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A note on fractional stochastic convergence

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Abstract

We show that a class of non-stationary stochastic processes exhibiting long-range dependence satisfies one definition of time series convergence proposed in the literature. We also show explicitly the relationship between two time series concepts convergence proposed in the literature. Furthermore, we assess income per capita convergence for a sample OECD of economies using time series based tests. When we allow income shocks to exhibit long-range dependence, generalizing previous specifications, we find ample evidence of pairwise convergence among OECD economies. This finding is contrary to the literature that uses unit roots and cointegration tests.

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1. Introduction

Bernard and Durlauf (1995, 1996), henceforth BD, proposed time series tests to assess income convergence based on the ideas of unit roots and cointegration. Their proposed tests impose restrictions on the stochastic process for income per capita differentials. In particular, two economies converge in a time series sense if their income per capita process cointegrates, and the cointegrating vector has no intercept and a unitary slope. BD (1995) apply these time series test of convergence to a sample of 15 OECD economies using data for the period 1900-1987. They find no evidence of income per capita convergence.

BD's finding of no convergence is in sharp contrast with the cross-section literature that indicates strong evidence of convergence among OECD economies; see Baumol (1986), among others. Moreover, BD´s finding is economically unintuitive, after all rich OECD economies are rich and, in some sense, their income levels already converged. De Long (1988) made this point long ago in his reply to Baumol's (1986) seminal article on convergence. Furthermore, BD´s finding can be misleading, especially for the growth economist worried about policy making. It is important to know if economies are converging or not; this information could be helpful to the policy maker in determining, for instance, the amount of financial aid that should be sent to a slow growing economy.

In our view, BD's finding of no convergence for the OECD sample is a statistical artifact due to misspecification of their convergence tests. We argue that the presence of long-range dependence[1] in income per capita differentials, which translates into highly persistent income shocks, might have led to the rejection of the convergence hypothesis based on standard unit roots and cointegration tests, such as the ones used by BD. The specifics of our argument are as follows. First, the empirical growth literature suggests that the speed of economic convergence is quite low. In a time series sense, this means that income shocks are highly persistent but eventually die out. To illustrate the point, assume that the speed of convergence is 2% per year, as suggested by the cross-section literature, then it would take 34.7 years for an economy to transit half way to its steady-state equilibrium. Assuming that income differentials can be described by an AR(1) process, $y_t = \rho y_{t-1} + \varepsilon_t$, a half-life[2] of 34.7 years is associated with a stationary AR(1) process with $\rho = 0.98$, that is, a near-unit root process, where shocks are persistent but eventually die out. Even with a speed of convergence of 5%, the implied autoregressive coefficient is $\rho = 0.95$, again a near-unit root process. Second, if income per capita differentials processes have autoregressive coefficients so close to unity, given the well-known low power problem of unit roots tests and the fact that unit root tests have the no-convergence as the null hypothesis, it is not surprising that BD could not reject the presence of a unit root in the data and, consequently, reject the convergence hypothesis.

Research by Michellacci and Zaffaroni (2000) reinforces our suspicions that the tests used by BD are misspecified. They provide evidence suggesting that the *level* of GDP per capita for the period 1885-1994 for OECD countries can be well represented by a long range dependence process. If indeed output per capita can be described by a long range dependence process then cointegration tests of income convergence like the ones in BD (1995) are misspecified since variables can only be cointegrated if they have the same order of integration.

[1] In section 3, we provide a brief discussion of long range dependence processes.

[2] For an AR(1) process the half-life is computed as $\ln(1/2)/\ln(\rho)$.

We model long range dependence in income per capita *differentials* as an ARFIMA (Autoregressive-Fractionally-Integrated-Moving-Average) process[3]. For an ARFIMA process the parameter of integration, d, can take on non-integer values. The ARFIMA process is part of a class of long range dependence processes, also known as long memory processes, which are characterized by having slowly decaying covariance function so that observations widely separated over time can exhibit strong dependence. For certain values of the integration parameter an ARFIMA process can exhibit non-stationarity but mean-reverting behavior. For our purposes, this is the key property of ARFIMA processes, because it captures the slow speed of income convergence observed in the data[4].

Given the persistence of income shocks, using fractional integration to model income differentials seems to be a more flexible modeling strategy than traditional ARMA processes. Unit roots and cointegration tests can only distinguish between an I(1) process (zero speed of convergence) and an I(0) process (exponential speed of convergence). Therefore, given the well known problem of low power of unit root tests[5], they might not be able to distinguish between I(1) and I(d) processes, especially when the autoregressive parameter is close to one, which seems to be the case for income per capita differential processes. The flexibility provided by the fractionally integrated series can overcome the knife-edge behavior of unit root tests and, therefore, provide a more accurate picture of the convergence dynamics.

In order to assess our conjecture, we use BD's data set and apply their time series test criteria to assess income convergence. However, as discussed above, we use a more general specification of the data generating process, which nests BD´s specification. In particular, we model income per capita *differentials* as an ARFIMA process to capture the observed slow speed of income convergence. Our estimates suggest that there is ample evidence of mean reversion in income per capita differentials processes, and, as shown in this article, mean reversion in income differentials satisfies one of the time series convergence criterion proposed by BD.

This article is divided as follows. In section 2, we discuss the concepts of time series convergence, and present a proposition showing how these concepts are related. In section 3, we briefly review the properties of long range dependence processes, and show its relationship with one of the criterion of time series convergence. In section 4, new evidence is presented on time series convergence by estimating the fractional integration parameter for the OECD sample used by BD (1995). Section 5 concludes.

2. Tests of the Convergence Hypothesis

The definition below follows BD (1991).

Definition 1: Stochastic Convergence in per capita income. The logarithm of income per capita for economies *i* and *j*, denoted by $Y_{i,t}$ and $Y_{j,t}$, respectively, is said to converge in a time series sense if their difference is a stationary stochastic process with zero mean and constant variance. That is, if $Y_{i,t} - Y_{j,t} = \varepsilon_{ij,t} \approx I(0)$, where $\varepsilon_{ij,t} \sim (0, \sigma_{\varepsilon}^2)$, then economies *i* and *j* converge in a time series sense.

[3] In section 3 we provide a brief description of ARFIMA processes.

[4] It is important to emphasize that we are interested in modeling income per capita *differentials*, and not the *level* of income per capita, as Michellacci and Zaffaroni (2000).

[5] Recall that the null hypothesis in unit root tests is that the series is I(1), that is, the null is of no convergence. On the power properties of unit root tests, see Campbell and Perron (1991), and Diebold and Rudebusch (1991).

The above definition implies that pairwise time series convergence is equivalent to cointegration between two economies' income per capita when the cointegrating vector has no intercept and a unitary slope coefficient. BD (1991) defines pairwise convergence by first requiring cointegration, and then restricting the cointegrating vector to have a zero intercept and a unitary slope coefficient. Definition 1 directly imposes the restriction on the cointegrating vector. This seems to be a superior strategy because it avoids the possible shortcomings of the cointegration approach. First, even when a pair is cointegrated, one cannot be sure that the cointegrating vector estimate satisfies the economic hypothesis. Second, the cointegrating vector may suffer from finite sample bias, and, even worse, the tests may have size distortions, thus leading to spurious inference. Definition 1 implies that we can test for pairwise convergence by performing unit root tests on the income per capita differential process between two economies.

Definition 1 can be considered too strict, in the sense it only verifies if countries' income per capita already converged[6]. It is not able to capture any transitional dynamics. Identical countries with the same steady-state level of income per capita, but at different points in their transition path, might not pass the convergence criterion in definition 1. This problem might be amplified given that data in growth studies spans a relatively short time horizon. If indeed this is the case, definition 1 may fail to identify converging economies. A weaker time series convergence criterion is proposed by BD (1996), and is reproduced below as definition 2.

Definition 2: Convergence as equality of long-run forecasts at a fixed time. The logarithm of income per capita for countries *i* and *j*, denoted by $Y_{i,t}$ and $Y_{j,t}$, respectively, is said to converge in time series sense if their long-run forecast of the log of income per capita for both countries are equal at a fixed time *t*. This condition can be written as $\lim_{k\to\infty} E(Y_{i,t+k} - Y_{j,t+k} | \Im_t) = 0$, where $\Im_t$ is the information set at time t.

It is relatively straightforward to show the relationship between the two time series criteria above. Nevertheless, we present it, under general conditions, in the form of a proposition.

Proposition 1: If the logarithm of income per capita for economies *i* and *j* are stationary, ergodic, and satisfy definition 1, then they also satisfy definition 2.

Proof: By Wold's decomposition theorem any covariance stationary stochastic process x_t has a moving average representation given by $x_t = \sum_{j=0}^{\infty} d_j \varepsilon_{t-j}$, where $\{\varepsilon_t\}$ is the sequence consisting of one-step-ahead linear least square forecasting innovations, that is, $\varepsilon_t = x_t - P[x_t | x_{t-1}, x_{t-2}, ...]$, where P[.] denotes the linear least squares projection operator. Let $x_t = \Delta Y_{ij,t}$, that is, x_t (suppressing the i and j subscripts) is the difference of the logarithm of the income per capita

[6] BD (1995) are aware of that. For instance, on p. 100, they write "One potential difficulty with the use of unit root tests to identify convergence is the presence of a transitional component in the aggregate output of various countries…. If the countries in our sample start at different initial conditions and are converging to, but are not yet at a steady-state output distribution, then the available data may be generated by a transitional law of motion rather than by an invariant stochastic process. Consequently, unit root tests may erroneously accept a no-convergence null." They go on and write that: "Simulations using data from a calibrated Solow growth model suggest that the size distortions are unlikely to be significant for the time span we consider (Bernard and Durlauf, 1992)". Unfortunately, the article cited in this last quote is not listed in BD's (1995) references.

for economies *i* and *j*. By ergodicity, the coefficients on the $MA(\infty)$ are absolutely summable (see Hamilton 1994, p. 70), that is $\sum_{j=0}^{\infty}\left|d_j\right| < \infty$, which implies that $d_j \to 0$, as $j \to \infty$. Note that, $x_{t+k} = \varepsilon_{t+k} + d_1\varepsilon_{t+k-1} + ... + d_k\varepsilon_t + d_{k+1}\varepsilon_{t-1} + ...$, and $E(x_{t+k}\left|\Im_t\right.) = d_k\varepsilon_t + d_{k+1}\varepsilon_{t-1} + ...$ Therefore, we have that $\lim_{k\to\infty} E(x_{t+k}\left|\Im_t\right.) = E(\Delta Y_{ij,t+k}\left|\Im_t\right.) = 0$. □

3. Fractional Integration and Stochastic Convergence

Long-range dependent processes were introduced in the literature by Granger (1980), and Granger and Joyeux (1980). In this section, we briefly review the theory of a class of long-range dependent processes, the so-called ARFIMA processes. Moreover, we present a proposition showing how fractionally integrated processes relate to one of the time series convergence criterion proposed by BD (1996). For a more complete treatment of ARFIMA models, the reader is referred to Baille (1996). Consider the stochastic process below.

$$(1-L)^d y_t = u_t \qquad (1)$$

where L is the lag operator, and u_t is a zero-mean constant variance and serially uncorrelated error term. The parameter of integration *d* is allowed to assume non-integer values. The process in (1) is called ARFIMA(0,d,0). For values of $d > -1$, the term $(1-L)^d$ has a binomial expansion given by $(1-L)^d = 1 - dL + d(d-1)L^2/2! - d(d-1)(d-2)L^3/3! + ...$. Invertibility is obtained whenever $-1/2 < d < 1/2$, in which case the process in (1) can be rewritten in moving average form as follows.

$$y_t = (1-L)^{-d} u_t = \sum_{j=0}^{\infty}\psi_j u_{t-j} \qquad (2)$$

where $\psi_j = \Gamma(j+d)/\Gamma(d)\Gamma(j+1)$, and $\Gamma(.)$ is the gamma function given by $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1}e^{-t}dt$.

The process in (1) is stationary for values of the parameter *d* lying in the interval $(-1/2, 1/2)$. A key property of the process in (1) is that for values of *d* in the interval $(1/2, 1)$ the process is non-stationary, but mean-reverting. Mean reversion means that the cumulative impulse response function at infinite is zero. More specifically, given the moving average parameters ψ_j, the cumulative impulse response, which gives the effect of a unit shock on the level of the series after N periods, is given by $c_N = \sum_{j=0}^{N}\psi_j$, $N = 0,1,2,...$,. It can be shown that if $d < 1$, then $c_\infty = 0$, there is, the process exhibit mean reversion. If $d > 1$, then $c_\infty = \infty$, and in the unit root case, when $d = 1$, c_∞ is constant and finite. In conclusion, for values of the fractional differencing parameter less than the unity income shocks die out, even when the y_t process is non-stationary. This is the key issue in our exercise. We formalize the relationship between mean reversion and time series convergence in proposition 2 below.

Proposition 2 shows that for d lying on the interval $(\frac{1}{2},1)$, that is, in the non-stationary mean-reverting region, income differential processes pass the convergence criterion in definition 2. This implies that the time series convergence criterion 2 is satisfied whenever estimates of the parameter d for income differential processes lie on the interval $(-\frac{1}{2},1)$.

Proposition 2: Let $X_{ij,t} = Y_{i,t} - Y_{j,t}$ be the income per capita differential process for economies i and j. If $X_{ij,t}$ can be described by an ARFIMA(0,d,0) process, that is, $X_t = (1-L)^{-d}\varepsilon_t$, then for values of parameter of integration in the interval $(-1/2,1)$ the time series convergence criteria in definition 2 is satisfied, that is, $\lim_{k\to\infty} E(X_{t+k}|\Im_t) = 0$.

Proof: The operator $(1-L)^{-d}$ is well defined for values of $d > -1/2$, and it can be represented as: $(1-L)^{-d} = \sum_{j=0}^{\infty}\psi_j L^j$, where $\psi_0 = 1$ and $\psi_j = (\frac{1}{j!})(d+j-1)(d+j-2)...(d+1)(d)$. Using Stirling's formula, for large j, we can write $\psi_j \approx (1+j)^{d-1}$ or $\psi_j \approx (\frac{1}{1+j})^{1-d}$. Note that we can write $X_{t+k} = \sum_{j=0}^{\infty}\psi_j\varepsilon_{t+k-j} = \psi_0\varepsilon_{t+k} + \psi_1\varepsilon_{t+k-1}... + \psi_{k-1}\varepsilon_{t+1} + \psi_k\varepsilon_t + \psi_{k+1}\varepsilon_{t-1} + ...$ Applying the conditional expectation operator we have that $E(X_{t+k}|\Im_t) = \psi_k\varepsilon_t + \psi_{k+1}\varepsilon_{t-1} + ...$ Clearly, if $-1/2 < d < 1$, then $\lim_{j\to\infty}\psi_j = \lim_{j\to\infty}\left(\frac{1}{1+j}\right)^{1-d} = 0$, which implies that $\lim_{k\to\infty} E(X_{t+k}|\Im_t) = 0$, which proves the claim. □

In the next section, we use obtain estimates of the fractional differencing parameter using three estimators. First, we use the log-periodogram estimator proposed by Geweke and Porter-Hudak (1983), henceforth GPH, as a first pass on the estimates of d. Second, we use Robinson's multivariate semi-parametric method, which can be seen as a generalization of GPH's estimator. Both GPH and Robinson's estimators are only applicable on stationary time series, that is, when $-1/2 < d < 1/2$. However, it would be also interesting to run the log-periodogram regression for the unit root case, that is, when $d = 1$. Phillips (1999) proposes an estimator that is consistent and asymptotic normal for values of $d \geq 1$. Phillips´ estimator is called the modified GPH.

Estimators of the fractional integration parameter require the choice of a bandwidth parameter, which determines the number of ordinates in the log periodogram regression. There is no complete theory on how to optimally choose the bandwidth parameter. To check for robustness of our estimates presented in section 4, we use alternative bandwidth parameters, which is the strategy followed in the literature (see, for instance, Michelacci and Zaffaroni, 2000). The estimates we present in tables III, IV, and V use a bandwidth of 0.5, but we also estimated the fractional differencing parameter for bandwidths 0.6, 0.7, and 0.8 (all estimates are available upon request). Estimates based on alternative bandwidths confirm our findings.

4. New Evidence on Time Series Convergence

Our data set is the same as the one used by BD (1995). It can be obtained at http://qed.econ.queensu.ca/jae/1995-v10.2/bernard-durlauf/. It consists of income per capita measured as the annual logarithm of the real GDP per capita in 1980 PPP adjusted dollars for 15 OECD countries (see table I below for the list of countries). Table 1 presents t-statistics based on ADF tests for the 15 OECD economies. The results below suggest ample evidence of non-stationarity.

[Insert table I here]

Based on the results contained in table I, we run the pairwise convergence tests following the methodology in BD. First, we test for a unit root in income per capita differentials. Definition 1 implies that two economies converge in times series if their income per capita differentials is a stationary stochastic process with zero mean, that is, two converging economies must satisfy the following criterion $Y_{i,t} - Y_{j,t} = \varepsilon_{ij,t} \approx I(0)$, where $\varepsilon_{ij,t} \sim (0, \sigma_{\varepsilon}^{2})$. In order to perform this test, we estimate the following ADF equation $\Delta X_{ij,t} = \delta X_{ij,t-1} + B(L)\Delta X_{ij,t-1} + \varepsilon_t$, where $X_{ij,t} = Y_{i,t} - Y_{j,t}$ is the income per capita differentials process between economies *i* and *j*, and $B(L)$ is a finite order polynomial lag. Table II displays our results.

[Insert table II here]

At 5% level of significance, for 30 pairs out of 105 pairs of economies we reject the null of no convergence, that is, only 30/105 pairs of economies pass the convergence test according to criterion in definition 1. At 10% level of significance, we reject a unit root in income per capita differentials for 23 out of 105 pairs of economies. That is, at a 10% level of significance, only 23 pairs of economies satisfy the convergence criterion in definition 1.

Not surprisingly, so far our results confirm BD's (1991, 1995) initial findings. That is, we find very weak evidence of income per capita convergence in a time series sense for OECD economies. As discussed above, we believe that the finding of no convergence for the OECD sample is a statistical artifact due to the knife-edge behavior of the unit roots test and the slow speed of income convergence. If this is really the case, a more flexible data generating process should be able to capture the convergence pattern among converging economies. Using BD's data set, we estimate the fractional differencing parameter in an attempt to capture the slow rate of income convergence. It is important to emphasize that our data generating process nests BD's specification. Table III reports the estimates for the parameter *d* in equation (1) using Robinson's (1995) estimator.

[Insert table III here]

Among the 105 pairs of countries, the parameter *d* lies in the non-stationary/mean reverting region for 66 pairs, it lies in the stationary region for 26 pairs, and it lies in the non-stationary/explosive region for 13 pairs. Hence, according to definition 2, 92(=66+26) pairs of countries exhibit pairwise time series convergence. The estimates in tables II and III are largely consistent, that is, when the series is found to be stationary or non-stationary in table II, it is also found to be stationary in table III. In only 19 cases we found inconsistent results, where a series is stationary according to the ADF tests in table II, and is found to be non-stationary according to the estimates in table III, or vice-versa, i.e., it is found to be non-stationary according to table II, and it is found to be stationary according to table III.

Table IV displays the estimates of the parameter *d* using the GPH´s estimator. The estimates suggest that the parameter *d* lies in the stationary region for 26 pairs out of the 105, it lies in the non-stationary/mean-reverting region for 67 pairs, and in the non-stationary/explosive region for 12 pairs. Therefore, there are a total of 93 converging pairs of economies according to time series convergence criterion in definition 2. These findings are consistent with the ones in table III.

[Insert table IV here]

Table V displays estimates of the parameter *d* using Phillips modified GPH estimator. According to Phillips´ estimator, the parameter *d* lies in the non-stationary/explosive for 10 pairs, it lies in the non-stationary region for 61 pairs, and it lies in the stationary region for 34 pairs. In this case, there are a total of 95 converging pairs according to convergence criterion in definition 2. Again, these findings are consistent with the ones in tables III and IV.

[Insert table V here]

For the estimated d, according to Robinson´s estimator, at a 10% significance level, 42 out of the 105 estimated *ds* are significant against a two-sided alternative hypothesis of irrelevance. At a 5% significance level, the number of estimated *ds* that are significant is 30. For estimated *ds* obtained with GPH´s estimator, at a 10% level of significance, 38 estimated *ds* are significant, while at 5% level 26 are significant. For the estimates of the parameter of fractional integration obtained with Phillips's estimator, at a 10% level of significance, 75 out of 105 coefficients are significant, and at a 5% level, 64 of the estimated *ds* are significant. We take this observation as evidence in our favor, especially the level of significance of the estimates based on Phillips's estimator.

Furthermore, we run alternatives tests to assess stochastic convergence (not shown here; available upon request). First, we conduct the above time series tests on the cross-sectionally demeaned individual income per capita series, following Evans and Karras (1996). Second, we run panel unit roots tests on the income per capita differential processes following Im, Shin, and Pesaram (2003), and Maddala and Wu (1999). The results largely confirm our initial findings.

Finally, based on the evidence presented above, we conclude that the findings obtained with time series convergence tests using unit roots and cointegration concepts are be misleading. At best, it seems premature to conclude that for this set of OECD economies there is no evidence of economic convergence.

5. Conclusion

We assess income convergence using the time series criteria proposed by BD (1991, 1996). We use a more flexible data generating process that nests previous specifications. In particular, we model income per capita differentials as an ARFIMA process. Our specification captures slow rates of income convergence and, at the same time, avoids the perverse effects of the knife-edge behavior of unit root tests.

New interpretations of previous time series convergence criteria are suggested and a proposition relating their definitions is presented. We also show that a class of stochastic process satisfies one criterion of pairwise time series convergence. In particular, we show that when the income per capita differentials can be well described by a long-range dependence process, then for certain values of the fractional differencing parameter, the series satisfies one criteria of stochastic convergence. This is an important result in the light of the empirical evidence presented in this article. Finally, we conclude that the initial rejections of the convergence hypothesis by BD (1995) may be due by a statistical artifact, and therefore, are misleading.

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Table I – ADF Unit Root Tests for OECD countries – 1900-87

	Australia	*Austria*	*Belgium*	*Canada*	*Denmark*
ADF-BIC	-1.4520	-1.8852	-1.4109	-2.5812	-1.8381
DF-GLS	-1.2032	-1.5511	-1.0864	-1.2722	-1.3767
	Finland	*France*	*Germany*	*Italy*	*Japan*
ADF-BIC	-2.1649	-2.1285	-2.7223	-1.7668	-1.3094
DF-GLS	-1.0746	-1.2056	-2.0434	-1.4384	-1.0457
	Netherlands	*Norway*	*Sweden*	*UK*	*US*
ADF-BIC	-2.4476	-1.9966	-2.1594	-2.1826	-3.7276**
DF-GLS	-1.8290	-1.3886	-1.3054	-1.0234	-2.7852

Note: For each country the first row shows the t-statistics for the ADF test with the lag length chosen by the BIC. The time trend was included in the ADF equation. The critical values for the ADF test are: at 1% -4.07; at 5% -3.46; and at 10% -3.16. The second row shows the t-statistic for the Elliot, Rothemberg, and Stock (1996) DF-GLS test with the lag length chosen by MAIC, as suggested by Ng and Perron (2001). The critical values for the DF-GLS test are: at 1% -3.65; at 5% -3.09 and at 10% -2.80.
* Reject Ho at 10%. ** Reject Ho at 5%. *** Reject Ho at 1%.

Table II – DF-GLS Test on Income differentials

	Australia	Austria	Belgium	Canada	Denmark	Finland	France
Austria	-1.53						
Belgium	-1.96**	-2.75***					
Canada	0.29	-1.61	-1.05				
Denmark	-0.50	-1.86*	-0.48	-2.84**			
Finland	0.72	-1.27	0.83	-1.50	-1.22		
France	-0.86	-2.97***	-0.59	-2.61***	-1.63*	-0.57	
Germany	-0.12	0.06	-0.65	-1.43	-1.35	-2.22**	-1.17
Italy	-0.73	-2.15**	-1.31	-2.34**	-1.26	-0.90	-2.80**
Japan	0.85	0.05	-0.27	-0.65	-1.19	-1.73*	-1.37
Netherlands	-2.13**	-2.85***	-2.14**	-2.01**	-3.74***	0.28	-0.02
Norway	0.32	-1.42	0.68	-2.64***	-1.40	-2.36**	-0.81
Sweden	0.38	-1.58	-0.16	-2.48**	-1.14	-0.86	-1.76*
UK	-2.65***	-1.55	-1.79*	0.19	-2.15**	0.02	-0.73
USA	-0.98	-1.66*	-1.51	0.17	-1.20	-0.28	-2.01**

Table II, cont. – DF-GLS Test on Income differentials

	Germany	Italy	Japan	Netherlands	Norway	Sweden	UK
Italy	-0.98						
Japan	-1.10	-0.10					
Netherlands	-0.48	-1.51	-0.49				
Norway	-1.90*	-1.27	-1.38	-0.16			
Sweden	-1.51	-1.73	-1.03	-1.25	-0.63		
UK	-0.23	-0.45	0.68	-1.92	0.32	-0.11	
USA	-0.65	-1.94**	0.02	-2.59**	-0.11	-1.55	-0.28

Note: The lag length was chosen according to the MAIC. The critical values for the DF-GLS test are: at 1% -2.59; at 5% -1.94; and at 10% -1.61. The ADF equation does not include any deterministic components. *Reject the null at 10%. **Reject the null at 5%. *** Reject the null at 1%.

Table III – Estimates of the fractioning differencing parameter *d* using Robinson's estimator

	Australia	Austria	Belgium	Canada	Denmark	Finland	France
Austria	0.5975 (0.1106)						
Belgium	0.8498 (0.2130)	0.0790 (0.3173)					
Canada	0.2151 (0.2031)	0.7632 (0.1486)	0.7043 (0.2804)				
Denmark	0.6156 (0.2206)	0.3066 (0.3023)	1.3007 (0.2936)	0.1779 (0.4168)			
Finland	0.9313 (0.3206)	0.4590 (0.1841)	1.2857 (0.2627)	0.6595 (0.2631)	0.9918 (0.6760)		
France	0.5580 (0.1604)	0.1400 (0.1311)	0.4513 (0.1045)	0.6749 (0.2271)	0.7797 (0.2576)	0.6790 (0.0953)	
Germany	0.6532 (0.1415)	1.1380 (0.2051)	0.6011 (0.1755)	0.8303 (0.1409)	0.3001 (0.3951)	0.4530 (0.1484)	0.5677 (0.1381)
Italy	0.7634 (0.2276)	0.4606 (0.2379)	0.5702 (0.3221)	0.8373 (0.1697)	0.6896 (0.3314)	0.7028 (0.3968)	0.5257 (0.3061)
Japan	0.9889 (0.1453)	0.9579 (0.2986)	0.6856 (0.1761)	1.0211 (0.2499)	0.7853 (0.1376)	0.8974 (0.2211)	0.5253 (0.1814)
Netherlands	0.2197 (0.2359)	0.3910 (0.1450)	0.2659 (0.1981)	0.4182 (0.2712)	0.4708 (0.1008)	0.4845 (0.1955)	0.3979 (0.2716)
Norway	0.4244 (0.7808)	0.5301 (0.3881)	0.9297 (0.2232)	0.3560 (0.4318)	0.2926 (0.3451)	0.6508 (0.2408)	1.1553 (0.5149)
Sweden	0.9337 (0.1783)	0.5954 (0.1494)	1.3432 (0.1793)	1.0665 (0.2645)	1.2819 (0.2645)	0.8957 (0.2993)	0.8741 (0.1111)
UK	0.1932 (0.2236)	0.6644 (0.1605)	0.8626 (0.3152)	0.9219 (0.3670)	0.4485 (0.2673)	0.4895 (0.2489)	0.7013 (0.2693)
USA	0.8600 (0.2508)	0.8217 (0.1568)	0.8938 (0.1749)	0.5056 (0.2778)	0.5982 (0.2264)	0.8258 (0.2379)	0.7059 (0.1293)

Table III, cont.

	Germany	Italy	Japan	Netherlands	Norway	Sweden	UK
Italy	0.9901 (0.4245)						
Japan	1.4649 (0.1218)	0.6562 (0.1554)					
Netherlands	0.6317 (0.0696)	0.6179 (0.2747)	0.6231 (0.1376)				
Norway	0.6251 (0.2982)	1.0897 (0.2844)	0.8795 (0.0703)	0.7314 (0.1555)			
Sweden	0.4137 (0.0950)	0.9923 (0.2940)	0.9409 (0.1923)	0.7691 (0.1199)	1.0257 (0.3801)		
UK	0.6894 (0.2012)	0.8961 (0.1295)	1.1405 (0.3698)	0.5108 (0.2533)	0.3686 (0.3065)	0.8608 (0.3781)	
USA	0.7817 (0.1326)	0.8719 (0.1400)	1.2057 (0.2613)	0.5651 (0.1102)	0.4544 (0.2121)	0.8962 (0.2119)	0.8252 (0.2931)

Note: The standard errors appear in parenthesis. Robinson´s estimator is only applicable when the series is stationary. In order to ensure stationary we first-difference the series and then apply Robinson´s estimator. See equation (3) in the text. The above estimates were generated using a bandwidth parameter of N=0.5 (recall that $g(T) = T^N$), however, our conclusions remain unchanged if different values of N are used. Estimates for $N \in \{.60,.70,.80,.90\}$, are available from the authors upon request.

Table IV – Estimates of the fractioning differencing parameter *d* using GPH's estimator

	Australia	Austria	Belgium	Canada	Denmark	Finland	France
Austria	0.5948 (0.1115)						
Belgium	0.8470 (0.2144)	0.0726 (0.3198)					
Canada	0.2087 (0.2043)	0.7615 (0.1497)	0.7000 (0.2821)				
Denmark	0.6105 (0.2217)	0.3023 (0.3048)	0.6993 (0.2962)	0.1680 (0.4188)			
Finland	0.9287 (0.3230)	0.4562 (0.1860)	1.2881 (0.2646)	0.6558 (0.2648)	0.9928 (0.6811)		
France	0.5535 (0.1611)	0.1332 (0.1317)	0.4468 (0.1050)	0.6704 (0.2283)	0.7783 (0.2596)	0.6767 (0.0961)	
Germany	0.6508 (0.1426)	1.1403 (0.2065)	0.5983 (0.1768)	0.8287 (0.1419)	0.2955 (0.3981)	0.4497 (0.1499)	0.5642 (0.1390)
Italy	0.7617 (0.2293)	0.4566 (0.2397)	0.5657 (0.3243)	0.8345 (0.1708)	0.6872 (0.3339)	0.7023 (0.3999)	0.5200 (0.3079)
Japan	0.9887 (0.1463)	0.9552 (0.3008)	0.6823 (0.1771)	1.0208 (0.2518)	0.7834 (0.1385)	0.8968 (0.2228)	0.5208 (0.1824)
Netherlands	0.2126 (0.2370)	0.3866 (0.1462)	0.2610 (0.1999)	0.4118 (0.2726)	0.4671 (0.1017)	0.4810 (0.1971)	0.3950 (0.2741)
Norway	0.4188 (0.2203)	0.5281 (0.3912)	0.9290 (0.2249)	0.3474 (0.4342)	0.2855 (0.3471)	0.6482 (0.2427)	1.1565 (0.5187)
Sweden	0.9330 (0.1796)	0.5931 (0.1508)	1.3455 (0.1807)	1.0654 (0.2665)	1.2837 (0.2665)	0.8949 (0.3015)	0.8729 (0.1118)
UK	0.1863 (0.2247)	0.6623 (0.1618)	0.8605 (0.3174)	0.9180 (0.3696)	0.4421 (0.2686)	0.4841 (0.2503)	0.6982 (0.2376)
USA	0.8581 (0.2526)	0.8208 (0.1581)	0.8916 (0.1760)	0.5028 (0.2801)	0.5929 (0.2275)	0.8227 (0.2395)	0.7027 (0.1300)

Table IV, cont.

	Germany	Italy	Japan	Netherlands	Norway	Sweden	UK
Italy	0.9917 (0.4277)						
Japan	1.4676 (0.1231)	0.6535 (0.1565)					
Netherlands	0.6291 (0.0702)	0.6152 (0.2768)	0.6202 (0.1386)				
Norway	0.6232 (0.3006)	1.0897 (0.2865)	0.8784 (0.0708)	0.7293 (0.1566)			
Sweden	0.4091 (0.0955)	0.9931 (0.2962)	0.9410 (0.1937)	0.7676 (0.1208)	1.0255 (0.3830)		
UK	0.6876 (0.2028)	0.8959 (0.1305)	1.1424 (0.3725)	0.5065 (0.2550)	0.3623 (0.3083)	0.8576 (0.3808)	
USA	0.7800 (0.1336)	0.8704 (0.1410)	1.2072 (0.2633)	0.5611 (0.1105)	0.4484 (0.2130)	0.8935 (0.2133)	0.8212 (0.2950)

Note: Same as above in table 4a

Table V – Estimates of the fractioning differencing parameter *d* using Phillips´ estimator

	Australia	Austria	Belgium	Canada	Denmark	Finland	France
Austria	0.5376 (0.1521)						
Belgium	0.8882 (0.2363)	0.1883 (0.2044)					
Canada	-0.0807 (0.2420)	0.7289 (0.2104)	0.6029 (0.1723)				
Denmark	0.5521 (0.2556)	0.1705 (0.4197)	0.8676 (0.2182)	0.2673 (0.3157)			
Finland	0.5160 (0.4162)	0.4697 (0.1974)	1.3985 (0.3337)	-0.2869 (0.3630)	0.5547 (0.3349)		
France	0.5415 (0.1632)	0.1300 (0.1345)	0.3458 (0.1294)	0.5561 (0.1756)	0.6668 (0.1719)	0.6763 (0.0972)	
Germany	0.4267 (0.2081)	1.0902 (0.2591)	0.5462 (0.0783)	0.5275 (0.3022)	0.4043 (0.2218)	0.3659 (0.1415)	0.4978 (0.0934)
Italy	0.7121 (0.1872)	0.2976 (0.1689)	0.4640 (0.3675)	0.8318 (0.1591)	0.6352 (0.2894)	0.6294 (0.3557)	0.1908 (0.4278)
Japan	0.9678 (0.1493)	0.8905 (0.2888)	0.6668 (0.1795)	1.3188 (0.5575)	0.7488 (0.1104)	0.8769 (0.2082)	0.5154 (0.1967)
Netherlands	0.2266 (0.2297)	0.4035 (0.1817)	0.3104 (0.2109)	0.3692 (0.2451)	0.4150 (0.0901)	0.4305 (0.2288)	0.5766 (0.3338)
Norway	0.4300 (0.2138)	0.4781 (0.3143)	0.8823 (0.2043)	0.4089 (0.2528)	0.7596 (0.4141)	0.4539 (0.2025)	1.0216 (0.3927)
Sweden	0.8151 (0.2043)	0.5866 (0.1675)	1.2993 (0.1456)	0.5641 (0.2223)	1.3689 (0.2979)	0.7481 (0.1746)	0.8837 (0.1199)
UK	0.1808 (0.1947)	0.6364 (0.1838)	0.7854 (0.2871)	0.5655 (0.3664)	0.3412 (0.3104)	0.1212 (0.2290)	0.6709 (0.2469)
USA	0.6201 (0.2277)	0.7597 (0.1904)	0.8123 (0.1429)	0.5362 (0.2583)	0.5970 (0.1992)	0.5812 (0.2062)	0.6608 (0.1225)

Table V, cont.

	Germany	Italy	Japan	Netherlands	Norway	Sweden	UK
Italy	0.7498 (0.1552)						
Japan	1.3616 (0.1581)	0.7138 (0.1339)					
Netherlands	0.5426 (0.0673)	0.5410 (0.3205)	0.6193 (0.1784)				
Norway	0.5340 (0.4108)	1.1073 (0.3034)	0.8673 (0.0682)	0.7026 (0.1553)			
Sweden	0.3194 (0.1229)	0.9233 (0.2904)	0.9282 (0.1578)	0.7594 (0.1234)	1.0633 (0.2855)		
UK	0.5353 (0.2366)	0.8918 (0.1474)	1.0672 (0.2842)	0.5097 (0.2467)	0.3529 (0.2444)	0.2856 (0.5378)	
USA	0.5628 (0.2575)	0.8553 (0.1238)	0.0941 (0.2994)	0.5573 (0.0901)	0.4667 (0.1909)	0.4768 (0.2440)	0.7049 (0.3017)

Note: The standard errors appear in parenthesis. The above estimates were generated using a bandwidth parameter of N=0.5 (recall that $g(T) = T^N$), however, our conclusions remain unchanged if different values of N are used. Estimates for $N \in \{.60,.70,.80,.90\}$, are available from the authors upon request.