frac{dp}{d\lambda} < 0$, the consumer surplus is unambiguously decreasing in λ.

Part D): Welfare is defined as the sum of consumer surplus, the fringe's surplus and the predator firm's profit $W = \int_0^{D(p)} \left(D^{-1}(q+x)\right) d(q+x) - F(x) - F(q) + \lambda q - R(q+x)$ where $q \equiv D(p) - S(p,r)$ and $x \equiv S(p,r)$. The envelope theorem can again be invoked to show that welfare increases in response to partial integration, but instead we rely on a more intuitive proof. If the quantity marketed of the final good remained constant, consumer surplus would be unaffected by the partial integration and welfare changes would be driven solely by sourcing cost considerations: $R\left(D(p)\right) + F\left(D(p) - S(p,r)\right) + F\left(S(p,r)\right) - \lambda\left(D(p) - S(p,r)\right)$. The fringe could produce the same output at the same cost, but the predator firm would benefit from its cost advantage to produce the same output. Thus, welfare would increase if total output and each firm's output were to remain constant. But the outputs do change. At the new equilibrium, total output is higher because the predator firm fully exploits its cost advantage. As a result the more efficient firm produces more and the less efficient fringe produces less.[11] Therefore, there is a cost rationalization and an increase in consumer surplus as *p-r* gets smaller. Consequently, welfare unambiguously increases upon partial integration. **QED**

[11] The increase in total output and in the output of the dominant firm cannot increase cost too much because from the first order conditions, it must be a positive margin on the product sold in the downstream market: $p > R_D + F_q - \lambda$.