E C O N O M I C S B U L L E T I N

The Dynamics of Growth and Migrations with Congestion Externalities

Orlando Gomes
Escola Superior de Comunicação Social

Abstract

The paper develops a simple Solow-like growth model, with two independent geographical spaces, where migration is possible and it is stimulated by wage differences. The model assumes a congestion externality: high concentration of individual agents in one of the economic spaces implies losses in the ability to accumulate physical capital. Combining wage incentives, negative externalities of excessive concentration of people and a mechanism of discrete choice that governs the decisions concerning migrations, the analysis reveals that for some combinations of parameter values strange dynamics arise. A Neimark-Sacker bifurcation takes place, leading to endogenous cycles that describe the long term evolution of the capital accumulation and consumption variables. Also, the steady state will be characterized by never ending fluctuations on the share of individuals remaining in each one of the two assumed regions.

Acknowledgements: Financial support from the Fundação Ciência e Tecnologia, Lisbon, is grateful acknowledged, under the contract No POCTI/ECO/48628/2002, partially funded by the European Regional Development Fund (ERDF).
Citation: Gomes, Orlando, (2007) "The Dynamics of Growth and Migrations with Congestion Externalities." *Economics Bulletin*, Vol. 15, No. 1 pp. 1-8
Submitted: November 17, 2006. **Accepted:** January 16, 2007.
URL: http://economicsbulletin.vanderbilt.edu/2007/volume15/EB-06O40015A.pdf

1. Introduction

Studies on geography and growth tend to stress a relevant dichotomy. Free movements of capital and labor allow for a better allocation of inputs and therefore economic efficiency will rise. On the other hand, economic growth in one region may flourish at the expenses of the other (capital and skilled labor may move solely in one direction, leading to widening regional discrepancies). Because regional cohesion is socially desirable, the equitable distribution of economic activity across locations is a reasonable policy goal, alongside with the growth objective. This argument is widely stressed in Fujita, Krugman and Venables (1999), Gallup, Sachs and Mellinger (1999) and Fujita and Thisse (2002).

In this paper, we consider a two-location economy, where, in each location, a different final good is produced. Assuming that different forms of capital are necessary to produce different goods, there is no need to consider capital mobility. Thus, we concentrate on the mobility of the labor input.[1]

In our analysis, a pair of Solow capital accumulation difference equations is considered. Therefore, one might associate the model in this note to the strand of literature that explains urban and regional growth through the simple capital accumulation equation of the neoclassical growth model. See Miyao (1987) and Anas (1992) for the analysis of the dynamic properties of such one-sector simple model. We refer the reader to Berliant and Wang (2004), who present a thorough discussion on dynamic growth models (neoclassical and of the endogenous growth type) under a spatial perspective.

The main assumption underlying the proposed framework relates to how labor is allocated to each one of the two geographical points. We assume that the share of labor in each region is determined endogenously over time, given two central assumptions: first, there is a congestion external effect that disturbs capital accumulation, when population becomes too concentrated in one of the regions; second, migration decisions are explicitly governed by wage differentials, but implicitely they are also dependent on other non specified factors, and this is captured by a discrete choice mechanism, as the one we find in many heterogeneous agents analyses, like Brock and Hommes (1998), Barucci (1999), Negroni (2003), De Grauwe and Grimaldi (2005) and Gomes (2005).

The setup allows to find various qualitative long term stability outcomes ranging from fixed point stability to cycles of low periodicity, limit cycles, a-periodicity (chaos) and instability.[2] The different results are found for different combinations of parameters. We give particular attention to one parameter: the intensity of choice underlying the discrete choice rule. We construct a bifurcation diagram regarding the referred constant, and we find that a fixed point gives place to limit cycles and chaos. Therefore, our setup is able to identify, under specific conditions, an everlasting process

[1] The relation between migration and growth is also the subject of analysis of Palivos and Wang (1996), Walz (1996), Black and Henderson (1999), Baldwin and Forslid (2000) and Rossi-Hansberg and Wright (2005). The models therein address labor mobility in endogenous growth frameworks as a way to identify patterns of migration.

[2] While the notions of fixed point stability and instability are trivial and well known (they just refer to the convergence to or divergence from unique long term values of the endogenous variable), cycles and chaos involve less straightforward stability analysis. For instance, it may be hard to distinguish between a limit cycle and a chaotic attractor in some specific cases. In this paper, we apply some of the concepts and tools of nonlinear theory, but we refer the reader to detailed analysis of nonlinear dynamics, that can be found for instance in Alligood, Sauer and Yorke (1997), Lorenz (1997) and Medio and Lines (2001).

of migration (households' shares in each region will not become constant in the steady state), and this process implies that capital, output and consumption aggregates will not assume constant long term values as well. Thus, one can identify the presence of endogenous business cycles. In this sense, it is possible to attach this analysis also to the literature on endogenous business cycles [that has as first fundamental references Stutzer (1980), Benhabib and Day (1981), Day (1982) and Grandmont (1985), and that has continued with the important work on increasing returns in RBC deterministic models by Christiano and Harrison (1999), Schmitt-Grohé (2000) and Guo and Lansing (2002). Other approaches to endogenous business cycles, involving overlapping generations, firms expectations about demand and learning can be found in Aloi, Dixon and Lloyd-Braga (2000), Gomes (2006) and Cellarier (2006)].

Differently from other studies, here the cause of endogenous fluctuations is not production externalities, imperfect expectations or learning. The cause of cycles is the endogenous migration process that is triggered by an economic environment where wages determine location decisions, where congestion externalities are present and where a discrete choice mechanism governs the choices of rational agents.

2. A Two-Location Environment

Consider an economy, with a constant population level L that is geographically separated into two autonomous regions. There are no barriers to the circulation of goods, capital and labor between the two regions; nevertheless, capital does not flow from one region to the other, because each region produces a final good using as input the form of capital available in the corresponding location. Labor is used in the production process in each region, and the only way to increase the participation of labor in one of the regions is by a migration process from one region to the other.

The output levels in each location are given by conventional Cobb-Douglas production functions that exhibit constant returns to scale and decreasing marginal returns, that is, $Y_{1t} = AK_{1t}{}^{\alpha} \cdot (a_t L)^{1-\alpha}$ and $Y_{2t} = AK_{2t}{}^{\alpha} \cdot [(1-a_t) \cdot L]^{1-\alpha}$. Aggregates Y_{1t}, Y_{2t}, K_{1t} and K_{2t} refer, respectively, to output in each one of the regions and the amount of accumulated capital also in each region. Note that the production functions share common features: technological capabilities are the same in both regions ($A>0$ represents a technological index) and the output – capital elasticity is also identical ($0<\alpha<1$).

Each region will have a given share of labor allocated to production (a_t and $1-a_t$, respectively). To simplify the discussion, we assume that L represents simultaneously the population level and the amount of available labor. Thus, a_tL is at the same time the part of the population of the economy living and working in the first region (we do not assume as possible for people to live in one region and to work in the other).

A central assumption of our framework is related to the means through which capital loses value in time. We assume a usual constant depreciation rate, $\delta>0$; to this we add a negative externality effect caused by population congestion. The argument is that overcrowded locations will suffer a faster loss of value of physical infrastructures; traffic and pollution, for instance, will contribute to the degradation of the accumulated social capital stock (like roads and other collective equipment that depreciates faster, when overused). Therefore, instead of a simple constant rate of depreciation, one assumes a depreciation function, for each region, that reflects the negative externality produced by congestion; these functions are: $f(K_{1t}, a_t L) = \delta K_{1t} + \xi(a_t L)$ and $f[K_{2t}, (1-a_t)\cdot L] = \delta K_{2t} + \xi[(1-a_t)\cdot L]$. We will soon describe the properties that functions ξ should obey to. For now, we just assume processes of capital accumulation given by simple Solow-type equations; taking a constant marginal propensity to consume, $c \in (0,1)$,

$$K_{1t+1} - K_{1t} = (1-c)\cdot Y_{1t} - f(\delta K_{1t}, a_t L),\ K_{10} \text{ given.} \quad (1)$$

$$K_{2t+1} - K_{2t} = (1-c)\cdot Y_{2t} - f[\delta K_{2t}, (1-a_t)\cdot L],\ K_{20} \text{ given.} \quad (2)$$

Equations (1) and (2) describe growth in each one of the two regions. As it is straightforward to perceive, the growth process in each region is independent from the other, except in one central detail: people can migrate and, thus, the amount of labor available to produce in each region might vary over time.

Relatively to the effect of population congestion over capital accumulation, this is intuitively depicted in figure 1.

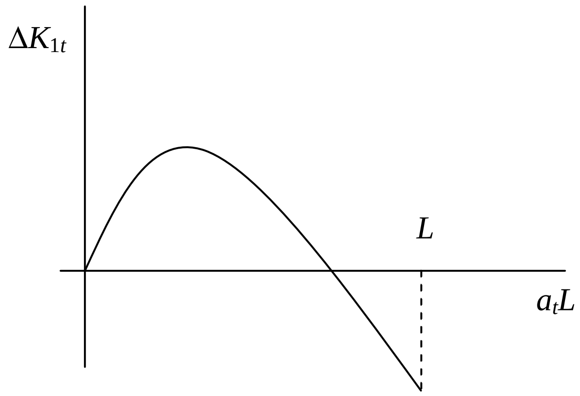

Figure 1 – The congestion externality in region 1.

We should expect that low levels of population would not imply a negative effect over capital accumulation, because no congestion is yet present. Thus, in a first phase, the capital stock will vary increasingly as more labor is introduced in production. After a given point, additional population / labor in the location begins to imply a penalty over the growth of the capital stock: two conflicting forces will collide; the production effect related with labor as an input and the congestion effect, linked with the disruption of social infrastructures. For extremely high levels of population the second effect may clearly dominate implying that the large amount of labor available to produce does not compensate the losses that the negative externality of congestion imposes. The stock of capital will decline for extremely high population levels. Note that figure 1 represents the externality effect for region 1, but a similar figure could be presented to characterize labor and capital dynamics in region 2.

The following functional forms will serve our purpose, in the sense they are in accordance with the capital dynamic features just described:

$\xi(a_t L) = \theta\cdot(a_t L)\cdot\ln(a_t L)$ and $\xi[(1-a_t)\cdot L] = \theta\cdot[(1-a_t)\cdot L]\cdot\ln[(1-a_t)\cdot L]$.

Parameter $\theta > 0$ will be designated as the congestion parameter.

Hereafter, we will deal with equations (1) and (2) in intensive form, that is, defining $k_{1t} \equiv K_{1t}/L$, $k_{2t} \equiv K_{2t}/L$, $\lambda \equiv \ln L$,

$$k_{1t+1} - k_{1t} = (1-c)\cdot Ak_{1t}^{\ \alpha} \cdot a_t^{\ 1-\alpha} - \delta k_{1t} - \theta a_t \cdot (\lambda + \ln a_t) \tag{3}$$

$$k_{2t+1} - k_{2t} = (1-c)\cdot Ak_{2t}^{\ \alpha} \cdot (1-a_t)^{1-\alpha} - \delta k_{2t} - \theta \cdot (1-a_t)\cdot [\lambda + \ln(1-a_t)] \tag{4}$$

Which factors do the households take in consideration when deciding where to locate? Under our simplified framework such decisions are determined explicitly only by the wage rate. Considering a competitive market structure, wage rates coincide with the marginal productivity of labor; thus, $w_{1t} = (1-\alpha)\cdot A \cdot (k_{1t}/a_t)^{\alpha}$ and $w_{2t} = (1-\alpha)\cdot A\cdot [k_{2t}/(1-a_t)]^{\alpha}$.

Households attribute utility values to accumulated wage rates. Variables u_{1t} and u_{2t} represent these utility values, which evolve according to rules (5) and (6)

$$u_{1t+1} - u_{1t} = u(w_{1t}) - \rho u_{1t},\ u_{10} \text{ given.} \tag{5}$$

$$u_{2t+1} - u_{2t} = u(w_{2t}) - \rho u_{2t},\ u_{20} \text{ given.} \tag{6}$$

Parameter $\rho>0$ can be interpreted as a rate at which past wage utility levels lose value and $u(w_t)$ are the utilities of the contemporaneous wage rates (in each region) that are added to previously accumulated location utility variables. We consider that wage has a positive but diminishing contribution to the utility of staying in a region and, therefore, we adopt the following functional forms: $u(w_{1t}) = \ln w_{1t}$; $u(w_{2t}) = \ln w_{2t}$.

Finally, one must recognize that there are some inertia factors that lock the individual to the location where she is, independently of wage differentials. This idea is captured by adopting a discrete choice rule. In this rule, parameter $b\geq 0$, that is known as the intensity of choice, will govern the willingness with which each worker responds to changes in the utility of the wage levels. In the extreme cases, if $b=0$ individuals will not move, even though wages might be systematically higher in the other region (we can call this case 'full cultural inertia'); when $b\to\infty$, the agent will react solely to the utility withdrawn from the wage, and change location immediately if this is advantageous from an income point of view. The discrete choice rule takes the form

$$a_t = \frac{\exp(bu_{1t})}{\exp(bu_{1t}) + \exp(bu_{2t})} \tag{7}$$

Share a_t in (7) has two possible interpretations. It reflects the percentage of individuals choosing to stay in region 1 in moment t; it can also be seen as the probability of a single agent choosing to remain in that region.

Simple algebra allows us to take (5), (6) and (7) to arrive to a dynamic equation describing the motion of share a_t; the calculus leads to

$$a_{t+1} = \frac{1}{\left\{1 + \left(\frac{1-a_t}{a_t}\right)^{1-\rho} \cdot \exp\left[b \cdot \left(u(w_{2t}) - u(w_{1t})\right)\right]\right\}}, \quad a_0 \text{ given.} \tag{8}$$

With equation (8), our dynamic setup is complete. System of equations (3), (4) and (8) is a two-region growth model, where growth is described by simple neoclassical capital accumulation equations, and location decisions, that determine labor force availability, depend on wages; these, in turn, are conditioned by the potential of production that is strongly limited by congestion externalities that arise when population exceeds some threshold level. Particularly interesting in this model, is the fact that regions are modelled as perfectly symmetric: they share the same parameters regarding production and location decisions of households. The next section finds some interesting dynamic results for this setup.

3. Global Dynamics

The nonlinear nature of system (3)-(4)-(8) introduces important obstacles into the dynamic analysis of the long run behaviour of the considered economic aggregates. In particular, it is not feasible to compute steady state results or to undertake a local analysis in the steady state vicinity. Only through numerical simulation one may withdraw some meaningful conclusions. To keep the analysis synthetic, we concentrate the study on the intensity of choice, letting all the other parameters assume reasonable values. In what follows, we consider a_0=0.6, k_{10}=k_{20}=1 and the vector of parameters [c A α δ ρ λ θ]=[0.75 0.1 0.25 0.1 0.1 0.4861 3].

Figure 2 draws a bifurcation diagram for $9<b<10$ (considering variable a_t).[3] In this case, one identifies a bifurcation process that transforms a fixed point result into limit cycles and eventually chaos. The figure indicates the presence of a Neimark-Sacker bifurcation or a Hopf bifurcation in discrete time [the transition from a fixed point to a-periodicity displayed in the figure is characteristic of this type of bifurcation. Nevertheless, given the sophistication of the system under analysis, a rigorous proof of the presence of this type of bifurcation is not feasible; see Medio and Lines (2001), page 158, for a rigorous statement of the Neimark-Sacker theorem].

[3] This and all the following figures are drawn using IDMC software (interactive Dynamical Model Calculator). This is a free software program available at www.dss.uniud.it/nonlinear, and copyright of Marji Lines and Alfredo Medio.

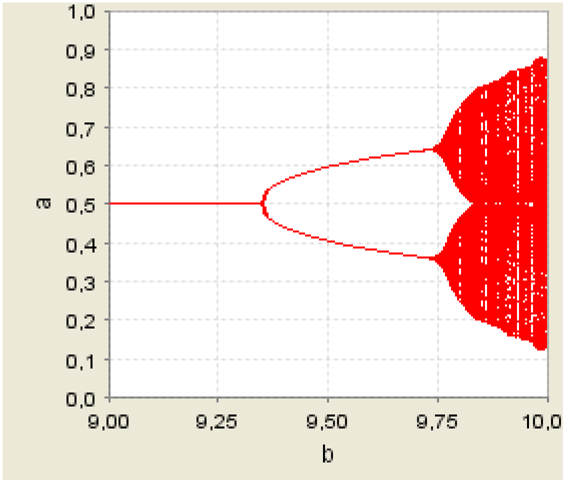

Figure 2 – Bifurcation diagram (a_t,*b*)

The referred bifurcation may eventually lead to chaos. We compute Lyapunov characteristic exponents to infer about the presence of chaos. These exponents are a measure of divergence of nearby orbits, and it is accepted that the presence of a positive Lyapunov exponent indicates that chaotic motion exists. Figure 3 displays the Lyapunov exponents of our system, and we effectively regard that for values of *b* near 10 chaos exists. For lower values, we have invariant limit cycles, and before this long term state even lower values of the intensity of choice imply the fixed point result observed in figure 2.

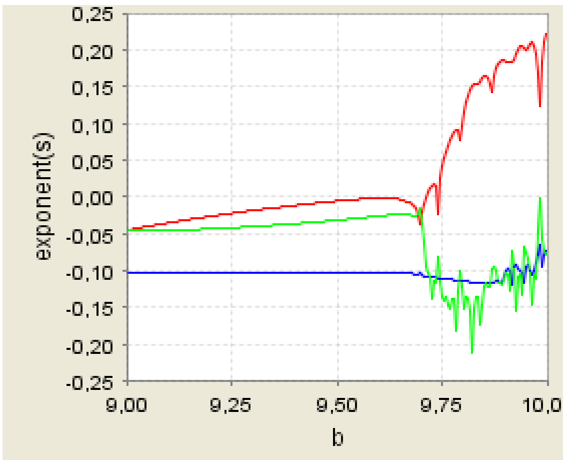

Figure 3 – Lyapunov characteristic exponents (9<*b*<10).

Some attractors confirm the previous results. Figure 4 characterizes the long run relationship between capital variables, in the presence of chaos (a strange attractor is observed); figure 5 presents the same relation, but for an intensity of choice where two invariant limit cycles are found (*b*=9.8). For the same *b* as in figure 4, figures 6, 7 and 8 give attractors for the relation between capital and consumption in each region and for the long term relation between total consumption (the sum of the consumption aggregates relating the production in each location) and the population share a_t. Considering once again the same set of parameter values, figures 9 to 11 represent the long term time trajectories of the main variables in our system.

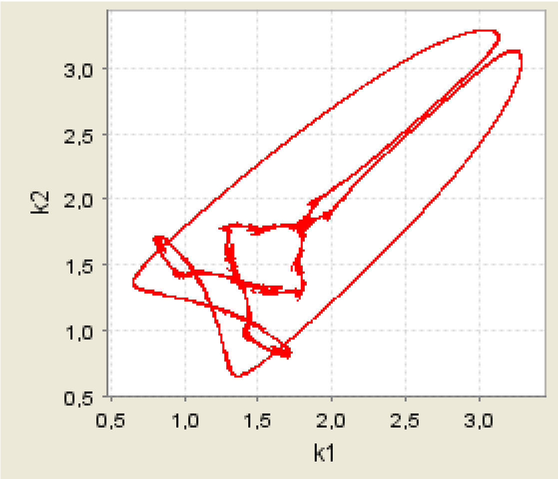

Figure 4 – Attractor (k_{1t}, k_{2t}; b=10).

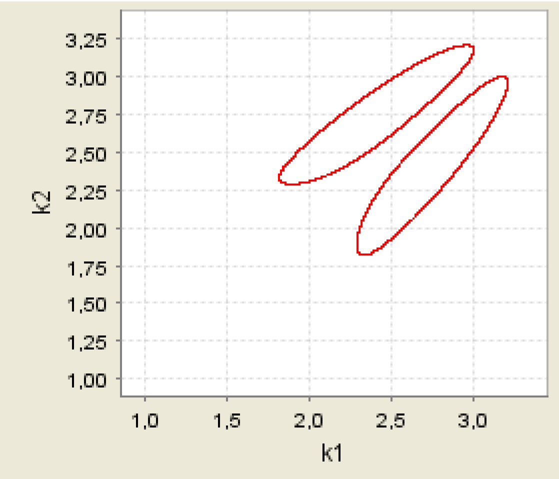

Figure 5 – Attractor (k_{1t}, k_{2t}; b=9.8).

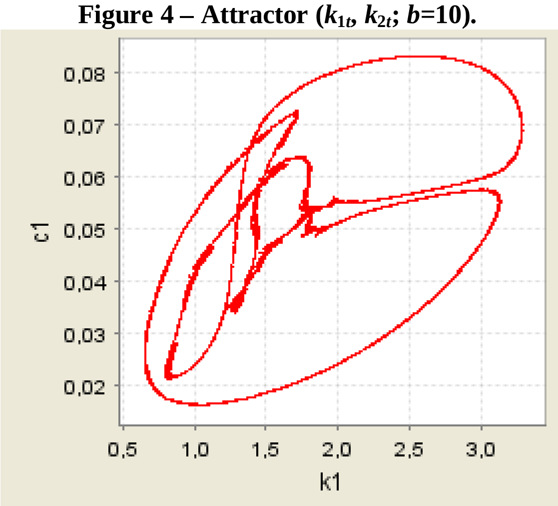

Figure 6 – Attractor (k_{1t}, c_{1t}; b=10).

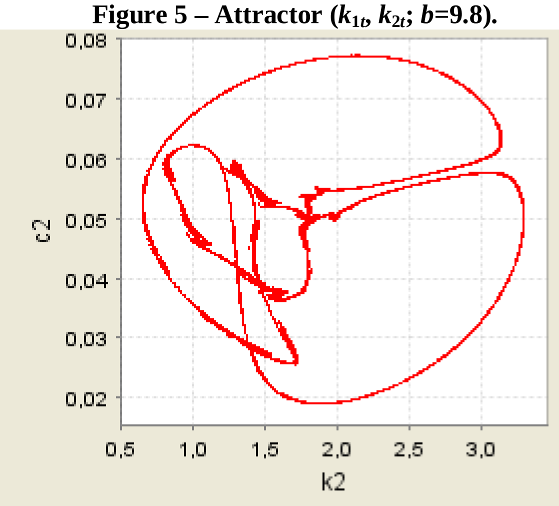

Figure 7 – Attractor (k_{2t}, c_{2t}; b=10).

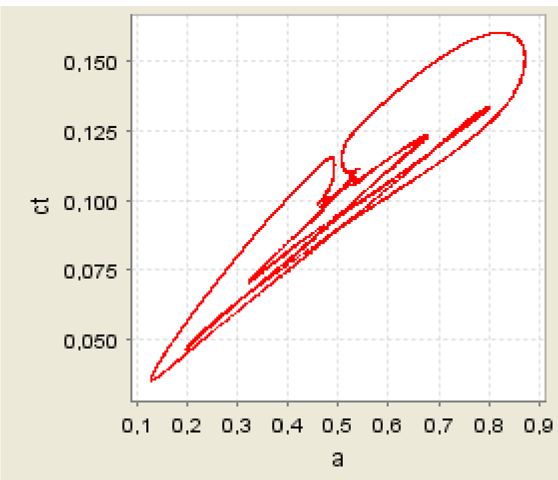

Figure 8 – Attractor (a_t, c_t; b=10).

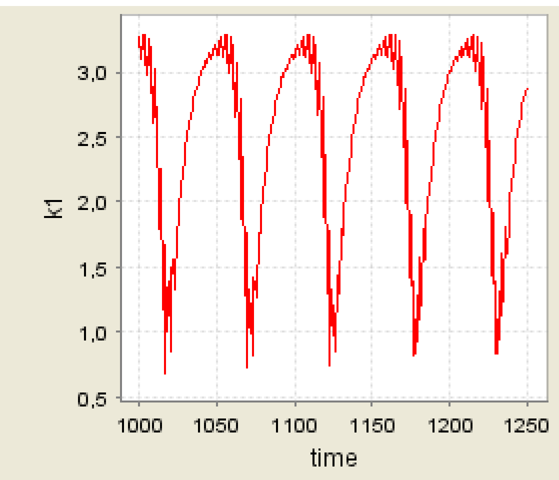

Figure 9 – Long term time series (k_{1t}, b=10).

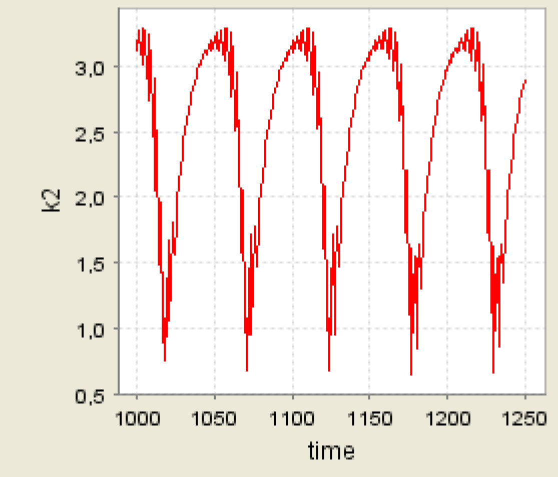

Figure 10 - Long term time series (k_{2t}, b=10).

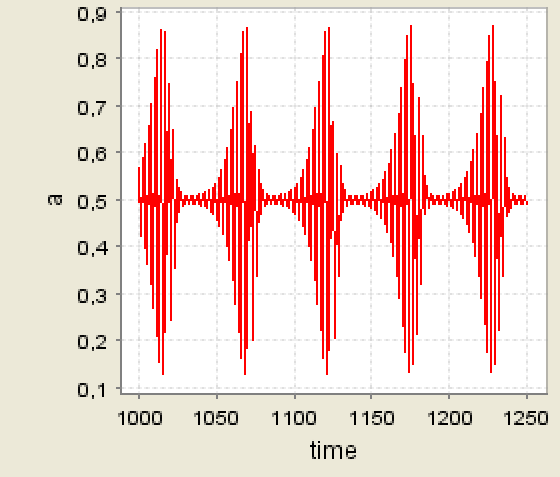

Figure 11 – Long term time series (a_t, b=10).

The graphical analysis indicates that for a specific set of parameter values, one finds endogenous cycles characterizing the long run behaviour of the stock of capital, consumption and labor availability. In this case, we can justify processes of never ending migrations, with impact over the paths of production and consumption. Business cycles are determined by changes in wages over time, that lead people to change location systematically, responding to these monetary incentives.

4. Policy Implications and Conclusions

The proposed framework allows to jointly approach growth phenomena and household decisions regarding location, in an environment of congestion externalities. The two regions have identical productive processes (use the same technology and have identical output elasticities) and the same parameters governing capital depreciation, congestion externalities, utility withdrawn from wages, and the percentage of savings out of income. The regions are independent in the sense they produce different goods with different forms of capital, although they are linked through the labor marker: people migrate, looking for higher wages and this determines the amount of labor available in each one of the locations.

For reasonable values of parameters, one identifies cases of stability (the share of labor remaining in one region stays, in the long term, unchangeable, in a given value between zero and one), instability (the system diverges for a full concentration of individuals, and thus economic activity, in one of the regions), and it is found that a bifurcation leads to endogenous cycles that are eventually chaotic. The presence of endogenous cycles supports the view that this simple framework may contribute to explain macroeconomic fluctuations: cycles are triggered by two conflicting forces, which are a positive stimulus of agglomeration implied by increasing wages in regions where high amounts of labor exist and a negative factor that is congestion external effects.

The main policy implication comes from the intensity with which agents react to wage differentials. Stability requires relatively low intensity of choice, meaning that the political ability to keep people in one region even though this is less developed than the other is crucial for stable growth. Moreover, given that instability implies the full concentration of activity in one of the regions, guaranteeing a low intensity of choice is not only the way for economic long term predictability but also for regional cohesion. This, in turn, helps as well to avoid the perverse effects of congestion externalities, not only over the productive system but also over the households' quality of life.

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