E C O N O M I C S B U L L E T I N

# Final Goods Substitutability and Economic Growth

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## *Abstract*

In this paper, I investigate the effect of substitutability among final goods on welfare growth under the environment that productivity growth in each industry is not independent of one another. In such an environment, less substitutability is favorable to the welfare growth rate and the steady state welfare level, contrasting to Baumol (1967) and Lucas (1988).

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# 1 Introduction

In this paper, I investigate the effect of substitutability among final goods on welfare growth under the environment that productivity growth in one industry is not independent of that in another industry. Baumol (1967) analyzes the effect of substitutability when one sector has no productivity growth, and Lucas (1988) examines it with industry-specific human capital accumulation in the context of international trade. In both models, substitutability is good for growth because more resources are put into the more productive sector. However, if productivity growth is not independent across sectors, the answer is easily reversed: complementarity is good for growth. Research which is aimed for productivity improvement in an industry could provide useful knowledge, directly or indirectly, to production of other goods. If more research is made in the economy where consumer's utility depends more on variety and it is general in the sense of the previous sentence, less substitutability is favorable for welfare growth. To see this story, I present a model in which many industries share a common research sector that provides research contributing to technical efficiency in production process.

# 2 The Model Setting

*Consumers.* — There exists a continuum variety of final goods, $i \in [0, 1]$. Consumers have a Dixit-Stiglitz utility function:

$$u = \left( \int_0^1 c_i^\rho \, di \right)^{1/\rho}, \quad \rho < 1. \tag{1}$$

There exists a continuum of consumers and firms (both are unit masses). Consumers share all firms evenly, so their incomes are the expected profit of an average firm, $\pi^e$. There is neither wage income nor method of saving. They maximize their utility under the appropriate budget constraint. Then, the relative price of final goods in industry $i$, $q_i$, must satisfy

$$q_i = \left( \frac{c_i}{c_0} \right)^{-\frac{1}{\eta}},$$

where $\eta = 1/(1 - \rho)$ is the elasticity of substitution, and the final good produced in industry 0 is the numeraire.

*Industries and firms.* — Each final good is produced by a distinct industry. I use the same subscript $i$ to describe industries. Industries are identical except the uncertainty levels, $\{s_i\}_{i=0}^1$, such that $s_0 > 0$ and $s_i \geq s_j$ if $i > j$.

Each firm has limited resource of 1 and may be a producer in an industry or a researcher. I assume that there is no switching cost. If a firm $j$ is a producer in industry $i$, it has the following production function, similarly to Jovanovic and Nyarko (1996):

$$y_{ij} = 1 - s_i(\theta_{ij} - z_{ij})^2.$$

The shock parameter $\theta_{ij}$ is drawn from a standard normal distribution, which is common knowledge, and is idiosyncratic across firms. $z_{ij}$ is the control variable. If a manager, by chance, sets $z_{ij}$ as the same as $\theta_{ij}$, production reaches the maximum level for any $s_i$. In other words, $s$ represents the importance of manager's prediction and planning ability.

With the quadratic form and risk neutrality, the optimal $z_{ij}$ equals the expected value of $\theta_{ij}$. To refine the belief about $\theta$, a producer can obtain a signal according to its amount of purchase of research. The signal is generated from the rule: $\theta_{ij} + \epsilon_{ij}/\sqrt{n_{ij}}$, where $\epsilon_{ij} \sim N(0, \sigma_r^2)$.[1]

Denoting $p$ as the research price, the expected profit function of a producer is

$$\pi^e_{P,ij} = q_i\,(1 - h(n_{ij}, s_i)) - pn_{ij}. \tag{2}$$

$h(n_{ij}, s_i)$ is the posterior variance of $\theta_{ij}$:

$$h(n_{ij}, s_i) = \frac{s_i}{1 + n_{ij}/\sigma_r^2}.$$

I call this function as the expected technical inefficiency. Assuming that the research market is competitive, producer $(i,j)$ maximizes (2) about $n_{ij}$ with taking $p$ and $\{q_i\}$ as given.

$$n_{ij}(p, q_i; s_i) = \begin{cases} 0 & \text{for } p > \frac{s_i q_i}{\sigma_r^2}, \\ \sigma_r^2\left(\sqrt{\frac{s_i q_i}{\sigma_r^2 p}} - 1\right) & \text{otherwise.} \end{cases} \tag{3}$$

$$\pi^e_P(p, q_i; s_i) = \begin{cases} q_i(1 - s_i) & \text{for } p > \frac{s_i q_i}{\sigma_r^2}, \\ q_i - 2\sqrt{\sigma_r^2 s_i p q_i} + \sigma_r^2 p & \text{otherwise.} \end{cases}$$

*Research sector and knowledge accumulation.* — I define $k$ as the existing knowledge in the society which is available for any researcher. Based on the idea that an input of knowledge creation is also knowledge, I would formulate research productivity as an increasing function of $k$. But, for simplicity, I assume it is the identical function. (I should note that all arguments in this paper hold if research productivity is strictly increasing in $k$.) If a firm (with the limited resource of 1) chooses to be a researcher, it creates $k$ research without any cost. So the researcher's profit is

$$\pi_R(p, k) = pk.$$

I assume that new knowledge created in one instant become publicly available in the next instant. Letting $\lambda$ be the measure of researchers, knowledge accumulation is set as $\dot{k} = (\lambda - \delta)k$, where $\delta$ is the obsolescence rate of knowledge. I also assume that the initial knowledge capital is sufficiently large:

$$k > \frac{\sigma_r^2(1 - s_0)}{s_0}. \tag{4}$$

This assumption simplifies the following analyses by eliminating equilibrium in which producers in some industries do not require any research.

[1] This setting avoids a non-integer number of signals. If I allow a continuous number of signals, $n$ is the number of signals and each signal has the value of $\theta + \epsilon$.

# 3 Instantaneous Equilibrium

For given $k$ and $\{s_i\}_{i=0}^1$, instantaneous equilibrium is a bundle of $p$, $\{q_i\}_{i=0}^1$, $\{n_{ij}\}$, $\lambda$, $\{\mu_i\}_{i=0}^1$ (measures of producers), and $\{c_i\}_{i=0}^1$ that solve consumer's utility maximization, solve firms' profit maximization in expectation, and satisfy the following conditions (omitting firm subscripts):

$$\pi_{P,i}^e = \pi_R \quad \forall i; \tag{5}$$

$$c_i = \mu_i(1-h_i) \quad \forall i; \tag{6}$$

$$\lambda k = \int_0^1 \mu_i n_i \, di; \tag{7}$$

$$1 = \lambda + \int_0^1 \mu_i di. \tag{8}$$

These conditions represent no switching between producers and researchers, goods market clearing, research market clearing, and feasibility, respectively. In the rest of this section, I derive the equilibrium bundle of the above variables.

*Equilibrium prices.* — A firm may choose in which industry it operates and also may choose to be a researcher. Because there is no switching cost in the model, every producer in every industry and every researcher must have the same expected profits in equilibrium. Therefore,

$$p = \frac{h_0^2}{s_0 \sigma_r^2} \quad \text{and} \quad q_i = \frac{s_i h_0^2}{s_0 h_i^2}, \tag{9}$$

where $h_i$ is the expected inefficiency in industry $i$:

$$h_i = \frac{1}{1 + \sqrt{\frac{k-\sigma_r^2}{s_i \sigma_r^2} + 1}}. \tag{10}$$

*Determination of industry sizes.*— From the first-order conditions of the consumer's maximization problem and (6),

$$q_i = \left(\frac{c_i}{c_0}\right)^{-\frac{1}{\eta}} = \left(\frac{\mu_i(1-h_i)}{\mu_0(1-h_0)}\right)^{-\frac{1}{\eta}}.$$

Therefore,

$$\mu_i = \frac{1-h_0}{1-h_i} \mu_0 q_i^{-\eta}. \tag{11}$$

Industry size, $\mu_i$, is non-monotonic in $i$ in general. When the final goods are highly substitutable, it is strictly decreasing in $i$ because consumers consume few or non of expensive goods. When they are poor substitutes, $\mu_i$ is strictly increasing because more resources are required to produce highly uncertain goods. However, in between, it draws a

U-shape curve. In small $s$ industries, there is high demand because of low prices, and large $s$ industries require many producers to satisfy the demand because of high inefficiency in production.

The measure of producers in industry 0 is pinned down by (5), (7), (8), and (11) as follows:

$$\mu_0 = \left[ \int_0^1 q_i^{1-\eta} \, di \right]^{-1} . \tag{12}$$

As knowledge accumulates, the measure of firms in each industry changes, depending on the elasticity of substitution. When $\eta < 1$ (poor substitutes), an increase in $k$ leads to an increase in $\mu_0$. They have to spend a lot of resources on highly uncertain goods when $k$ is small. However, if $k$ becomes higher, they can afford to spend more resources on more certain goods production. If $\eta > 1$ (highly substitutable), $\mu_0$ is decreasing in $k$. The reason is analogous.

# 4 The Effect of Substitutability on Welfare Growth

Since $\eta$ determines the industries' and research sector's size profile, knowledge accumulation is also affected by the elasticity of substitution. As an extreme example, when the final goods are perfect substitutes ($\eta \to \infty$), all industries other than industry 0 collapse because it is the cheapest goods. Since the level of uncertainty in industry 0 is the smallest, there is only a little research done in the economy.

On the contrary, if they are perfect complements ($\eta = 0$), the measures of producers $\mu_i$ are strictly increasing in $i$ to produce the same amount of output despite the differences in inefficiency. The research demand must be high, so the accumulation of knowledge capital is rapid.

From these extreme examples, it is predicted that there exists a monotonic relation between the elasticity of substitution and the knowledge capital accumulation. The next lemma and proposition show that it is indeed true.

**Lemma 1** *There exists a unique threshold industry $\hat{\imath} \in (0, 1)$ such that $\mu_i$ increases as $\eta$ increases if and only if $i < \hat{\imath}$.*

**Proof.**

$$\frac{\partial \mu_i}{\partial \eta} = \frac{1 - h_0}{1 - h_i} \mu_0 q_i^{-\eta} \left( \frac{\partial \mu_0 / \partial \eta}{\mu_0} - \ln q_i \right) .$$

Since $\frac{\partial \mu_0/\partial \eta}{\mu_0}$ is a positive constant and $\ln q_i$ is strictly increasing with the minimum at 0, there exists a unique threshold $\hat{\imath}$ that divides the sign of $\partial \mu_i / \partial \eta$ if $\ln q_i$ crosses the line of $\frac{\partial \mu_0/\partial \eta}{\mu_0}$. Suppose $\frac{\partial \mu_0/\partial \eta}{\mu_0} - \ln q_i$ is positive for all $i$. Then the above equation is positive for any $i$, implying that the measure of producers $\mu_i$ increases in all industries. Because the research demand from each producer is unchanged by a shift in $\eta$, the total demand for research is also increased. However, this cannot happen in equilibrium since there is now a smaller measure of researchers than before the change. Hence, there exists unique $\hat{\imath} \in (0,1)$ such that $\mu_i$ increases as $\eta$ increases if and only if $i < \hat{\imath}$. ■

This lemma says that industries with lower uncertainty have more producers when the substitutability among final goods is high. This is simply because the final good of a more inefficient industry is more expensive. The next proposition shows that knowledge accumulation is faster under less substitutability.

**Proposition 2** *For given $k$, the growth rate of knowledge capital is higher when the elasticity of substitution is lower.*

**Proof.** Suffice it to show that $\lambda$ is decreasing in $\eta$ for given $k$ by the assumption about knowledge accumulation. Suppose $\lambda$ increases in response to an upward shift of $\eta$. The total measure of producers must be decreased, so

$$\int_0^{\hat{\imath}} \mu_i'(\eta) di < \int_{\hat{\imath}}^1 |\mu_i'(\eta)| di,$$

where $\hat{\imath}$ is defined in Lemma 1 above. Now, denote $\hat{n} = n_{\hat{\imath}}$. Since $n_i$ is increasing in $i$, $n_i \leq \hat{n}$ for $i \leq \hat{\imath}$, and vice versa. Then,

$$\int_0^{\hat{\imath}} \mu_i'(\eta) n_i di \leq \int_0^{\hat{\imath}} \mu_i'(\eta) \hat{n} di < \int_{\hat{\imath}}^1 |\mu_i'(\eta)| \hat{n} di < \int_{\hat{\imath}}^1 |\mu_i'(\eta)| n_i di.$$

This implies that the total demand for research shifts downward. This is a contradiction because the supply of research increased under the current supposition. Therefore, an upward shift of $\eta$ must reduce $\lambda$. ■

*Welfare growth.*— The growth rate of welfare measured by representative consumer's utility is positively related with the growth rate of knowledge capital. I show it in Proposition 3.

**Proposition 3** *For given $k$, the growth rate of welfare of the representative agent is increasing in the knowledge capital growth rate:*

$$g^u = \left[ \lambda + \frac{h_0}{2(1-h_0)} \right] g^k. \tag{13}$$

**Proof.** From (1), (11), and (12),

$$u = (1-h_0) \left[ \int_0^1 q_i^{1-\eta} di \right]^{-\frac{1}{1-\eta}}.$$

On the equilibrium path,

$$g^u = \left[ \frac{h_0}{2(1-h_0)} + \mu_0 \int_0^1 q_i^{1-\eta} \left( -\frac{q_i'(k)k}{q_i} \right) di \right] g^k,$$

where

$$-\frac{q_i'(k)k}{q_i} = 1 - \frac{1-h_0}{q_i(1-h_i)} = 1 - q_i^{\eta-1}\frac{\mu_i}{\mu_0}.$$

Combination of the above equations with (8) yields (13). ■

The accumulation of knowledge reduces the final goods prices through cheaper research prices or more efficient production processes. So, as in the above equation, the welfare growth rate is increasing in the knowledge growth rate. Then, from Proposition 2 and 3, the following corollary holds.

**Corollary 4** *For given $k$, the growth rate of welfare is higher if the final goods are less substitutable, as long as the growth rate of knowledge is positive.*

In Baumol (1967) and the autarchy model of Lucas (1988)'s Section 5, higher substitutability among final goods is favorable for growth because, under complementarity, they spend more resources on the less productive sector. In the current model, more resources are spent on less productive industries as well. However, producers in those industries demand more research, stimulating knowledge accumulation and thus welfare growth. Therefore, less substitutability is favorable for growth.

# 5 Steady State Analysis

Because there is no growth in the steady state of this model, I have analyzed the transitional equilibrium path so far. Here, I show that the steady state welfare level is higher if the elasticity of substitution is lower, i.e., final goods are less substitutable.

*Steady state.*— Steady state is defined by $\lambda = \delta$ from the law of motion of knowledge capital. For a steady state knowledge to exist, it is sufficient to assume that the rate of obsolescence of knowledge, $\delta$, is sufficiently small and the initial $k$ is sufficiently large because $\lambda$ is converging to zero as $k$ goes to infinity.[2] Under such assumptions, the next proposition holds.

**Proposition 5** *The steady state welfare level is higher if the elasticity of substitution is lower.*

**Proof.** Suppose there are two economies, 1 and 2, which are identical except $\eta_1 > \eta_2$. In the proof of Proposition 2, I have shown $\lambda$ is decreasing in $\eta$ for given $k$. Since the steady states (expressed by asterisks) are defined by $\lambda_1(k_1^*) = \delta = \lambda_2(k_2^*)$, $k_1^* < k_2^*$. Appealing to (10), one can see $h_{1,i}^* > h_{2,i}^*$ for any $i$, and thus $q_{1,i}^* \geq q_{2,i}^*$ for any $i$ from (9) and the fact that $h$ is concave in $k$ (the equality holds only for $i = 0$). Now we know final goods are

[2] Oikawa (2007) includes steady state analyses of this economy in more detail.

cheaper in the economy 2. To see the higher welfare level in the steady state of economy 2, suffice it to show that income $\pi^e = p^* k^*$ is greater in economy 2. From (9),

$$\frac{\partial \pi^e}{\partial k} > 0 \quad \Leftrightarrow \quad \frac{k}{s_0 \sigma_r^2} < \sqrt{\frac{k}{s_0 \sigma_r^2} - \frac{1-s}{s}} \left( 1 + \sqrt{\frac{k}{s_0 \sigma_r^2} - \frac{1-s}{s}} \right).$$

After some calculation, one can show that $\partial \pi^e / \partial k > 0$ for any $k$ under the assumption (4). ■

The favorable effect of less substitutability is not only on the point-wise growth rates, but also on the steady state level of welfare. Because knowledge capital is accumulated more under less substitutability, such an economy becomes more able to resolve uncertainty.

# 6 Concluding Remarks and Further Research

The knowledge capital in this paper may be interpreted as a general purpose technology, e.g., information technology, which affects productivity not only in the industry in which the technology is developed, but also in many other industries (Stiroh (2001), etc.). The result of Baumol (1967) and Lucas (1988) comes from the setting that productivity improvement is independent across sectors. If the productivity growth in one sector is not independent of that in another sector, as in this paper, the conclusion is reversed. Unfortunately, it is hard to measure the elasticity of substitution in a utility function, and I have not yet found a way to test empirically which story is more likely. This is the empirical part of the further research. Theoretically, a shortcoming of this model is that all uncertainty is idiosyncratic. Industry-level, macro-level, and/or technology-specific risks should be included to make the model closer to the reality.

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