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Scale invariance in financial time series

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Abstract

We focus on new insights of scale invariance and scaling properties usefully applied in the framework of a statistical approach to study the empirical finance. Two stock returns of Sri Lankan stock market indices All Share Price Index and Milanka Price Index index were considered. Central parts of the probability distribution function of returns are well fitted by the Lorentzian distribution function. However, tail parts of the probability distribution function follow a power law asymptotic behavior. We found that the probability distribution function of returns for both All Share Price Index and Milanka Price Index , is outside the L´evy stable distribution. Sri Lankan stock market is not described by the random Gaussian stochastic processes.

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Editor's Comment
on
Scale Invariance in Financial Time Series
by
Ranasinghe P. K. C. Malmini

General: Unfortunately, from time to time it comes to our attention that the scientific standards we strive to maintain at the Economics Bulletin may have been compromised. Rather than sweep such things under the rug, it is our policy to lay out what facts we have and let the broader scientific community judge for itself.

Comment: There appear to strong similarities between:

Ranasinghe Malmini, (2007) ''Scale invariance in financial time series'', Economics Bulletin, Vol. 3 no.24 pp. 1-7 (http://www.accessecon.com/pubs/EB/2007/Volume3/EB-07C50001A.pdf)

and two previously published papers:

Kyuong Eun Lee, Jae Woo Lee, (2004) "Scaling Properties of Price Changes for Korean Stock Indices", J. Korean Phys. Soc. 44, 668(http://arxiv.org/PS_cache/cond-mat/pdf/0407/0407418v1.pdf)

Rama Cont Marc Potters Jean-Philippe Bouchaud (1977) Scaling in stock market data: stable laws and beyond arXiv:cond-mat/9705087v1 (http://arxiv.org/pdf/cond-mat/9705087v1)

The reader is invited to look at the conclusions of all three papers, and the main bodies of the first and the third. It appears that significant sections of text have been taken more or less verbatim from the earlier works. While the *Economics Bulletin* paper does study new data and is not simply a copy of these works, it is our opinion that such strong textual similarities could be not be an accident. The author should certainly have cited these works if he was familiar with them.

We do not pass judgment on the author's motivations. However, this paper does not meet the standard standards of the *Economics Bulletin*, and had we been aware of the similarities between Dr. Malmini's submission and the two older works, we would not have accepted it for publication.

Scale Invariance in Financial Time Series

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Abstract

We focus on new insights of scale invariance and scaling properties usefully applied in the framework of a statistical approach to study the empirical finance. Two stock returns of Sri Lankan stock market indices All Share Price Index and Milanka Price Index index were considered. Central parts of the probability distribution function of returns are well fitted by the Lorentzian distribution function. However, tail parts of the probability distribution function follow a power law asymptotic behavior. We found that the probability distribution function of returns for both All Share Price Index and Milanka Price Index , is outside the L´evy stable distribution. Sri Lankan stock market is not described by the random Gaussian stochastic processes.

Introduction

In recent years scaling concept is increasingly applied to financial markets[1,2,3,4,5,6,7] The behavior of some forms of volatility measure (variance of returns, absolute value of returns) as a function of the time interval on which the returns are measured lead to the estimation of a scaling exponent related to the Hurst exponent and the behavior of the tails of the distribution of returns as a function of the size of the movement but keeping the time interval of the returns constant are two types of scaling behavior studying in financial market. To map out the statistical properties of financial markets considered as complex systems is an attempt by physicists and mathematicians . A lot of economic data have been reanalyzed recently [8, 9, 10, 11]. Time series of stock market around the world have rich behaviors. The time series deviate from the EMH(efficient market hypothesis). Indices of stock market show scaling behaviors in the well developed market. It is very difficult to understand the dynamics of financial systems because there are many factors among interacting agents.

As in the case of other types of complex systems with universal characteristics, a stochastic approach has proved to be fruitful in this case. The main object of study in this framework is the probability density function (PDF) of the increments at a given time scale T i.e. the probability distribution of $Z(t+T) - Z(t)$ where $Z(t)$ is the price of the asset at time t. This approach was inaugurated by Louis Bachelier who first introduced the idea that stock market prices behave as a random walk [4], and who considered Brownian motion as a candidate for modeling price fluctuations.

Bachelier's model, applied to the logarithm of the prices (to ensure positivity of the price!), became very popular in the 1950s [12] and it is one of the main ingredients of the famous Black Scholes option pricing formula [6]. This model implies that the increments of asset returns (or asset prices) are independent identically distributed (iid) Gaussian variables. Indeed if one considers each price change as a sum of many small and independent random contributions from various market factors, the Central Limit Theorem suggests the Gaussian as a natural candidate. Mandelbrot analyzed a relatively short time series of cotton prices and observed that returns have L´evy stable symmetric distribution with Pareto fat tail[13, 14, 15, 16].

By analyzing the high frequency data observed that away from Levy stable distribution of returns and power law behaviour of the probability density function (pdf) of returns with the fat tail exceeding the Levy distribution[5].

In this article we consider two stock market indices, All Share Price Index and Milanka Price index in the Sri lankan Financial market from January 1985 to March 2005.

Data

The Colombo Stock exchange has two main price indices and twenty sector price indices based on the business activities of companies. In addition the Colombo stock exchange publishes a series of Total return indices. The market has been divided into 20 sectors and the price index for each sector is calculated on a daily basis. Each index indicates the direction of the price movements of the sector. There are two main price indices as All Share price index and Milanka price index.

These price indices are capital weighted indices. The weight of any company is taken as the number of ordinary shares listed on the market. This weighting system allows the price movements of large companies to have a greater impact on the index. Such a weighting system was adopted on the assumption that the general economic situation has a greater influence on large companies than on small ones.

The daily trading activity results in market price changes market capitalization, which affects price indices. There are however, exceptions other than market price changes. Some times the number of listed shares will increase or decrease and this will affect the market capitalization of companies. In order to avoid the fluctuation of price indices, the base market capitalization and market capitalization are adjusted in such instances.

We count the time during trading hours and remove closing hours, weekends and holidays from data sets. For a time series $Z(t)$ of stock market index values, the return $G_T(t)$ over a return time T is defined as

$$G_T(t) = \ln \frac{Z(t+T)}{Z(t)} \qquad (1)$$

For small changes in $Z(t)$, the return is approximately

$$G_T(t) \cong \frac{Z(t+T) - Z(t)}{Z(t)} \qquad (2)$$

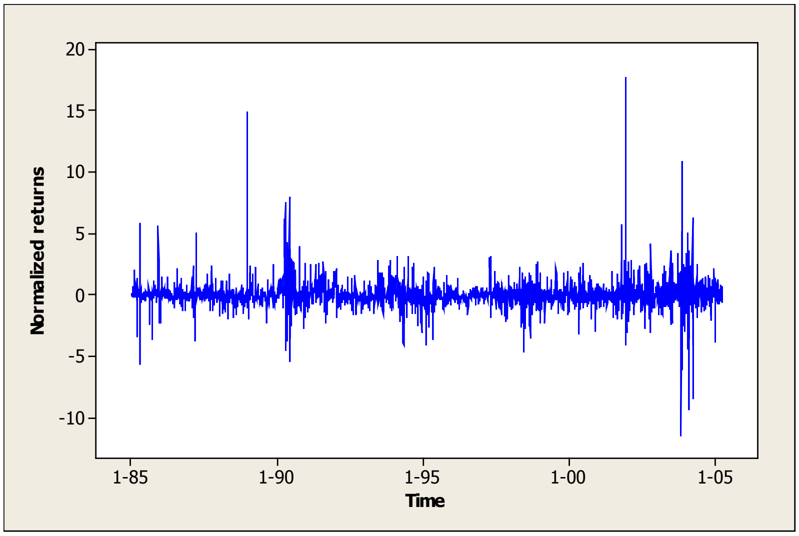
The normalized return is defined as

$$g_T(t) \cong \frac{G_T(t) - \langle G_T(t) \rangle}{\sigma(G_T(t))} \qquad (3)$$

where $\sigma(G)$ is the standard deviation and $<\ >$ de notes averaging over time variable.

RESULTS AND DISCUSSIONS

A Gaussian pdf systematically underestimates the probability of a large price fluctuations an issue of utmost importance is financial risk management. The normalized return presented in Fig.1 for All Share Price Index ansd Fig 2. for Milanka Price Index.



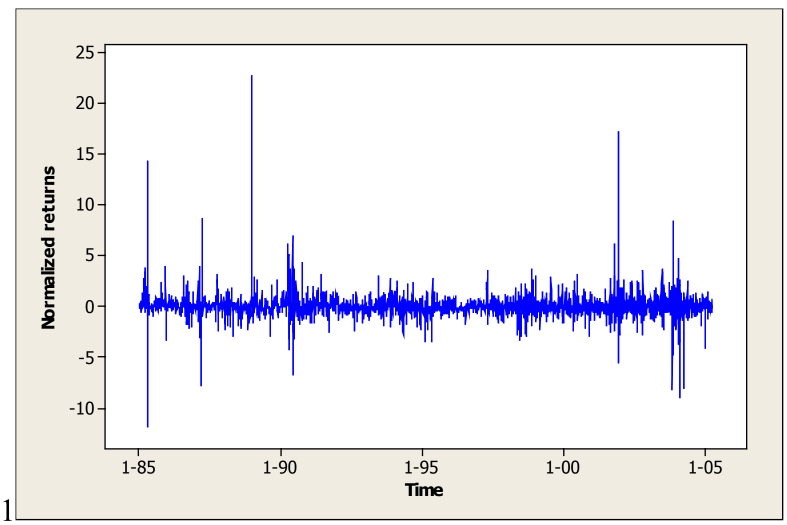


Fig 1

Fig2

We observed large price changes around the period of Asian economic crisis in September 1997. We consider the logarithmic return with the return time T=1day, 7day, and 30day. The pdf for the return presented in Fig.3 (All Share Price Index) and Fig.4 (Milanka). For the short return time T=1day, the pdf of the return has long tails with very large fluctuations.

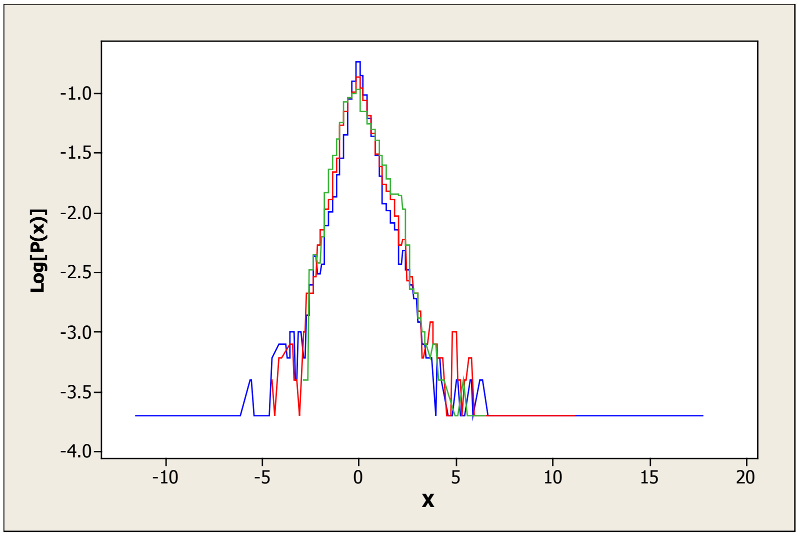


Fig 3

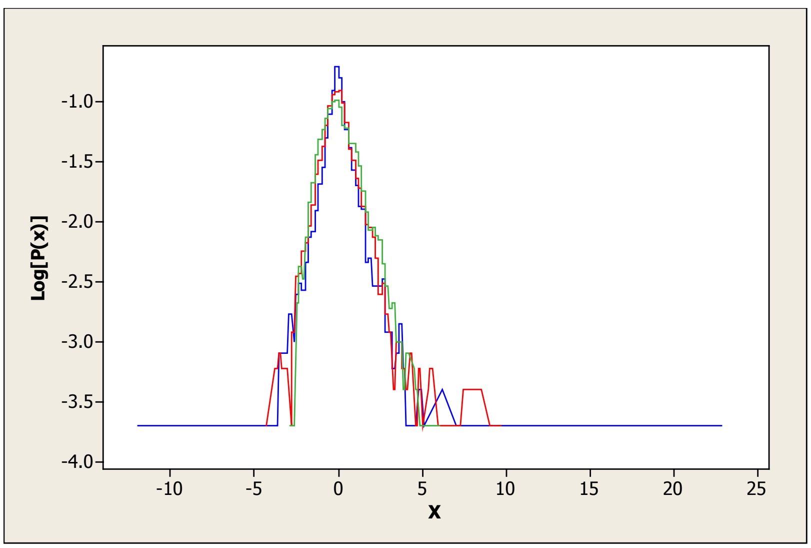


Fig4

The peak of the pdf of the return decreases as the return time T increases. We fit the pdf with the Gaussian distribution function and Lorentzian distribution function for the return time T=1day in Fig.5(All Share Price Index) and Fig.6(Milanka).

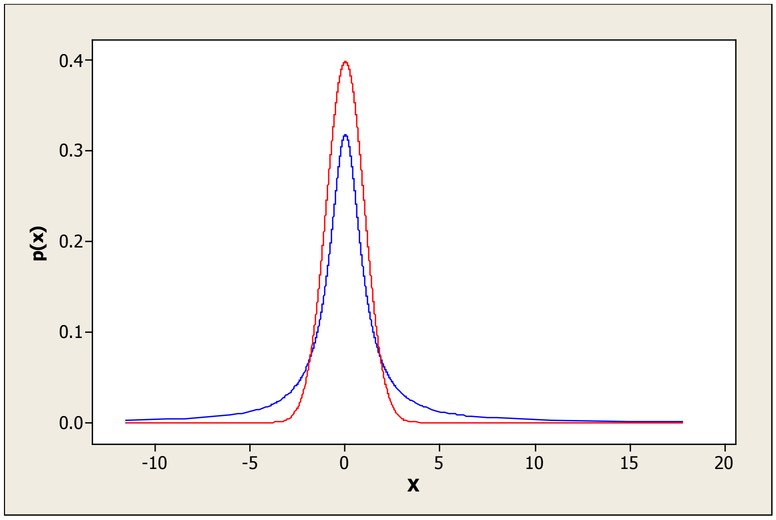


Fig 5

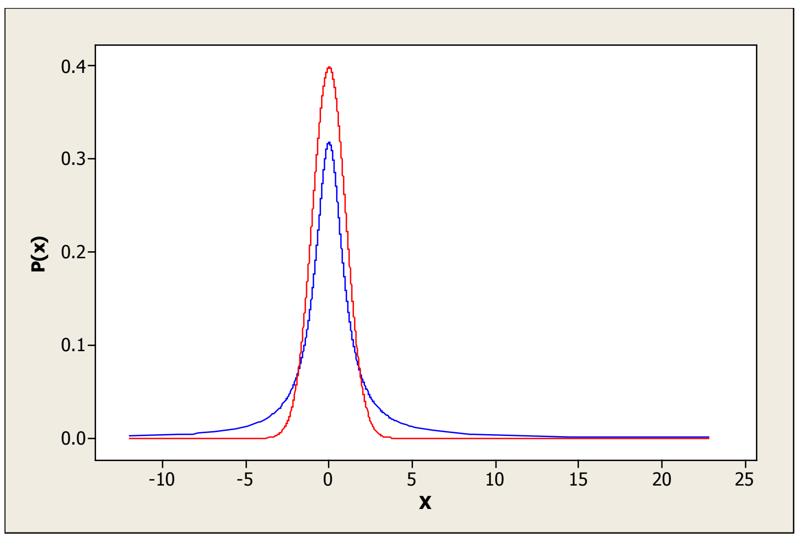


Fig6

Stock price fluctuations have scale invariant properties. The distribution of he sum a large number of independent identically distributed random variables belong to a family if distribution known as stable L´evy distribution .The central region of the pdf is fitted better by Lorentzian than by Gaussian[20]. However, the positive and negative tail region of the pdf deviates from Gaussian and Lorentzian. The tail of the pdf of returns decays according to a power law as

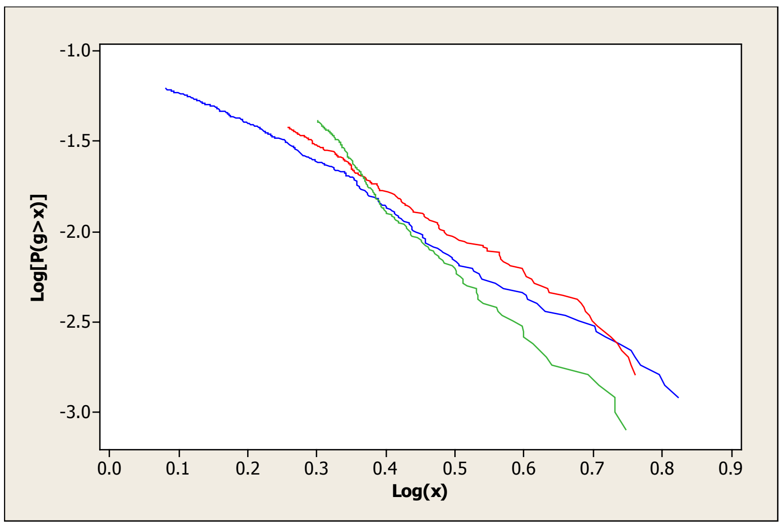
$$p(x) \approx x^{-(1+\alpha)} \qquad (4)$$

time T=10min for ASPI with the exponent $\alpha > 2$. The accumulated pdf of returns is defined as

$$P(g > x) = \int_x^\infty p(x)dx \qquad (5)$$

The accumulated pdf follows a power-law behavior as

$$P(g > x) \approx \frac{1}{x^\alpha}$$



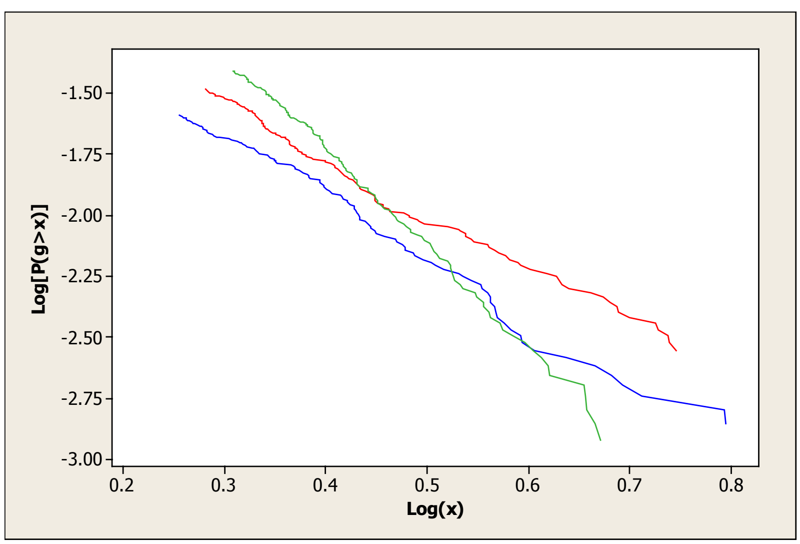


Fig 8

In Fig.7(ASPI) and Fig.8(Milanka), we presented the accumulated probability distribution function for the return time T=10min. We observed that the exponents are greater than 2 which means that the pdf of returns deviated from the stable L´evy distribution with 0 < α < 2. We present the exponents for the different return time T in the table 1.

We summarized the measured exponent α in the table 1 for many different stock indices. Exponents of ASPI and Milanka increase when the return time increases[1]. We also observed that the range of the power law diminish as the return time increases. The pdf of the return deviated from the stable L´evy distribution for all different stock indices. The exponents α of the positive and negative tail are greater than 2. The exponent α of the positive tail of ASPI and Milanka are slightly less than one of the negative

From the analysis of the stock market indices, we observed that the pdf of ASPI index is well outside the stable L´evy distribution. The exponent α depends on the return time T . The observed exponent α increased when the return time increased for both positive and negative tail. The pdf of Milanka index also is outside the stable. L´evy distribution with α > 2 for the positive tail.

Positive tail		
	α	range
ASPI	2.11 (T=1day) 2.31 (T=7day) 2.43 (T=30day)	1.2 < g < 6.7 1.8 < g < 5.9 2.0 < g < 5.6
MPI	2.01 (T=1day) 2.23 (T=7day) 2.51 (T=30day)	1.8 < g < 7.0 1.9 < g < 5.9 2.0 < g < 4.8

Negative tail		
	α	range
ASPI	2.61 (T=1day) 2.78 (T=7day) 3.01(T=30day)	1.6 < g < 6.1 1.8 < g < 4.3 2.0 < g < 3.0

MPI	2.66 (T=1day) 2.76 (T=7day) 2.81 (T=30day)	$1.8 < g < 3.6$ $1.9 < g < 2.8$ $2.1 < g < 2.6$

CONCLUSIONS

We consider the probability density function (pdf) of the Sri Lankan stock market indices, ASPI and Milanka indices. We observe that the pdf for both ASPI and Milanka indices fit neither Gaussian nor Lorentzian distribution function. Central parts of indices are well fitted by the Lorentzian distribution function. However, the tail parts of the pdf deviate strongly form Gaussian and Lorentzian distribution function. The tail part of the pdf follows a power-law asymptotic behavior. We observe that exponents of the power-law of the accumulated probability distribution function are well outside the stable L´evy distribution. Sri Lankan financial market is not described by the random Gaussian stochastic processes. The scientific study of financial market proves to be a fascinating subject itself, particular whose theoretical tools may prove to be uncovering new properties and mechanism in financial data.

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