E C O N O M I C S   B U L L E T I N

On the Distribution of the Break-Date Estimator Implied by the Perron-Type Statistics When the Form of Break is Misspecified

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Abstract

Montañés, Olloqui, and Calvo (2005, Journal of Econometrics) argue that use of the Perron-type minimum t-statistics will lead the practitioner to incorrectly assess the time series properties of the variable under investigation when the form of break is misspecified. However, their simulations do not provide insight into the distribution of the estimated break-date implied by the unknown break-date Perron-type statistics when the form of break is misspecified. Using finite sample simulations, we show that the break-date implied by the Mixed model will tend to estimate the break-date consistently even when the form of break is misspecified. The practitioner should, therefore, use the Mixed model as the appropriate trend-break stationary alternative when testing for a unit root with an endogenous break-date.

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1. Introduction

It has been long recognized that conventional unit root tests, such as the Dickey and Fuller (1979) t-statistic and the normalized estimator, fail to reject the null hypothesis if the true data generating process evolves according to a trend-break stationary process. The behaviour of the Dickey-Fuller statistics under the trend-break stationary alternative was originally studied by Perron (1989).[1] According to Perron, visual inspection of several U.S. macroeconomic time series revealed a break in the trend component during the Great Crash of 1929 or the Oil Price Shock of 1973. Perron (1989) suggested three different characterizations of the break or `form of break' under the alternative, namely, (a) the Crash model that allows for a break in the intercept alone, (b) the Changing Growth model that allows for a break in the slope with the two segments joined at the break-date, and (c) the Mixed model that allows for a simultaneous break in the intercept and slope.[2] Perron (1989) devised unit root statistics that have power against the trend-break stationary alternative of choice when the location of break or break-date is assumed to be known a priori. In order to implement Perron's (1989) methodology, the practitioner estimates a regression that nests the unit root null and the alternative of choice. The unit root statistic is the t-statistic on the first lag of the dependent variable, denoted by $t_{DF}^{i}(T_b^c)$, where T_b^c is the correct break-date, and i=A, B, C corresponds to the Crash model, the Changing Growth model, and the Mixed model respectively. We note that the limiting null distribution of $t_{DF}^{i}(T_b^c)$ (i=A, B, C) is indexed by the location of break and the form of break.

An aspect of Perron's (1989) methodology that has drawn criticism pertains to the pre-specification of the break-date. As pointed out by Christiano (1992), the choice of the break-date is invariably correlated with the data and this `pretest examination of data' is not accounted for in Perron's (1989) testing procedure. As a consequence, the unit root statistics $t_{DF}^{i}(T_b^c)$ (i=A, B, C) will reject the null hypothesis far too often. Several studies have extended Perron's (1989) methodology to allow for an unknown break-date. See, for example, Perron and Vogelsang (1992), Banerjee, Lumsdaine and Stock (1992), Zivot and Andrews (1992), Perron (1997), and Vogelsang and Perron (1998). These studies suggest some variant of a minimum t-statistic. The minimum t-statistics are based on the sequence of t-statistics $(\{t_{DF}^{i}([\lambda T])\}_{\lambda\in\Lambda}$, i=A, B, C) obtained by implementing Perron's (1989) methodology for each possible break-date $[\lambda T]$ that corresponds to a break-fraction λ in a suitably defined choice set $\Lambda=[\lambda_0, 1-\lambda_0]\subset(0,1)$, where [.] is the smallest integer function. The minimum t-statistic is then constructed by choosing the t-statistic from $\{t_{DF}^{i}([\lambda T])\}_{\lambda\in\Lambda}$, based on some algorithm, that maximizes evidence against the unit root null. For example, one may use the minimum of the sequence of t-statistics, denoted by $t_{DF}^{\min}(i)$, i=A, B, C. In the eventuality that the unit root null

[1] Details on the asymptotic behaviour of the Dickey-Fuller statistics can be found in Perron (1989) and Montañés and Reyes (1998, 1999). Details on the asymptotic behaviour of the Dickey-Fuller statistics can be found in Perron (1989) and Montañés and Reyes (1998, 1999).

[2] Specifically, Perron (1989) examined the Nelson and Plosser (1982) macroeconomic series and U.S. Postwar Quarterly Real GNP. Perron (1989) found the Changing Growth model suitable for Quarterly U.S. Real GNP, the Mixed model suitable for Common Stock Prices and Real Wages series, and the Crash model suitable for the remaining Nelson-Plosser (1982) series.

is rejected in favour of the chosen trend-break stationary alternative, one can obtain an estimate of the break-date as $\hat{T}_b(t_{DF}^{\min}(i)) = \arg\min_{T_b} \; t_{DF}^i(T_b)$, for i=A, B, C.

Sen (2003) argues that when the break-date is assumed to be unknown, the practitioner should specify the form of break according to the most general Mixed model. Sen (2003) presents simulation evidence pertaining to the minimum t-statistics $t_{DF}^{\min}(i)$, i=A,B,C that correspond to the unknown break-date), and finds that: (a) the power of the Crash (Changing Growth) model statistics is low and can be close to zero if the break occurs according to the Changing Growth (Crash) model or the Mixed model; and (b) there is not much loss in power if the Mixed model is used, when in fact the break occurs according to either the Crash model or the Changing Growth model. Therefore, Sen (2003) suggests that the practitioner should use the Mixed model as the appropriate trend-break stationary alternative so as to guard against possible misspecification of the form of break.

In a recent paper, Montañés, Olloqui, and Calvo (2005) argue that use of the minimum t-statistics will lead the practitioner to incorrectly assess the time series properties of the variable under investigation when the form of break is misspecified. They derive the limiting behaviour of Perron's (1989) t-statistics for both the correct break-date and incorrect break-dates when the form of break is misspecified, see their Propositions 1 and 2. Using finite sample simulations, they assess the power of the Perron (1989) statistics ($t_{DF}^i(T_b)$, i=A, B, C that correspond to a known break-date) evaluated at the true break-date (T_b^c) and at several incorrect break-dates ($T_b \neq T_b^c$), see their Tables 1 and 2. The simulation evidence shows that: (a) if the correct break-date is used, the Crash (Changing Growth) model statistic suffers from severe power loss if the break occurs according to the Changing Growth (Crash) model, but the Mixed model statistic has high power; and (b) if the incorrect break-date is used, both the Crash (Changing Growth) model and the Mixed model statistics will suffer serious power loss when the break occurs according to the Changing Growth (Crash) model. While the analytical results of Montañés, Olloqui, and Calvo (2005) imply that the minimum t-statistics will yield an inconsistent break-date estimator, their simulations do not provide insight into distribution of the estimated break-date implied by the minimum t-statistics under model misspecification.

In this paper, we study the effect of misspecification in the form of break on the distribution of the estimated break-date implied by the minimum t-statistics using finite sample simulations. We also consider the maximum F-statistic of Murray and Zivot (1998). Our results show that: (a) the estimated break-date implied by the Crash (Changing Growth) model statistic fails to identify the correct location of break when the true data generating process evolves according to the Changing Growth (Crash) model or the Mixed model; and (b) the estimated break-date from the Mixed model identifies the true break-date fairly accurately, even when the break occurs according to either the Crash model or the Changing Growth model. The latter result implies that the use of the Mixed model will reveal valuable information by accurately identifying the correct break-date, and also guard against power distortions owing to misspecification in the form of break. Our results regarding the estimated break-date, therefore, complement the analysis of both Montañés, Olloqui, and Calvo (2005) and Sen (2003).

This paper is laid out as follows. In Section 2, we briefly describe the unit root null and the trend-break stationary alternative hypotheses, the minimum t-statistic statistics, and the maximum F-statistic. In Section 3, we present simulation evidence regarding the distribution of the estimated break-date implied by the unit root statistics under model misspecification. Some concluding comments appear in Section 4.

2. Tests for the Unit Root Null Hypothesis

In this section, we describe the data generating process under the null hypothesis and the trend-break stationary alternative, the minimum t-statistics proposed by Zivot and Andrews (1992), and the maximum F-statistic of Murray and Zivot (1998). Our discussion follows the analysis in Zivot and Andrews (1992). Consider the time series $\{y_t\}_{t=1}^{T}$ where T is the available sample size. In this paper, we consider the Innovation Outlier (IO) model in which the change in the trend function evolves in the same manner as any other shock, see section 4.2 in Perron (1989) for further details. The data generating process under the Crash model, the Changing Growth model, and the Mixed model are respectively given by:

Model (A): $$y_t = \mu_0 + \mu_2 t + \psi(L)[\mu_1 DU_t(T_b^c) + \nu_t] \quad (1)$$

Model (B): $$y_t = \mu_0 + \mu_2 t + \psi(L)[\mu_3 DT_t(T_b^c) + \nu_t] \quad (2)$$

Model (C): $$y_t = \mu_0 + \mu_2 t + \psi(L)[\mu_1 DU_t(T_b^c) + \mu_3 DT_t(T_b^e) + \nu_t] \quad (3)$$

where $\psi(L) = A(L)^{-1}B(L)$, $A(L)e_t = B(L)\nu_t$, and ν_t is a sequence of i.i.d. $(0,\sigma^2)$ random variables, A(L) and B(L) are polynomials in the lag operator of order p and q respectively with all roots outside the unit circle. T_b^c is the correct location of the break (or break-date), $DU_t(T_b^c) = 1(t > T_b^c)$ is an intercept break dummy, $1(t > T_b^c)$ is an indicator function that takes on the value 0 if $t \le T_b^c$ and 1 if $t > T_b^c$, and $DT_t(T_b^c) = (t - T_b^c)1(t > T_b^c)$ is a slope break dummy. For the asymptotic results, we assume that the break-date is a constant fraction of the sample size, that is, $T_b^c = [\lambda^c T]$ for some $\lambda^c \in (0,1)$ where [.] is the smallest integer function. Model (A) is referred to as the Crash model as it allows for a break in the intercept alone, Model (B) is referred to as the Changing Growth model since it allows for a break in the slope with the two segments joined at the break-date, and Model (C) is referred to as the Mixed model as it allows for a simultaneous break in the intercept and the slope of the trend function.

Under the null hypothesis, the data generating process contains a unit root, that is:

$$y_t = \mu + y_{t-1} + \psi(L)\nu_t \quad (4)$$

where $\psi(L) = A(L)^{-1}B(L)$, $A(L) = (1-\alpha)A^*(L)$, and $\alpha = 1$. In order to test the unit root null against the alternatives specified in (1)-(3) when the location of break is not known, the following methodology has been prescribed by Zivot and Andrews (1992). Specify the interval $\Lambda = [\lambda_0, 1-\lambda_0] \subset (0,1)$ that is believed to contain the true break-fraction. For each possible $\lambda \in \Lambda$, estimate the following regression that nests the null and the appropriate alternative:

$$y_t = \hat{\mu}_0^A + \hat{\mu}_1^A DU_t(T_b) + \hat{\mu}_2^A t + \hat{\alpha}^A y_{t-1} + \sum_{j=1}^{k} c_j^A \Delta y_{t-j} + \hat{e}_t^A \quad (5)$$

$$y_t = \hat{\mu}_0^B + \hat{\mu}_2^B t + \hat{\mu}_3^B DT_t(T_b) + \hat{\alpha}^B y_{t-1} + \sum_{j=1}^{k} c_j^B \Delta y_{t-j} + \hat{e}_t^B \quad (6)$$

$$y_t = \hat{\mu}_0^C + \hat{\mu}_1^C DU_t(T_b) + \hat{\mu}_2^C t + \hat{\mu}_3^C DT_t(T_b) + \hat{\alpha}^C y_{t-1} + \sum_{j=1}^{k} c_j^C \Delta y_{t-j} + \hat{e}_t^C \quad (7)$$

where [.] is the smallest integer function. The `k' regressors $\{\Delta y_{t-j}\}_{j=1}^{k}$ in (5)-(7) are included in the regression to account for additional correlation in the time series. In practice, the value of the lag-truncation parameter (k) is unknown, and so we use the data-dependent method of Perron and Vogelsang (1992) for choosing the appropriate value of k is used, see discussion below. Based on the estimated regressions (5)-(7) for the break-dates $\{[\lambda_0 T], [\lambda_0 T]+1, ..., T-[\lambda_0 T]\}$, we calculate the sequence of t-statistics for $H_0 : \alpha = 1$, denoted by $\{t_{DF}^i(T_b)\}_{T_b=[\lambda_0 T]}^{T-[\lambda_0 T]}$ (i=A, B, C). This sequence of t-statistics can be used to obtain numerous minimum t-statistics by specifying a suitable algorithm to choose an appropriate break-date. We consider the algorithm proposed by Perron and Vogelsang (1992) and Zivot and Andrews (1992). The statistic is obtained by choosing the break-date that maximizes evidence against the unit root null, that is:

$$t_{DF}^{\min}(i) = Min_{T_b \in \{[\lambda_0 T], [\lambda_0 T]+1, ..., T-[\lambda_0 T]\}} \; t_{DF}^i(T_b) \quad (8)$$

for i=A, B, C. In the eventuality that the unit root null is rejected in favour of the chosen trend-break stationary alternative, one can obtain an estimate of the break-date as $\hat{T}_b(t_{DF}^{\min}(i)) = \arg\min_{T_b} \; t_{DF}^i(T_b)$, for i=A, B, C.

For the Mixed model, we also consider a version of the supWald statistic proposed by Murray and Zivot (1998) for the joint null hypothesis of a unit root and no break in the intercept and slope of the trend function, that is, $H_0^J : \alpha = 1, \mu_1 = 0, \mu_3 = 0$. We consider the maximum F-statistic characterization of the supWald statistic described in Sen (2003). In order to calculate the maximum F-statistic ($F_T^{\max}$), we estimate regression (7) for all possible break-dates $T_b \in \{[\lambda_0 T], [\lambda_0 T]+1, ..., T-[\lambda_0 T]\}$, and calculate the Wald statistic for H_0^J. Using the sequence $\{F_T(T_b)\}_{T_b=[\lambda_0 T]}^{T-[\lambda_0 T]}$, the maximum F-statistic is defined as:

$$F_T^{\max} = Max_{T_b \in \{[\lambda_0 T], [\lambda_0 T]+1, ..., T-[\lambda_0 T]\}} \; F_T(T_b) \quad (9)$$

If the unit root null is rejected, we can estimate the break-date as $\hat{T}_b(F_T^{\max}) = \arg\max_{T_b} \; F_T(T_b)$. The asymptotic distribution of $F_T^{\max}$ can be obtained easily using the results in Murray and Zivot (1998). Murray and Zivot (1998) present the asymptotic critical values for $F_T^{\max}$ without any trimming of the sample. The asymptotic and finite sample critical values for $F_T^{\max}$ for λ ={0.15, 0.10, 0.05} are reported in Table 1 in Sen (2003).

3. Estimated Break-Date When the Form of Break is Misspecified

In this section, we consider the effect of misspecification in the form of break on the estimated break-date implied by the unit root statistics $t_{DF}^{\min}(i)$ for i=A, B, C, and $F_T^{\max}$. We generate data according to the following simulation design:

$$y_t = \mu_1 DU_t^c + \mu_3 DT_t^c + \alpha y_{t-1} + e_t \qquad (10)$$

where $y_0 = 0$, e_t are i.i.d. N(0,1), $DU_t^c = 1(t > T_b^c)$, $DT_t^c = (t - T_b^c)1(t > T_b^c)$, T={50, 100}, $T_b^c = [\lambda^c T]$ implied by λ^c = {0.25, 0.5, 0.75}, $\alpha = \{0.8, 0.9\}$, μ_1={0, 1, 2, 3, 4, -1, -2, -3, -4}, and μ_3={0, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3}. The break occurs according to the Crash model when $\mu_1 \neq 0$ and $\mu_3 = 0$, according to the Changing Growth model when $\mu_1 = 0$ and $\mu_3 \neq 0$, and according to the Mixed model when $\mu_1 \neq 0$ and $\mu_3 \neq 0$. We consider all parameter combination that result from the specified values of μ_1 and μ_3. We use 10,000 replications for each parameter combination. We estimate regressions (5)-(7), and calculate $t_{DF}^{\min}(i)$ $ (i=A, B, C) and $F_T^{\max}$ with λ_0=0.15. For each statistic, we recorded the estimated break-dates, denoted by $\hat{T}_b(t_{DF}^{\min}(i))$ for i=A, B, C, and $\hat{T}_b(F_T^{\max})$. We used Perron and Vogelsang's (1992) method to determine the lag truncation parameter.

In what follows, we discuss the results regarding the distribution of the estimated break-dates implied by the unit root statistics $\hat{T}_b(t_{DF}^{\min}(i))$, i=A, B, C and $\hat{T}_b(F_T^{\max})$ when the form of break under the alternative is misspecified. In order to save space, we only report the results for the parameter combinations corresponding to T=100, $\tau^c = 0.5$, $\alpha = 0.8$, $\mu_1 \leq 0$, and $\mu_3 \geq 0$. However, the main conclusions discussed below are representative of the results based on all parameter combinations considered in our simulations.[3]

Figures 1-8 show the distribution of the estimated break-dates $\hat{T}_b(t_{DF}^{\min}(i))$ for i=A, B, C, and $\hat{T}_b(F_T^{\max})$ when the break evolves according to the Crash model with $\mu_1 < 0$ and $\mu_3 = 0$. The Crash model statistic $t_{DF}^{\min}(A)$ estimates the true break-date most accurately (Figures 1 and 5). The distribution of $\hat{T}_b(t_{DF}^{\min}(A))$ converges to the true break-date as the intercept-break magnitude increases. The Changing Growth model statistic $t_{DF}^{\min}(B)$ fails to identify the true break-date in most cases (Figures 2, 6). The distribution of the estimated break-date $\hat{T}_b(t_{DF}^{\min}(B))$ diverges away from the true break-date as the intercept break magnitude increases. However, the Mixed model statistics $t_{DF}^{\min}(C)$ and $F_T^{\max}$ identify the break-date accurately (Figures 3 and 7, and Figures 4 and 8 respectively), and the distribution of the estimated break-dates $\hat{T}_b(t_{DF}^{\min}(C))$ and $\hat{T}_b(F_T^{\max})$ converge to the true break-date as the intercept-break magnitude increases.

[3] A copy of the results for the distribution of the estimated break-dates corresponding to all parameter combinations is available from the author upon request.

In Figures 9-20, we plot the distribution of the estimated break-dates $\hat{T}_b(t_{DF}^{\min}(i))$ for i=A, B, C, and $\hat{T}_b(F_T^{\max})$ when the break evolves according to the Changing Growth model with $\mu_1 = 0$ and $\mu_3 > 0$. In this case, we find that the Changing Growth model statistic $t_{DF}^{\min}(B)$ estimates the break-date most accurately (Figures 10, 14, and 18), but the Crash model statistic $t_{DF}^{\min}(A)$ fails to identify the true break-date in most cases (Figures 9, 13, and 17). The Mixed model statistics $t_{DF}^{\min}(C)$ and $F_T^{\max}$, however, do a reasonably good job in identifying the true break-date (Figures 11, 15, and 19, and Figures 12, 16, and 20 respectively). As the slope-break magnitude increases, the accuracy with which the Changing Growth model and Mixed model statistics identify the correct break-date increases, but the estimated break-dates implied by the Crash model statistics diverges away from the true break-date.

Figures 21-32 show the distribution of the estimated break-dates $\hat{T}_b(t_{DF}^{\min}(i))$ for i=A, B, C, and $\hat{T}_b(F_T^{\max})$ when the break evolves according to the Mixed model with $\mu_1 < 0$ and $\mu_3 > 0$. In each case, we find that the Mixed model statistics identify the correct break-date most accurately, and the accuracy with which $\hat{T}_b(t_{DF}^{\min}(C))$ and $\hat{T}_b(F_T^{\max})$ estimate the true break-date increases with the size of both the intercept-break and slope-break.

While the distribution of the estimated break-date implied by the Crash model statistic ($\hat{T}_b(t_{DF}^{\min}(A))$) converges towards the true break-date as the intercept-break magnitude increases, it diverges away from the true break-date as the slope-break magnitude increases. For example, when the intercept-break magnitude is μ_1= -1 and the slope-break magnitude increases from μ_3= 0.1 to 0.3 (Figures 21 and 29), the distribution of $\hat{T}_b(t_{DF}^{\min}(A))$ diverges away from the true break-date. The distribution of the estimated break-date implied by $t_{DF}^{\min}(B)$ with a fixed intercept-break magnitude converges toward the true break-date as the slope-break magnitude increases. For example, with μ_1= -1, the distribution of $\hat{T}_b(t_{DF}^{\min}(B))$ gets closer to the middle of the sample as the slope-break magnitude increases from 0.1 to 0.3 (Figures 22 and 30). As expected, this convergence is slower for large μ_1. For a fixed slope-break magnitude, the distribution of the estimated break-date $\hat{T}_b(t_{DF}^{\min}(B))$ diverges away from the true break-date as the intercept-break magnitude increases (for example, Figures 22 and 26 show the distribution of $\hat{T}_b(t_{DF}^{\min}(B))$ when μ_3= 0.1 and μ_1 increases from -1 to -2). It is interesting, however, to note that $\hat{T}_b(t_{DF}^{\min}(B))$ tends to be in first half of the sample when the intercept-break and the slope-break have the same sign, but $\hat{T}_b(t_{DF}^{\min}(B))$ tends to be in second half of the sample when the intercept-break and the slope-break have opposite signs.

The results pertaining to the distribution of the estimated break-dates from the Crash and Changing Growth models ($\hat{T}_b(t_{DF}^{\min}(i))$, i=A, B) illustrate how misspecification in the form of break may lead to erroneous identification of the break-date. This result is consistent with the findings of Montañés, Olloqui, and Calvo (2005). However, the simulation evidence of Montañés, Olloqui, and Calvo (2005) does not clearly show that the estimated break-date implied by the Mixed model statistic ($\hat{T}_b(t_{DF}^{\min}(C))$) identifies the true break-date in most cases. Therefore, the practitioner should use the Mixed model statistics when the form of break is unknown.

4. Conclusion

In this paper, we consider a methodological issue concerning unit root tests designed to have power against the trend-break stationary alternative. Montañés, Olloqui, and Calvo (2005) show that the Perron-type statistics will yield an inconsistent break-date estimator when the form of break is misspecified. However, their simulations do not provide insight into distribution of the estimated break-date implied by the minimum t-statistics under model misspecification. Using finite sample simulations, we draw two main conclusions regarding the distribution of the estimated break-date implied by the minimum t-statistics. First, the Crash (Changing Growth) model statistics fail to identify the true break-date when the break evolves according to the Changing Growth (Crash) or the Mixed model. Second, the Mixed model statistics identify the true break-date accurately when the form of break occurs according to the Crash or Changing Growth model. Therefore, our results provide further justification for using the Mixed model as the appropriate trend-break stationary alternative when the form of break in unknown.

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Figure 1: Distribution of Break–Dates implied by $t_{DF}^{min}(A)$
$\mu_1=-1,\ \mu_3=0.0,\ \alpha=0.80,\ T=100,\ \lambda^c=0.50$

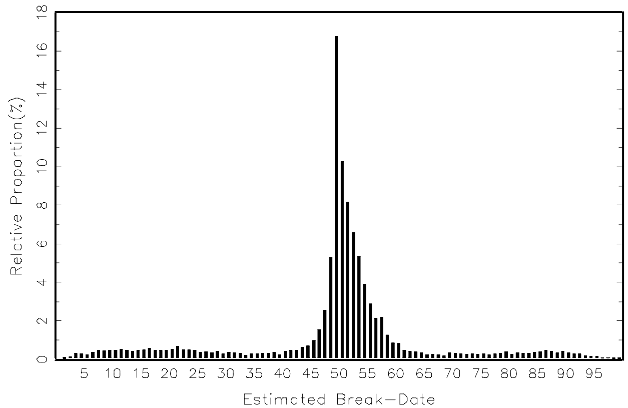


Figure 2: Distribution of Break–Dates implied by $t_{DF}^{min}(B)$
$\mu_1=-1,\ \mu_3=0.0,\ \alpha=0.80,\ T=100,\ \lambda^c=0.50$

Figure 3: Distribution of Break–Dates implied by $t_{DF}^{min}(C)$
$\mu_1=-1,\ \mu_3=0.0,\ \alpha=0.80,\ T=100,\ \lambda^c=0.50$

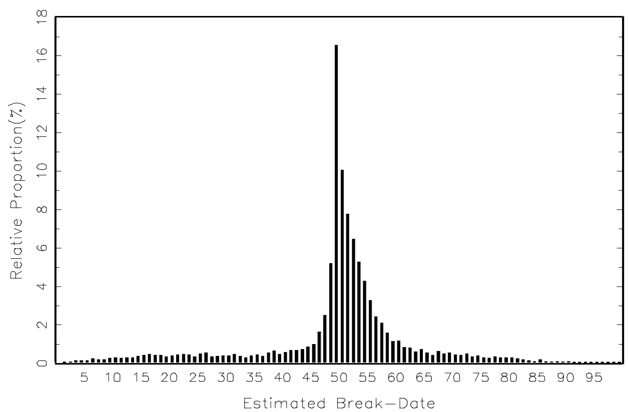


Figure 4: Distribution of Break–Dates implied by F_T^{max}
$\mu_1=-1,\ \mu_3=0.0,\ \alpha=0.80,\ T=100,\ \lambda^c=0.50$

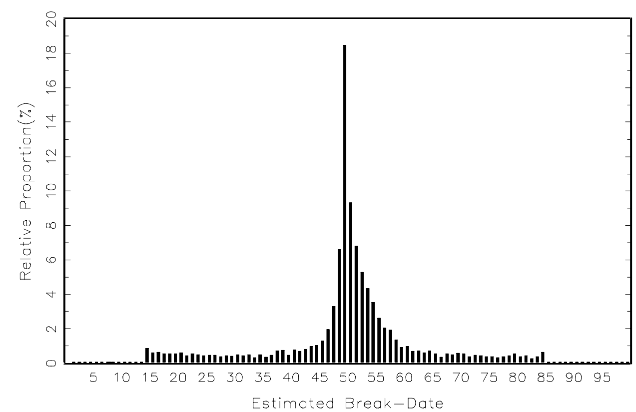


Figure 5: Distribution of Break-Dates implied by $t_{DF}^{min}(A)$
$\mu_1=-2$, $\mu_3=0.0$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

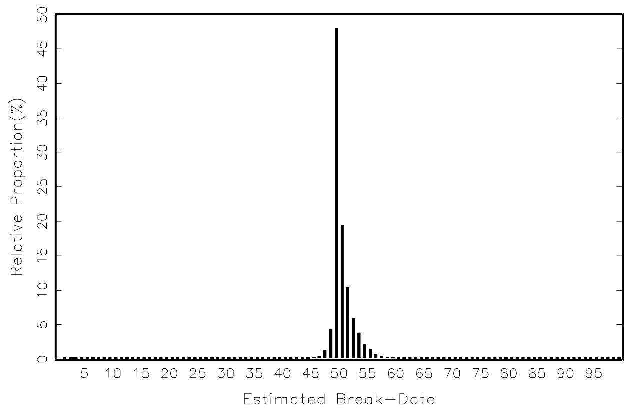


Figure 6: Distribution of Break-Dates implied by $t_{DF}^{min}(B)$
$\mu_1=-2$, $\mu_3=0.0$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

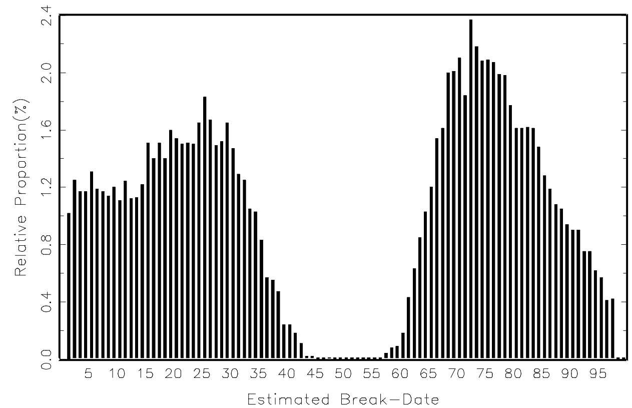


Figure 7: Distribution of Break-Dates implied by $t_{DF}^{min}(C)$
$\mu_1=-2$, $\mu_3=0.0$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

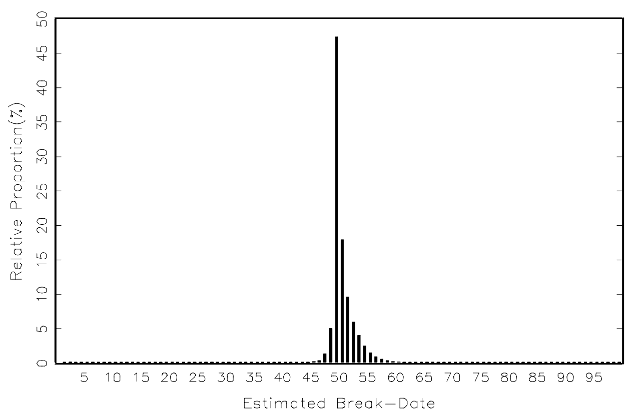


Figure 8: Distribution of Break-Dates implied by F_T^{max}
$\mu_1=-2$, $\mu_3=0.0$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

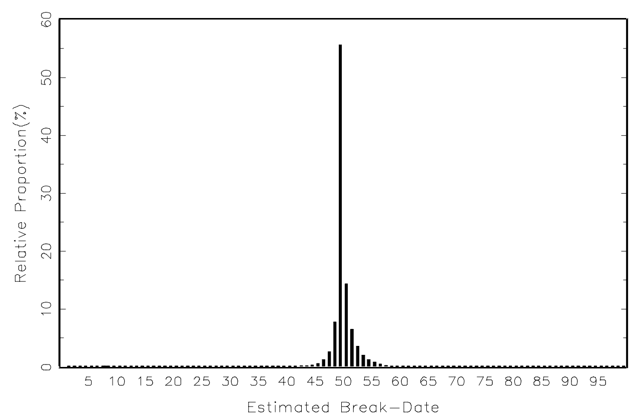


Figure 9: Distribution of Break–Dates implied by $t_{DF}^{min}(A)$
$\mu_1=0$, $\mu_3=0.1$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

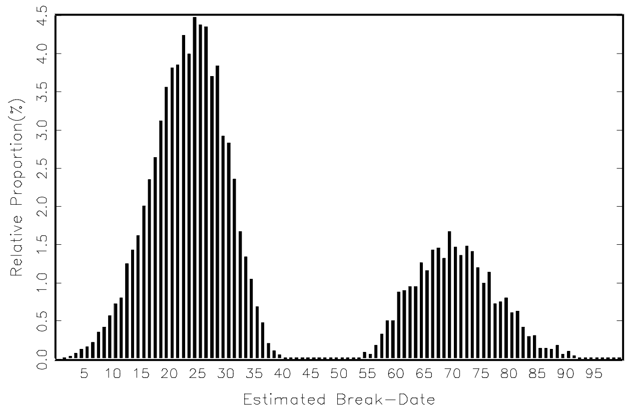


Figure 10: Distribution of Break–Dates implied by $t_{DF}^{min}(B)$
$\mu_1=0$, $\mu_3=0.1$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

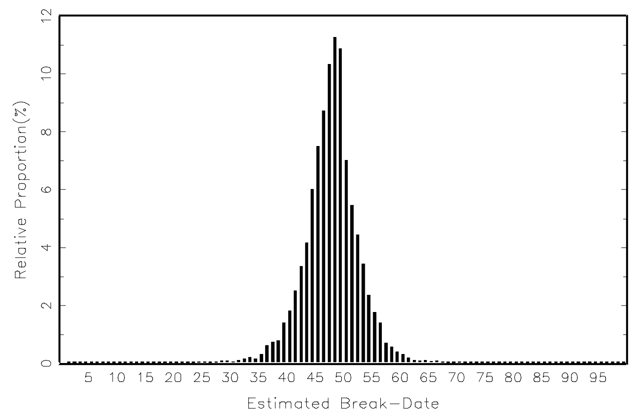


Figure 11: Distribution of Break–Dates implied by $t_{DF}^{min}(C)$
$\mu_1=0$, $\mu_3=0.1$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

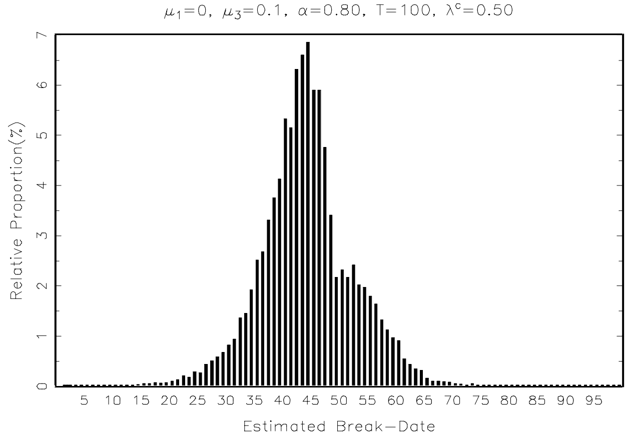


Figure 12: Distribution of Break–Dates implied by F_T^{max}
$\mu_1=0$, $\mu_3=0.1$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

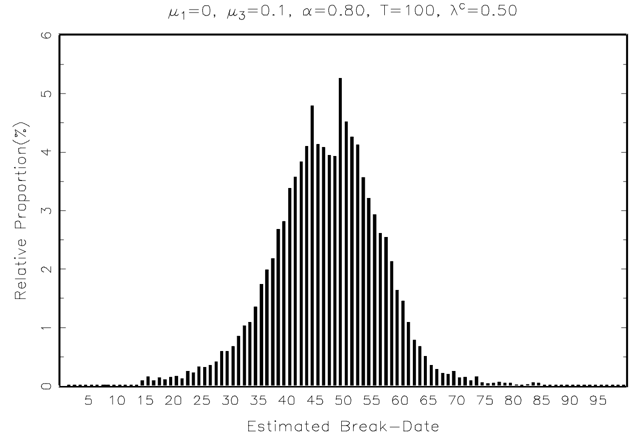


Figure 13: Distribution of Break–Dates implied by $t_{DF}^{min}(A)$
$\mu_1=0$, $\mu_3=0.2$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

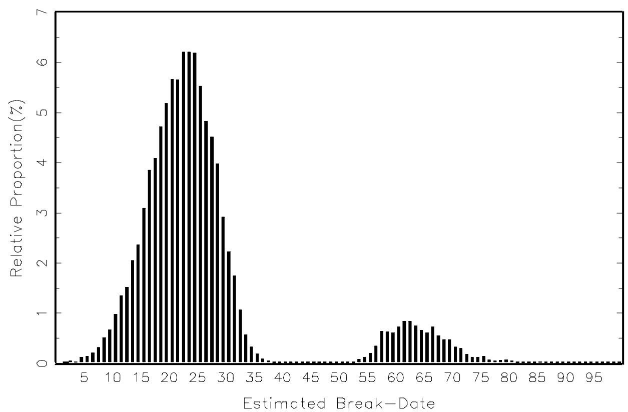


Figure 14: Distribution of Break–Dates implied by $t_{DF}^{min}(B)$
$\mu_1=0$, $\mu_3=0.2$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

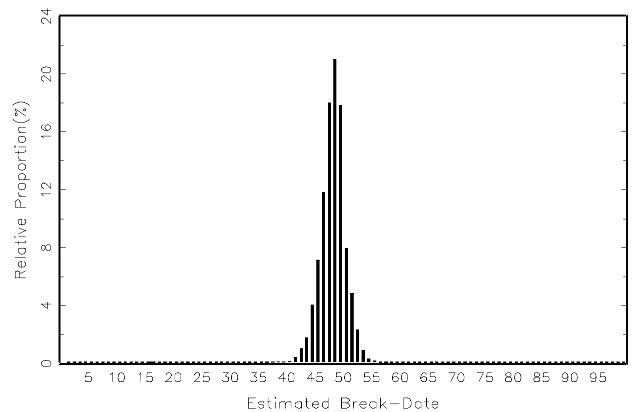


Figure 15: Distribution of Break–Dates implied by $t_{DF}^{min}(C)$
$\mu_1=0$, $\mu_3=0.2$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

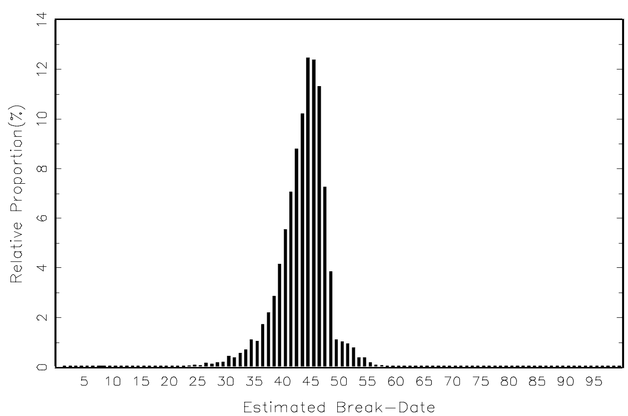


Figure 16: Distribution of Break–Dates implied by F_T^{max}
$\mu_1=0$, $\mu_3=0.2$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

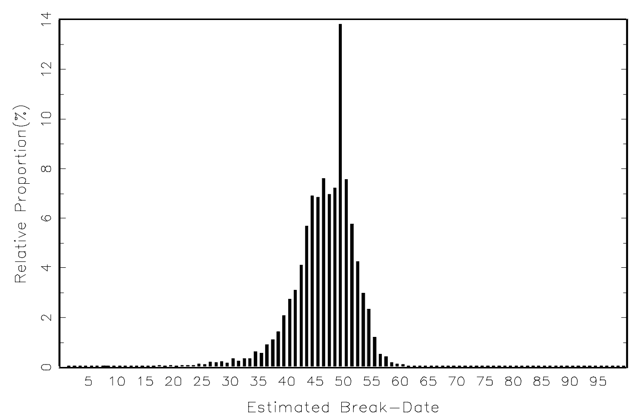


Figure 17: Distribution of Break−Dates implied by $t_{DF}^{min}(A)$
$\mu_1=0,\ \mu_3=0.3,\ \alpha=0.80,\ T=100,\ \lambda^c=0.50$

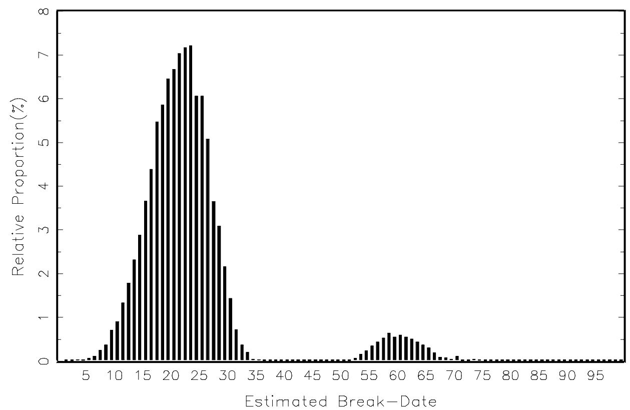


Figure 18: Distribution of Break−Dates implied by $t_{DF}^{min}(B)$
$\mu_1=0,\ \mu_3=0.3,\ \alpha=0.80,\ T=100,\ \lambda^c=0.50$

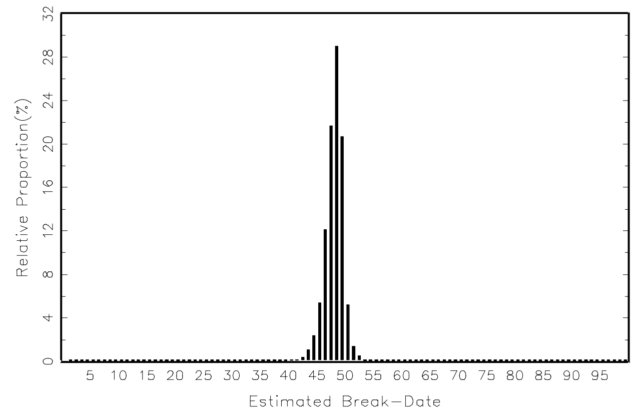


Figure 19: Distribution of Break−Dates implied by $t_{DF}^{min}(C)$
$\mu_1=0,\ \mu_3=0.3,\ \alpha=0.80,\ T=100,\ \lambda^c=0.50$

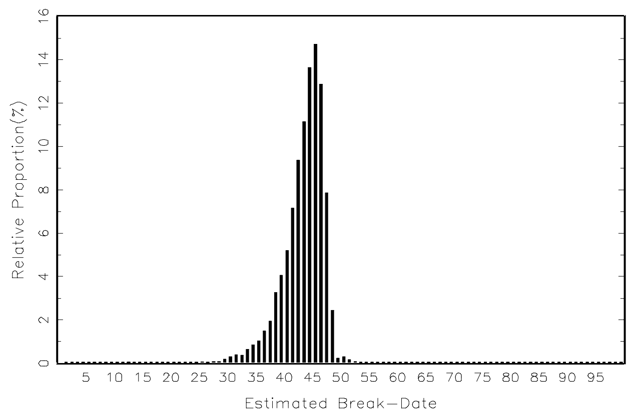
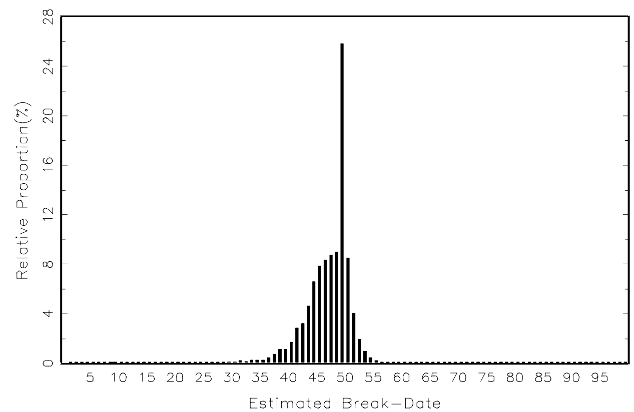


Figure 20: Distribution of Break−Dates implied by F_T^{max}
$\mu_1=0,\ \mu_3=0.3,\ \alpha=0.80,\ T=100,\ \lambda^c=0.50$



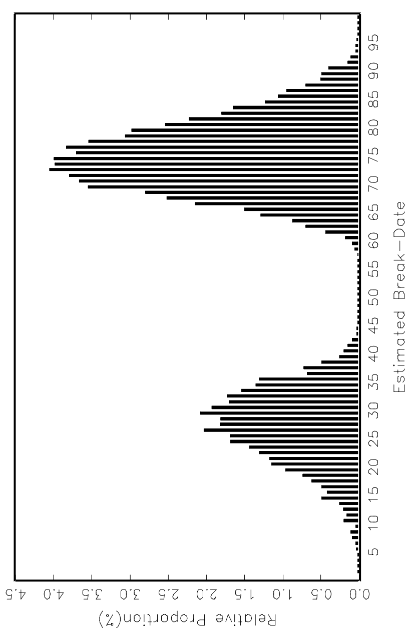


Figure 21: Distribution of Break-Dates implied by $t_{DF}^{min}(A)$
$\mu_1=-1$, $\mu_3=0.1$, $\alpha=0.80$, T=100, $\lambda^c=0.50$

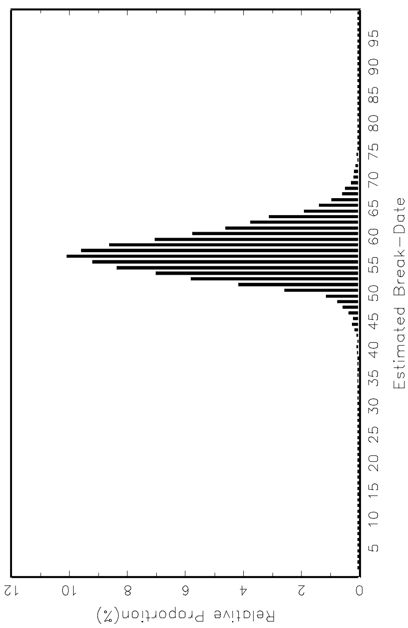


Figure 22: Distribution of Break-Dates implied by $t_{DF}^{min}(B)$
$\mu_1=-1$, $\mu_3=0.1$, $\alpha=0.80$, T=100, $\lambda^c=0.50$

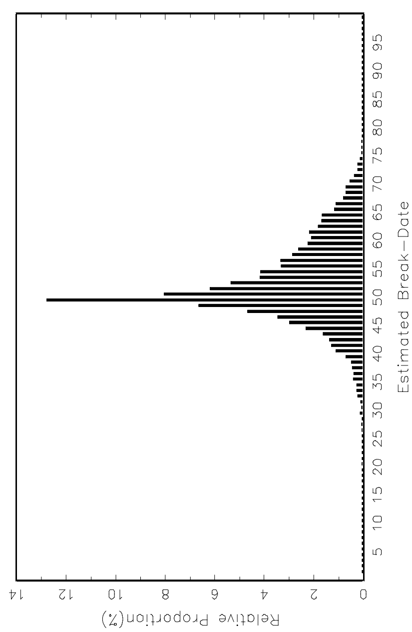


Figure 23: Distribution of Break-Dates implied by $t_{DF}^{min}(C)$
$\mu_1=-1$, $\mu_3=0.1$, $\alpha=0.80$, T=100, $\lambda^c=0.50$

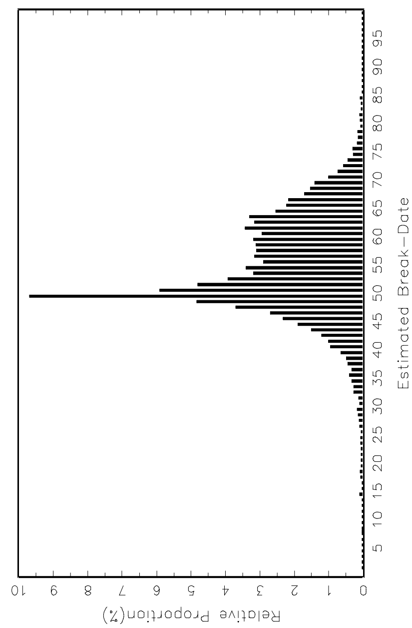


Figure 24: Distribution of Break-Dates implied by F_T^{max}
$\mu_1=-1$, $\mu_3=0.1$, $\alpha=0.80$, T=100, $\lambda^c=0.50$

Figure 25: Distribution of Break–Dates implied by $t_{DF}^{min}(A)$
$\mu_1=-2$, $\mu_3=0.1$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

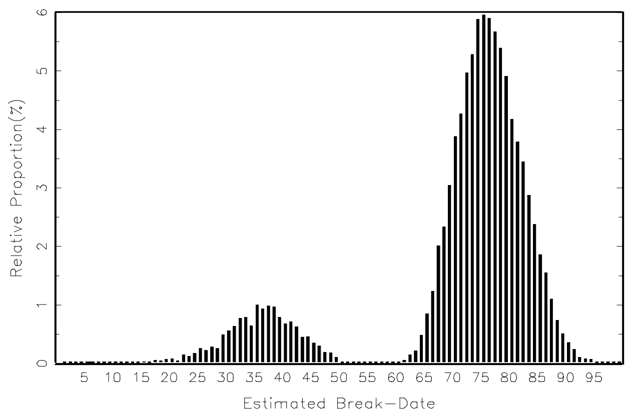


Figure 26: Distribution of Break–Dates implied by $t_{DF}^{min}(B)$
$\mu_1=-2$, $\mu_3=0.1$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

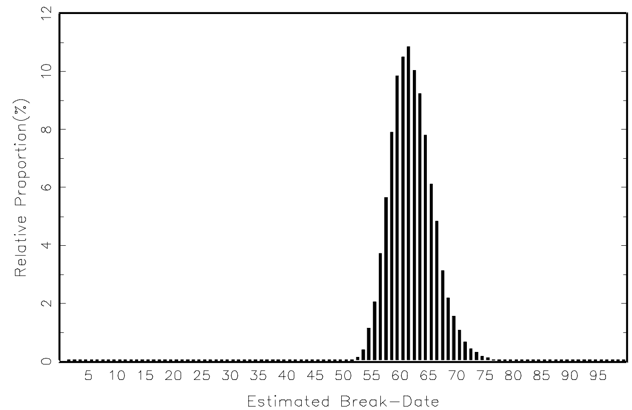


Figure 27: Distribution of Break–Dates implied by $t_{DF}^{min}(C)$
$\mu_1=-2$, $\mu_3=0.1$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

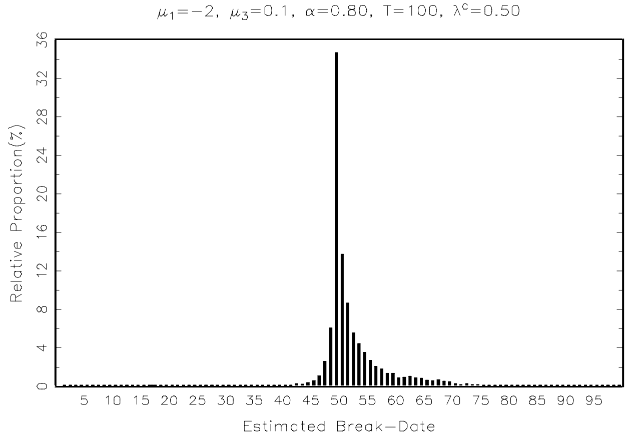


Figure 28: Distribution of Break–Dates implied by F_T^{max}
$\mu_1=-2$, $\mu_3=0.1$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

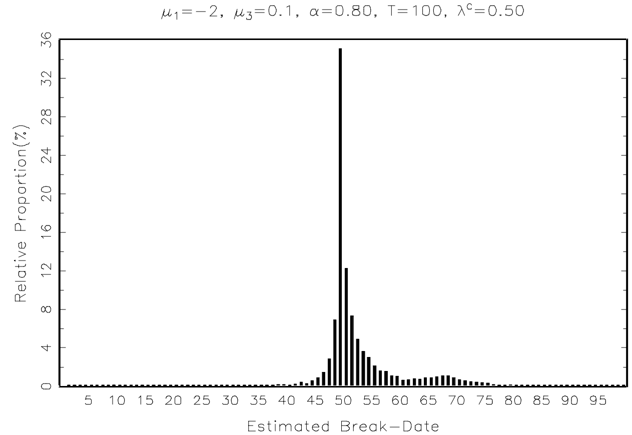


Figure 29: Distribution of Break−Dates implied by $t_{DF}^{min}(A)$
$\mu_1=-1$, $\mu_3=0.3$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

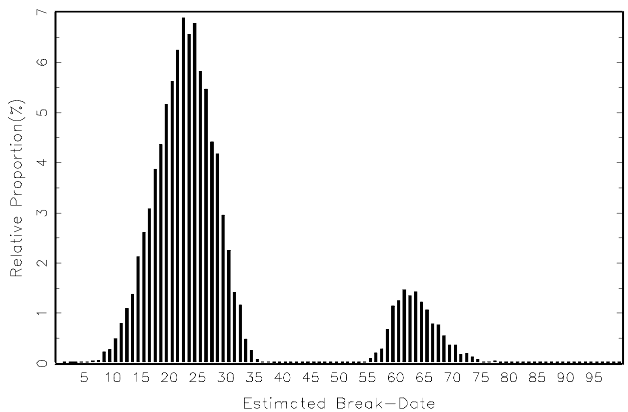


Figure 30: Distribution of Break−Dates implied by $t_{DF}^{min}(B)$
$\mu_1=-1$, $\mu_3=0.3$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

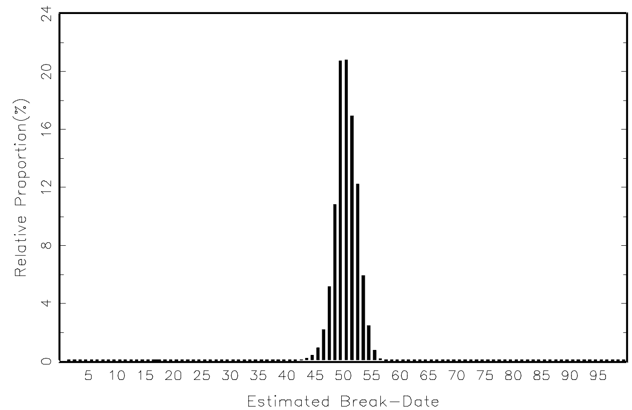


Figure 31: Distribution of Break−Dates implied by $t_{DF}^{min}(C)$
$\mu_1=-1$, $\mu_3=0.3$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$

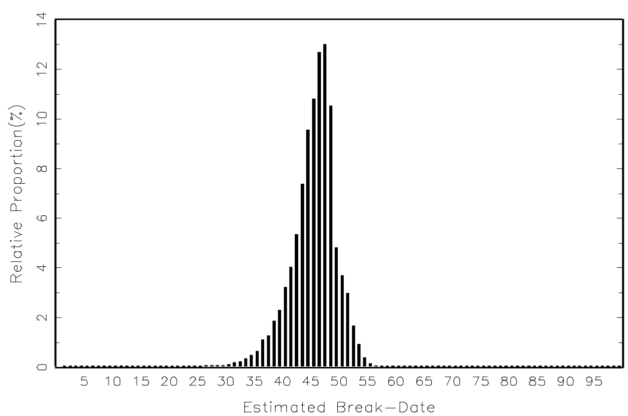


Figure 32: Distribution of Break−Dates implied by F_T^{max}
$\mu_1=-1$, $\mu_3=0.3$, $\alpha=0.80$, $T=100$, $\lambda^c=0.50$