E C O N O M I C S   B U L L E T I N

Hotelling's Location Model with Quality Choice in Mixed Duopoly

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Abstract

We investigate a mixed duopoly market by introducing quality choice into the Hotelling-type spatial competition model with linear transportation costs. We show that there does not exist a subgame perfect Nash equilibrium (SPNE) of location choice in the three-stage game that is location-then-quality choice and subsequent price choice. Moreover, we show the conditions of the existence of the quality equilibrium.

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1. Introduction

In the literature on Hotelling-type spatial competition model, Hotelling (1929) assumes that the transportation costs of consumers are linear, and characterizes the horizontal product differentiation as the *principle of minimum differentiation*. However, d'Aspremont, Gabszewicz and Thisse (1979) point out that Hotelling's research result is incorrect, and show that the existence of the equilibrium is provided if the transportation costs are quadratic[1]. Economides (1989) shows that the equilibrium of the game exists, by introducing quality choice into the model of Hotelling (1929) in pure strategies. His results are characterized by *minimum differentiation* (price and quality choice) and *maximum differentiation* (location choice).

As for studies of mixed markets, there is an enormous amount of literature examining mixed oligopoly markets, in which state-owned welfare-maximizing public firms compete against profit-maximizing private firms, in recent years[2]. In these studies, many papers assume that the transportation costs are not linear but quadratic. Cremer, Marchand and Thisse (1991), for example, develop a Hotelling-type spatial competition model of a mixed oligopoly with quadratic transportation costs[3]. In the most recent paper, however, Lu (2006) shows that there does not exist a subgame perfect Nash equilibrium (SPNE) in Hotelling's linear city location-then-price model when the transportation costs are linear in mixed duopoly.

Our main interest in this paper is whether there exists the equilibrium of the game, by incorporating the level of quality as a strategic variable which is similar to that of Economides (1989), when the transportation costs are linear. Therefore, the purpose of this paper is to investigate Hotelling's location model with quality choice, under the mixed duopoly. Using quality choice, we can analyze not merely location as horizontal product differentiation but also quality as vertical product differentiation. We show that there does not exist a SPNE of location choice in Hotelling's location model with quality choice while there exist the quality equilibrium and the price equilibrium in each of the subgames. In addition, we show the conditions of the existence of the quality equilibrium.

[1] For an excellent survey, see Gabszewicz and Thisse (1992).

[2] See De Fraja and Delbono (1990) and Nett (1993) for general reviews of the mixed oligopoly model.

[3] For related papers, see Nilssen and Sørgard (2002) and Matsumura and Matsushima (2003, 2004).

The remainder of this paper is organized as follows. In section 2, we present the model. Section 3 investigates the equilibrium outcomes of the model. Section 4 concludes the paper.

2. The model

We consider a Hotelling-type linear city of length 1 in a mixed duopoly market. There are two firms indexed by i $(= 0, 1)$. Firm 0 is a private firm. Firm 1 is a public firm. In the first stage of the game, firm i simultaneously chooses a location $l_i \in [0, 1]$. Here, firm 0 is located l_0 of distance from point 0. Firm 1 is located l_1 of distance from point 1. Therefore, we assume that $l_0 \geq 0,\ \ l_1 \geq 0$ and $l_0 + l_1 \leq 1$. In the second stage, the firms choose their quality level $q_i > 0$, simultaneously. Then, each firm i has a cost of producing quality $\delta(q_i)$ to achieve a quality level q_i[4]. For simplicity, we assume that $\delta(q_i) = \frac{1}{2}\theta q_i^2$, where $\delta'(\cdot) > 0, \delta''(\cdot) > 0$ and θ is a parameter of quality, $\theta > 0$. In the third stage, the two firms choose their price $p_i \in [0, \infty)$, with zero marginal cost, simultaneously. The game is solved by backward induction.

Consumers are uniformly distributed along the unit interval. A consumer lives at $x \in [0, 1]$. Each consumer demands one unit of the product, and derives a surplus from consumption equal to s. We assume that s is so large that each consumer consumes one unit of the product. To consume the product, a consumer living at x pays a transportation cost of $t|x - l_0|$ when purchasing the product from firm 0, or pays $t|x - (1 - l_1)|$ when purchasing from firm 1, where $t > 0$ is the unit cost of transportation. To hold the second-order conditions, we assume that $\theta t > 3/2$.

The utility of a consumer located at point x is given by

$$u_x = \begin{cases} s + q_0 - p_0 - t|x - l_0| & \text{if purchasing from firm 0,} \\ s + q_1 - p_1 - t|x - (1 - l_1)| & \text{if purchasing from firm 1.} \end{cases}$$

The location x of a consumer who is indifferent between purchasing the product from firm 0 and firm 1 is given by

$$x = \frac{(q_0 - p_0) - (q_1 - p_1)}{2t} + \frac{1 + l_0 - l_1}{2}. \tag{1}$$

[4]We assume that the product costs of the firms are quantitatively and qualitatively separable. This model setting is a widely used assumption in the literature (see e.g., Economides, 1989, 1993; Calem and Rizzo, 1995; Lyon, 1999; Gravelle and Masiero, 2000; Barros and Martinez-Giralt, 2002; Brekke, Nuscheler and Straume, 2006).

The demand for firm 0 is

$$D_0 = \begin{cases} 1 & \text{if } (q_0 - p_0) - (q_1 - p_1) > t(1 - l_0 - l_1), \\ 0 & \text{if } (q_0 - p_0) - (q_1 - p_1) < -t(1 - l_0 - l_1), \\ x & \text{otherwise.} \end{cases} \tag{2}$$

The demand for firm 1 is

$$D_1 = 1 - D_0. \tag{3}$$

The consumer surplus CS is

$$\begin{aligned} CS = & \int_0^x (s + q_0 - p_0 - t(y - l_0))\, dy + \int_x^1 (s + q_1 - p_1 - t(1 - l_1 - y))\, dy \\ = & s + (q_0 - p_0)x + (q_1 - p_1)(1 - x) - t\left\{x^2 - (1 + l_0 - l_1)x - l_1 + \frac{1}{2}\right\}. \end{aligned}$$

The two firms' profits are

$$\begin{aligned} \pi_0 &= p_0 D_0 - \delta(q_0) \\ &= p_0 x - \frac{1}{2}\theta q_0^2, \qquad (4) \\ \pi_1 &= p_1 D_1 - \delta(q_1) \\ &= p_1(1 - x) - \frac{1}{2}\theta q_1^2. \qquad (5) \end{aligned}$$

The social welfare W is given by

$$\begin{aligned} W = & CS + \pi_0 + \pi_1 \\ = & s + q_0 x + q_1(1 - x) - t\left\{x^2 - (1 + l_0 - l_1)x - l_1 + \frac{1}{2}\right\} \\ & - \frac{1}{2}\theta q_0^2 - \frac{1}{2}\theta q_1^2. \end{aligned} \tag{6}$$

3. Equilibrium

We solve the game by backward induction.

Third stage — Bertrand competition. First, we consider the third stage subgame. Since the concept of the equilibrium in this stage is the same as Lu (2006), then we briefly discuss as follows.

When both firms are located at the same place (i.e., $l_0 + l_1 = 1$), there are infinite equilibria $(p_1 - q_1 + q_0 - \epsilon, p_1)$ for small-enough $\epsilon > 0$ and given q_0 and q_1. This is obvious immediately. The public firm has no incentive to change its price p_1 for any $p_0 = p_1 - q_1 + q_0 - \epsilon$ since the total transportation costs are constant. Therefore, the private firm has no incentive to change its price p_0 as well.

When the two firms are located at the different places (i.e., $l_0 + l_1 < 1$), for a given pair of locations (l_0, l_1) and a given pair of quality levels (q_0, q_1) of firm 0 and firm 1, the first-order condition for firm 0 is given by maximizing π_0 with respect to p_0,

$$\frac{\partial \pi_0}{\partial p_0} = x + p_0 \frac{\partial x}{\partial p_0} = 0.$$

Hence, the best response function of firm 0 is given by

$$p_0 = \frac{p_1 + q_0 - q_1 + t(1 + l_0 - l_1)}{2}. \tag{7}$$

Similarly, the first-order condition for firm 1 is given by maximizing W with respect to p_1,

$$\frac{\partial W}{\partial p_1} = q_0 \frac{\partial x}{\partial p_1} - q_1 \frac{\partial x}{\partial p_1} - t \left\{ 2x \frac{\partial x}{\partial p_1} - (1 + l_0 - l_1) \frac{\partial x}{\partial p_1} \right\} = 0.$$

Hence, the best response function of firm 1 is given by

$$p_1 = p_0. \tag{8}$$

From (7) and (8), we obtain the price set of the two firms as follows:

$$p_0 = p_1 = q_0 - q_1 + t(1 + l_0 - l_1). \tag{9}$$

The market distribution is given by

$$x = \frac{q_0 - q_1}{2t} + \frac{1 + l_0 - l_1}{2}. \tag{10}$$

Second stage — quality choice. Next, we discuss the equilibrium in the second stage of the game.

When both firms are located at the same location, the following proposition is derived.

Proposition 1. *When $l_0 + l_1 = 1$, there are infinite quality equilibria if and only if*

$$0 < q_1 < q_0.$$

Proof. See Appendix.

When the two firms are located at the different locations, substituting (9) and (10) into (4) and (6), we have

$$\pi_0 = \frac{\{q_0 - q_1 + t(1 + l_0 - l_1)\}^2}{2t} - \frac{1}{2}\theta q_0^2, \tag{11}$$

and

$$W = s + \frac{\{q_0 - q_1 + t(1 + l_0 - l_1)\}^2}{4t} + q_1 + t\left(l_1 - \frac{1}{2}\right) - \frac{1}{2}\theta q_0^2 - \frac{1}{2}\theta q_1^2. \tag{12}$$

The first-order condition for firm 0 is

$$\frac{\partial \pi_0}{\partial q_0} = \frac{q_0 - q_1 + t(1 + l_0 - l_1)}{t} - \theta q_0 = 0.$$

Since $\partial^2 \pi_0 / \partial q_0^2 = -(\theta t - 1)/t < 0$, the second-order condition is satisfied. Hence, we have

$$q_0 = \frac{t(1 + l_0 - l_1) - q_1}{\theta t - 1}. \tag{13}$$

The first-order condition for firm 1 is

$$\frac{\partial W}{\partial q_1} = -\frac{q_0 - q_1 + t(1 + l_0 - l_1)}{2t} + 1 - \theta q_1 = 0.$$

Since $\partial^2 W / \partial q_1^2 = -(2\theta t - 1)/(2t) < 0$, the second-order condition is satisfied. Hence, we have

$$q_1 = \frac{t(1 - l_0 + l_1) - q_0}{2\theta t - 1}. \tag{14}$$

Solving (13) and (14) for q_0 and q_1, we derive

$$q_0 = \frac{2\theta t(1 + l_0 - l_1) - 2}{\theta(2\theta t - 3)}, \tag{15}$$

and

$$q_1 = \frac{\theta t(1 - l_0 + l_1) - 2}{\theta(2\theta t - 3)}. \tag{16}$$

Since we assume that $q_0 > 0, q_1 > 0$ and $\theta t > 3/2$, we could obtain the following conditions:

$$\theta t(1 + l_0 - l_1) - 1 > 0, \quad \theta t(1 - l_0 + l_1) - 2 > 0.$$

Proposition 2. *When $l_0 + l_1 < 1$, there is the quality equilibrium if and only if*

$$\frac{1}{\theta t} - 1 < l_0 - l_1 < 1 - \frac{2}{\theta t}.$$

Proof. See Appendix.

Substituting (15) and (16) into (9) and (10), we have

$$p_0 = p_1 = \frac{2t\{\theta t(1 + l_0 - l_1) - 1\}}{2\theta t - 3}, \tag{17}$$

$$x = \frac{1 + 3(l_0 - l_1)}{2(2\theta t - 3)} + \frac{1 + l_0 - l_1}{2}. \tag{18}$$

First stage — location choice. Finally, we consider the equilibrium in the first stage of the game. In this stage, we clearly investigate two cases.

(i) The case of the same location (i.e., $l_0 + l_1 = 1$)

When both firms are located at the same location, the public firm can lower the total transportation costs if the public firm moves away from the private firm. To maximize the social welfare, the public firm has an incentive of moving away from the private firm. Therefore, a subgame perfect equilibrium is not stable in this case.

(ii) The case of the different locations (i.e., $l_0 + l_1 < 1$)

We consider the case that there are not the two firms in the same location. Using (11), (15) and (16), firm 0's profit is given by

$$\pi_0 = \frac{2(\theta t - 1)\{\theta t(1 + l_0 - l_1) - 1\}^2}{\theta(2\theta t - 3)^2}. \tag{19}$$

Using (12), (15) and (16), the social welfare is given by

$$\begin{aligned} W =& s + \frac{t\{\theta t(1 + l_0 - l_1) - 1\}^2}{(2\theta t - 3)^2} + \frac{\theta t(1 - l_0 + l_1) - 2}{\theta(2\theta t - 3)} \\ & - \frac{2\{\theta t(1 + l_0 - l_1) - 1\}^2}{\theta(2\theta t - 3)^2} - \frac{\{\theta t(1 - l_0 + l_1) - 2\}^2}{2\theta(2\theta t - 3)^2}. \end{aligned} \tag{20}$$

Since $\theta t(1 + l_0 - l_1) - 1 > 0$ and $\theta t > 3/2$, differentiating π_0 with respect to l_0, we obtain

$$\frac{\partial \pi_0}{\partial l_0} = \frac{4t(\theta t - 1)\{\theta t(1 + l_0 - l_1) - 1\}}{(2\theta t - 3)^2} > 0. \tag{21}$$

Similarly, differentiating W with respect to l_1, we have

$$\frac{\partial W}{\partial l_1} = -\frac{t(2\theta t - 5)\{\theta t(1 + l_0 - l_1) - 1\}}{(2\theta t - 3)^2} \gtreqless 0. \tag{22}$$

The intuition behind these results is as follows. From (21), to maximize the profit, the private firm tends to move toward the right edge of the linear city. From (22), to maximize the social welfare, the public firm has three actions that are dependent on the value of θt as a strategy; (a) moving toward the center of the linear city if $3/2 < \theta t < 5/2$, (b) not moving if $\theta t = 5/2$, (c) moving toward the right edge of the linear city if $\theta t > 5/2$. As a result, the two firms are located at the same location. This result implies the contradiction since we assume that $l_0 + l_1 < 1$.

From (i) and (ii), Proposition 3 is derived.

Proposition 3. *When the transportation costs of consumers are linear in mixed duopoly, there does not exist a SPNE in Hotelling's location model with quality choice.*

4. Concluding remarks

In this paper, we investigated a mixed duopoly market where a welfare-maximizing public firm competes against a profit-maximizing private firm by using Hotelling-type spatial competition model with linear transportation costs. We introduced the level of quality as vertical product differentiation into the model. We showed that there is not a subgame perfect Nash equilibrium (SPNE) of location choice in the three-stage game that is location-then-quality choice and subsequent price choice. Furthermore, we showed the conditions of the existence of the quality equilibrium.

We find that the result of Lu (2006) that examines only location-then-price choice under the mixed duopoly is robust, while our result is a sharp contrast to Economides (1989) that investigates the three-stage sequential game in the pure duopoly.

Appendix

Proof of Proposition 1.

Following Lu (2006), we prove this proposition. Suppose that φ is sufficient small positive number. When both firms are located at the same place, the public firm has no incentive to change its quality q_1 for any $q_0 = q_1 + \varphi$ since the total transportation costs are constant. The private firm has no incentive to change its quality q_0 as well. $\square$

Proof of Proposition 2.

First, we show the necessary condition. Since we assume that $q_0 > 0$, from (15),

$$q_0 = \frac{2\theta t(1 + l_0 - l_1) - 2}{\theta(2\theta t - 3)} > 0.$$

Since we assume that $\theta t > 3/2$, the numerator is positive. Hence, $\theta t(1 + l_0 - l_1) - 1 > 0$. Therefore,

$$l_0 - l_1 > \frac{1}{\theta t} - 1.$$

Similarly, from (16), we obtain

$$l_0 - l_1 < 1 - \frac{2}{\theta t}.$$

Next, we show the sufficient condition. Suppose that $l_0 - l_1 \leq \frac{1}{\theta t} - 1$. Then $\theta t(1 + l_0 - l_1) - 1 \leq 0$, we obtain

$$q_0 = \frac{2\theta t(1 + l_0 - l_1) - 2}{\theta(2\theta t - 3)} \leq 0.$$

This result contradicts the assumption $q_0 > 0$. Similarly, we can also show that the case of $l_0 - l_1 \geq 1 - \frac{2}{\theta t}$. $\square$

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