E C O N O M I C S   B U L L E T I N

A synthesis of location models

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Abstract

This article considers a model of spatial competition where firms and consumers are located in a semicircular space rather than in the whole circle (Salop's model) or the linear city (Hotelling's model), under the assumptions of both, convex and concave, transportation costs. The paper tries to generalize the results of the two previous models. We find that for concave transportation costs the existence of a price equilibrium is warranted for every firms' location when the length of the semicircular space is greater than 3/4. For the convex case, perfect equilibrium is only obtained when the size of the market segment is equivalent to Hotelling's linear model.

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1.-Introduction

When studying spatial competition, two standard models have been considered in the literature, the linear model, first studied by Hotelling (1929), and the circular model as popularized by Salop's (1979).

The linear model is usually used when the problem under consideration is such that locations of firms is a priori heterogeneous, however, market boundaries do lead in certain situations to existence problems that do not appear in the circular model for the same type of assumptions[1]. On the other hand, the circular model is preferred when firms' locations can be considered homogeneous. It is used to study some market configurations such as locations of stores a long a city belt way, airlines choosing departure times on the dial of a clock, etc. However, in these types of circular market configurations, there are situations in which the market is discontinuous.

Many environmental policies involve the introduction of restrictions on market configuration by the regulating authorities. For example, in most urban designs, we observe the existence of portions of land located around the city belt-ways devoted to non-residential purposes. We can find environmentally protected areas, parks, recreational facilities, etc. When considering urban design, regulators have to decide whether they should leave some part of urban land for this type of recreational activities, and if they do so, what is the optimal size of these non-residential areas. Similarly, in order to control for noise pollution, many airports located near cities have introduced limits on take offs and landings during certain hours of the night, therefore, imposing time restrictions on the services offered by airlines.These physical or time zones represent a discontinuity in the market since neither consumers nor firms can be located within them; however, it is possible to find consumers and firms adjacent to both ends of these restricted areas.

In order to study the implications of this type of market configurations, we assume a circular spatial model, where there is a segment where people live and firms locate, that we shall refer to as the market, and another segment where no market activity takes place. In this context, a three-stage game can be considered in which in the first stage the regulator chooses the size of the market, in the second stage firms choose locations and in the third stage firms compete in prices. We will suppose that the regulator chooses the length of the market in a non-strategic manner[2]; therefore, the model we present, once market size is given, can be reduced to a two-stage game in which firms first choose location, and then compete in prices.

This model can be thought of as a synthesis of the circular and linear spatial models. If the size of the market is half of the circumference, $l/2$, the model is equivalent to Hotelling's (1929) linear city, while if the size of the market is the whole circle, l, we are in Salop's (1979) configuration. We analyze the existence of a price equilibrium when the market segment, h, is restricted to be less than l. (See figure 1)

We will make the standard assumptions of two firms selling a homogeneous product, consumers evenly distributed a long the market segment, and we will study the existence problem in our model, using a concave function that ensures the existence of a perfect equilibrium in pure strategies in the circular model (see De Frutos et al, 1999).

We find that there exists a subgame perfect price equilibrium for any location of firms provided the length of the market is greater than approximately $3/4$. Furthermore, this equilibrium is unique and implies firms locating opposite to each other. If h is

[1] For example, when 3 firms are considered, or when the transportation costs considered is concave.
[2] In the sense that the regulator does not optimize an objective function to choose market size.

smaller than $\frac{3}{4}$, we find that, for certain values of h and firms locations, there is a strip where no price equilibrium may exist. Nevertheless, there are many combinations of market sizes and firm locations for which equilibrium can be obtained. When we compare the intensity of competition, given the size of the market, we find that competition is more intense for low values of h, and the equilibrium region is smaller.

We also study the model under convex transportation costs and we find that perfect equilibrium can only be obtained for values of h for which our model is equivalent to Hotelling's linear city.

The paper is organized as follows, in section 2 we present the model, in section 3 we study the existence of equilibrium, section 4 contains the conclusions, and finally major proofs and graphs can be found in the appendix.

2.- The Model

We consider a circular city of length l where the regulator chooses the size of the market, h, so that firms and consumers can only be located on a certain segment of the circle $h \leq l$. There are two firms selling a homogeneous product, with zero production costs, located at x and y, with $0 \leq x \leq y \leq h$.

Consumers are evenly distributed along h, and each consumer buys a single unit of this product per unit of time, irrespective of its price. Since the product is homogeneous, consumers will buy from the firm who offers the least delivered price, that is, the mill price plus transportation costs. Let p_1, p_2, denote the mill prices charged by firms located at x and y, respectively. The distance between consumer z and firm i is given by $d_i = |z - i|$, $i = x, y$. We will consider a concave transportation costs function from the linear quadratic family: $C(d_i) = k(d_i - d_i^2)$ that have been shown to ensure existence of a perfect equilibrium in pure strategies in the circular model. Although firms and consumers can only be located within h, consumers can travel along the whole circle and they will always take the direction that implies the shorter distance to the chosen firm.

The model described above, given that the regulator behaves in a non-strategic manner, does give rise to a two-stage game in which firms first decide simultaneously their location and then simultaneously choose prices. It turns out that the solution to this game depends critically on the length of the market segment, h. In order to determine the market boundaries and derive the demands faced by each firm, we will have to find the indifferent consumers. A consumer is indifferent to buying from one firm or the other if and only if: $p_1 + C(d_1) = p_2 + C(d_2)$.

To analyze the problem, we will assume, without loss of generality that in the expression of the transportation costs functions $k = 1$, and the total length of the circle, l, is equal to1. When the market length considered is the whole circle (See figure 1), three types of possible indifferent consumers are found, each one belonging to a different segment of the circumference given by: $m_1 \in [x, y]$, $m_2 \in [y, 1]$, and $m_3 \in [0, x]$.

Taking into account that the market is equal to h $(h < 1)$, and depending on the price interval considered, we obtain the following demand function for firm1:

$$D_1 = \begin{cases} h & for \quad p_1 - p_2 \in I_1 \\ m_1 + (h - m_2) & for \quad p_1 - p_2 \in I_2 \\ m_1 & for \quad p_1 - p_2 \in I_3 \\ m_1 - m_3 & for \quad p_1 - p_2 \in I_4 \\ 0 & for \quad p_1 - p_2 \in I_5 \end{cases}$$

Where:
$I_1 = [-\infty, -z(1-z)]$, $I_2 = [-z(1-z), z(2h-q-1)]$, $I_3 = [z(2h-q-1), z(1-q)]$,
$I_4 = [z(1-q), z(1-z)], I_5 = [z(1-z), +\infty]$
Where $z = x_2 - x_1$ *and* $q = x_1 + x_2$
Demand for firm 2 can be obtained as $D_2 = h - D_1$.

3.- Equilibrium

Given the size of the market, *h*, and using the usual approach for a two-stage non-cooperative game in which firms select a position at the first stage and subsequently set their prices, we study the subgame perfect equilibrium. We recall that a perfect price-location equilibrium is defined as a pair (p_1^N, x^N), (p_2^N, y^N) such that:

(i) $p_1^N = p_1^N(x^N, y^N, h)$ and $p_2^N = p_2^N(x^N, y^N, h)$,

$$(ii) \quad \begin{cases} B_1(x^N, y^N, h, p_1^N(x^N, y^N, h), p_2^N(x^N, y^N, h)) \geq B_1(x, y^N, h, p_1^N(x^N, y^N, h), p_2^N(x^N, y^N, h)) \\ B_2(x^N, y^N, h, p_1^N(x^N, y^N, h), p_2^N(x^N, y^N, h)) \geq B_2(x^N, y, h, p_1^N(x^N, y^N, h), p_2^N(x^N, y^N, h)) \end{cases}$$

$\forall y, x \in [0, 1/2]$.

Where (p_1^N, p_2^N) is a Nash equilibrium in the price subgame when the locations choice is fixed.

The profit function for firm *i* is given by $B_i = p_i D_i$, $i = 1,2$. It can be easily observed that this profit function is not concave in prices (see the expression for the demand function above) and it may exhibit different configurations. In particular, it could exhibit several local maxima. Therefore, the sufficient condition for a price equilibrium for any possible location of firms is not satisfied, although depending on the values of *z*, *q* and *h*, we could find combinations of market sizes and firms locations for which equilibrium may be obtained. In order to explore this possibility we will reduce the number of parameters to two by assuming that: $x = 0$ and $0 \leq y \leq 1/2$. In this case, we only have two indifferent consumers $m_1 \in [x, y]$, $m_2 \in [y, 1]$ while $m_3 \in [0, x]$, disappears since $x = 0$. Consequently, the new demand function for firm 1 can be written as:

$$D_1 = \begin{cases} h & for \quad p_1 - p_2 \in I_1^1 \\ m_1 + (h - m_2) & for \quad p_1 - p_2 \in I_2^1 \\ m_1 & for \quad p_1 - p_2 \in I_3^1 \\ 0 & for \quad p_1 - p_2 \in I_4^1 \end{cases}$$

Where:
$I_1^1 = [-\infty, -y(1-y)]$, $I_2^1 = [-y(1-y), y(2h-y-1)]$, $I_3^1 = [y(2h-y-1), y(1-y)]$,
$I_4^1 = [y(1-y), +\infty]$

We will now compute the equilibrium of the price subgame.

The expression for the profit functions for firm 1 is given by:

$$B_1 = \begin{cases} hp_1 & for \quad p_1 - p_2 \in I_1^1 \\ p_1\left(\dfrac{p_2 - p_1}{2y(1-y)} + \dfrac{(2h-1)}{2} \right) & for \quad p_1 - p_2 \in I_2^1 \\ p_1\left(\dfrac{p_1 - p_2}{2(1-y)} + \dfrac{y}{2} \right) & for \quad p_1 - p_2 \in I_3^1 \\ 0 & for \quad p_1 - p_2 \in I_4^1 \end{cases}$$

The profit function is not concave in prices and it could exhibit different configurations. In particular, it may exhibit one or two local maxima. Depending on the values of y two cases may arise, one in which the global optimum belongs to region I_2^1, and another in which the global optimum belongs to I_3^1.

3.1.- Equilibrium in Region I_2^1

For this prices interval two indifferent consumers exist.Computing the first order conditions for the profit functions in I_2^1, we obtain:

$$p_1^{N1} = \frac{1}{3}y(1-y)(4h-1) \qquad \text{and} \qquad B_1(p_1^{N1}, p_2^{N1}) = \frac{1}{18}y(1-y)(4h-1)^2$$

$$p_2^{N1} = \frac{1}{3}y(1-y)(2h+1) \qquad \text{and} \quad B_2(p_1^{N1}, p_2^{N1}) = \frac{1}{18}y(1-y)(2h+1)^2$$

Proposition 1: The pair $(p_1^{N1}(0,y,h), p_2^{N1}(0,y,h))$ constitutes a Nash equilibrium if and only if $(h,y) \in R_1$, where: $R_1 = \left\{ (h,y) \;\middle|\; h \geq h_{12}, where, h_{12}(y) = \dfrac{1+2\sqrt{y}}{4-\sqrt{y}} \right\}$

Proof: See Appendix

Proposition 2: There exists a subgame perfect price-location equilibrium if and only if $h \geq 0{,}733$ and then, this equilibrium is unique and is given by:

$$y^N = \frac{1}{2}, \quad p_1^N(0, y^N, h) = \frac{4h-1}{12}, \quad p_2^N(0, y^N, h) = \frac{2h+1}{12}$$

Proof: See Apendix.

3.2.- Equilibrium in Region I_3^1

For this prices interval only one indifferent consumer exists.Computing the first order conditions for the profit functions in I_3^1, we obtain:

$$p_1^{N2} = \frac{1}{3}(1-y)(2h+y) \qquad \text{and} \qquad B_1(p_1^{N2}, p_2^{N2}) = \frac{1}{18}(1-y)(2h+y)^2$$

$$p_2^{N2} = \frac{1}{3}(1-y)(4h-y) \qquad \text{and} \qquad B_2(p_1^{N2}, p_2^{N2}) = \frac{1}{18}(1-y)(4h-y)^2$$

Proposition 3: The pair $(p_1^{N2}(0,y,h), p_2^{N2}(0,y,h))$ constitutes a Nash equilibrium if and only if $(h,y) \in R_2$, Where: $R_2 = \left\{(h,y) \middle| \; h \le h_{21}, where, h_{21} = \frac{y(5+y)}{2(2y+1)}\right\}$

Proof: See Appendix.

Corollary: if $h < 0{,}733$, there is no price equilibrium in region *R* defined by:
$R = \{(h,y)/h_{12}(y) < h < h_{21}(y)\}$

Proof: *R* is the intersection of the two complements of regions R_1 and R_2. Figure 6 combines the two equilibrium regions depicted in the previous two figures. As can be seen from the graph, there is only a narrow strip where no equilibrium exists.□

In figure 4, the equilibrium area for region I_2^1 is depicted, all points in the shaded area are possible equilibria. Note that for $h \ge 0{,}733$ there exists a price equilibrium for every possible location of firm 2. When we look at the optimal location of firm 2 in region I_2^1, when there are two indifferent consumers, we find that firm 2 benefits from moving away from firm 1 and locating at the opposite boundary of the market segment where m_1 exists. The result is equivalent to what we obtain when we consider the complete circular model: there are two indifferent consumers and firms locate opposite to each other and equidistant to the two indifferent consumers. However, in our model the competitive situation of the two firms is not the same, since we have had to fix the location of firm 1 in order to reduce the number of parameters. Firm 1 is located at the edge of the non-residential area, and therefore this side of its potential market is restricted, as a result, firm 2 may charge a larger price and obtain larger profits than firm 1.

Figure 5 shows the equilibrium area for region I_3^1. All points belonging to the shaded area (above the line $h = y$), are equilibrium candidates.
When we look at the optimal location of firm 2 in this region, we find that $\frac{\partial B_2}{\partial y} = \frac{1}{9}(4h-y) > 0$ and therefore, firm 2 will tend to move away from firm 1. In region I_3^1 there is only one indifferent consumer and the market resembles the linear city case, given that firm 1 is fixed at 0, firm 2 will choose to locate at the other market extreme.

If we look at the prices differences of the two equilibrium regions we obtain: $D^{N1} = p_1^{N1} - p_2^{N1} = \frac{-2y(1-y)(1-h)}{3} < 0$, $D^{N2} = p_1^{N2} - p_2^{N2} = \frac{-2y(1-y)(h-y)}{3} < 0$,
We can see that, in both cases, the price of firm 1 is smaller than that of firm 2, this can be explained, as mentioned above, in terms of market configuration since firm 2 has more potential customers than firm 1.

Also, when we compare the prices differences in both regions, D^{N1} and D^{N2}, we find that: $D^{N1} - D^{N2} = \frac{-2y(1-y)(h(1-y)+2y)}{3} > 0$ for all (h,y), so $D^{N1} > D^{N2}$. This implies that the intensity of competition is stronger in the second case than in the first one. This is not surprising since market size is smaller in the second case than in the first and this must induce stronger competition. This can explain why the equilibrium region of the second case is smaller than the first one.

Convex Transportation Costs

We have also studied the convex transportation costs function: $C(d) = kd^2$. This function was shown to be strategically equivalent to $C(d_i) = k(d_i - d_i^2)$ when the whole circle is being considered (see De Frutos et al, 1999, 2001). However, when the market is restricted to be a semicircle this result breaks down. We find that the structure of the demand function varies substantially from the concave to the convex case, as we will expose now:

Let $C(d_i) = kd_i^2$ and that $l = k = 1$, $x < y$. In this case, and for the whole circular market three types of indifferent consumers may be obtained: $n_1 \in \left[0, x+\frac{1}{2}\right]$, $n_2 \in \left[x+\frac{1}{2}, y+\frac{1}{2}\right]$ and $n_3 \in \left[y+\frac{1}{2}, 1\right]$. However, when we restrict the length of the market to h, we obtain the following cases:

Case 1: $h \in \left[y, x+\frac{1}{2}\right]$. Then the indifferent consumers n_2, n_3 do not belong to the market, therefore there is only one indifferent consumer, n_1. In this case we have a perfect equilibrium. This result is equivalent to Hotelling's linear model with quadratic transportation costs, as studied by D' Aspremont et al. (1979); where firms choose maximum differentiation.

Case 2: $h \in \left[x+\frac{1}{2}, y+\frac{1}{2}\right]$. Then there exist two possible indifferent consumers n_1 and n_2 and the demand function is piecewise linear with four different domains ([1] the whole market is for firm1,[2] market boundaries are determined by the indifferent consumers n_1 and n_2, [3] market boundaries are determined by only one indifferent consumer, n_1 or n_2. [4] the whole market is for firm 2). In this case, there is no price equilibrium for every possible firms' locations and every value of h.

Case 3: $h \in \left[y+\frac{1}{2}, 1\right]$. Then either two indifferent consumers will exist simultaneously n_1 and n_2 or n_2 and n_3, or just n_2 and the demand function is piecewise linear with five different domains ([1] the whole market is for firm1,[2] market boundaries are determined by the indifferent consumers n_1 and n_2, [3] market boundaries are determined by only one indifferent consumer, n_2. [4] market boundaries are determined by the indifferent consumers n_2 and n_3, [5]the whole market is for firm 2). As in case 2, there is no perfect equilibrium.

4.- Conclusions

In this article we propose a circular model of spatial competition in which the market is restricted in order to allow for a non-residential area. Aside from this discontinuity in the market, we make standard assumptions and use the two linear quadratic functions (concave and convex cases) that have been proven to ensure the existence of price equilibrium in the circular model.

We find that, unlike when the whole circle is considered, this two transportation costs functions are not strategically equivalent. When concave transportation costs are assumed, we obtain that provided the regulator chooses the size of the market segment to be greater than approximately $\frac{3}{4}$ $(h \geq 0{,}733)$, there is a subgame perfect price-location equilibrium and this equilibrium is unique. On the other hand, if market size is chosen to be less than $\frac{3}{4}$, for certain values of h and firms locations, there is a narrow strip where no price equilibrium may exist. As usual, equilibrium failure is due to the non-concavities exhibited by profit functions.

When convex transportation costs are assumed, perfect equilibrium can only be obtained for $h \in \left[y, x + \frac{1}{2}\right]$. This case is equivalent to Hotellings' linear model.

5.-Appendix

Proof of Proposition 1:

In order for (p_1^{N1}, p_2^{N1}) to be a price equilibrium, the following conditions must be met:

(i) $p_1^{N1} - p_2^{N1} \in I_2^1$

(ii) $B_1(p_1^{N1}, p_2^{N1}) \geq B_1(p_1, p_2^{N1}) \forall p_1$

(iii) $B_2(p_1^{N1}, p_2^{N1}) \geq B_2(p_1^{N1}, p_2) \forall p_2$

Condition (i) Implies that:

a) $-y(1-y) \leq p_1^{N1} - p_2^{N1}$ Which is always true, and

b) $p_1^{N1} - p_2^{N1} \leq y(2h-1-y)$

Which holds if and only if: $h \geq h_{11}$, where $h_{11} = \dfrac{(1+5y)}{2(2+y)}$

So (p_1^{N1}, p_2^{N1}) could be a Nash price equilibrium if *y* and *h* belong to the set:

$R_{11} = \{(h,y) \mid h \geq h_{11}\}$

In order to verify condition (ii) we have to check, for p_2^{N1} given, what is the maximum p_1^{**} for $B_1(p_1^{**}, p_2^{N1})$ in I_3^1 which is reached at:

a) $p_1^{**} = p_2^{N1} + y(2h-y-1)$, in this case we have $B_1\left(p_1^{N1}, p_2^{N1}\right) > B_1\left(p_1^{**}, p_2^{N1}\right)$

b) $p_1^{**} = \dfrac{1}{3} y(1-y)(2h+1)$, Where $p_1^{**} > p_2^{N1} + y(2h-y-1)$, and the profit function for firm 1 is given by: $B_1(p_1^{**}, p_2^{N1}) = \dfrac{1}{18}(1-y)[y(2+h)\ ]^2$, some simple calculation then show that $B_1\left(p_1^{N1}, p_2^{N1}\right) \geq B_1\left(p_1^{**}, p_2^{N1}\right)$ for all (h,y) such that $h \geq h_{12}$, where $h_{12} = \dfrac{1+2\sqrt{y}}{4-\sqrt{y}}$.

In order to verify condition (iii) we have to carry out the same analysis for firm 2:

Given p_1^{N1}, the profit function of firm 2, $B_2\left(p_1^{N1}, p_2\right)$ has only one maximum for $p_2 \in [o, +\infty)$, that is reached at $p_2^{**} = p_2^{N1}$, thus we have $B_2\left(p_1^{N1}, p_2^{N1}\right) \geq B_2\left(p_1^{N1}, p_2\right) \forall p_2$.

Therefore, combining conditions i, ii, iii, we obtain that (p_1^{N1}, p_2^{N1}) is a Nash price equilibrium for $h \geq h_{11}$ and $h \geq h_{12}$.

However, we also verify that $h_{12} > h_{11} > y$, (See figure 3), therefore (p_1^{N1}, p_2^{N1}) is a Nash price equilibrium in the region R_1 defined as:

$R_1 = \{(h, y) | \ h \geq h_{12}\}$ (See figure 4)

Proof of Proposition 2:

From proposition 1, there exists a price equilibrium if $h \geq h_{12}(y)$, however, simple calculations show that $h_{12}(y)$ is increasing $\left(\frac{\partial h_{12}}{\partial y} = \frac{9}{2\sqrt{y}(4-\sqrt{y})^2} > 0\right)$ and reaches a maximum for $y = \frac{1}{2}$ (see figure 4) and $h_{12}(\frac{1}{2}) = \frac{\sqrt{2}+2}{4\sqrt{2}-1} \approx 0{,}733$, therefore $\forall h \geq 0{,}733$ there exist a price equilibrium for any firms location.

If we look at the optimal location of firm 2 (given that the location of firm 1 is fixed at 0) we find that $\frac{\partial B_2}{\partial y} = 0 \quad \Rightarrow y = y^N = \frac{1}{2}$. Substituting y by $\frac{1}{2}$ in the expressions of

$p_1^{N1}(0, y, h)$ and $p_2^{N1}(0, y, h)$ we obtain $p_1^N(0, y^N, h) = \frac{4h-1}{12}$, $p_2^N(0, y^N, h) = \frac{2h+1}{12}$.□

Proof of Proposition 3:

In order for (p_1^{N2}, p_2^{N2}) to be a price equilibrium, the following conditions must be met:

(i') $p_1^{N2} - p_2^{N2} \in I_3^1$

(ii') $B_1(p_1^{N2}, p_2^{N2}) \geq B_1(p_1, p_2^{N2}) \forall p_1$

(iii') $B_2(p_1^{N2}, p_2^{N2}) \geq B_2(p_1^{N2}, p_2) \forall p_2$

Condition (i') implies that

a) $y(2h-1-y) \leq p_1^{N2} - p_2^{N2}$ Which holds if and only if, $h \leq h_{21}$ where $h_{21} = \frac{y(5+y)}{2(2y+1)}$

b) $p_1^{N2} - p_2^{N2} \leq y(1-y)$ Which is always true.

So (p_1^{N2}, p_2^{N2}) could be a Nash price equilibrium if *y* and *h* belong to the set:

$R_{21} = \{(h, y) | \ h \leq h_{21}\}$

In order to verify condition (ii'),

Given p_2^{N2}, and considering R_{21}, we verify that $B_1(p_1, p_2^{N2})$ admits a global maximum in $p_1 = p_1^{N2}$.

condition (iii’),

Similarly, given p_1^{N2}, and considering R_{21}, we verify that $B_2(p_1^{N2}, p_2)$ admits a global maximum in $p_2 = p_2^{N2}$.

Finally, (p_1^{N2}, p_2^{N2}) is a Nash price equilibrium in region R_2 defined as:

$R_2 = \{(h, y) | y \leq h \leq h_{21}\}$ (See figure 5)

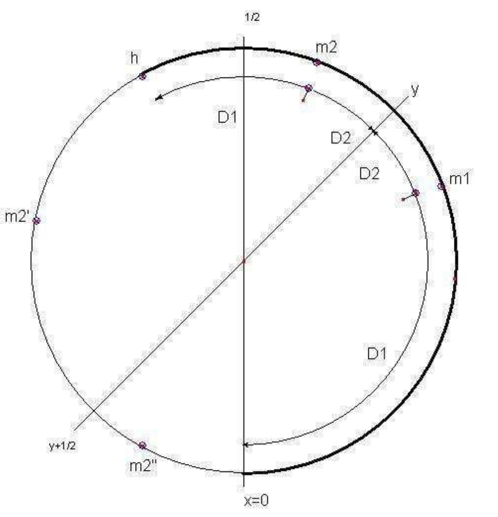

Figure 1

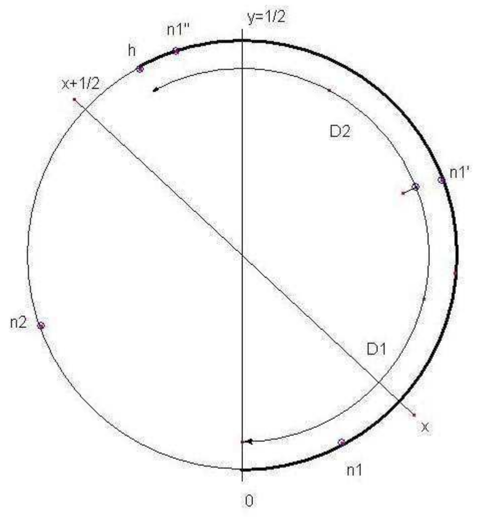

Figure 2

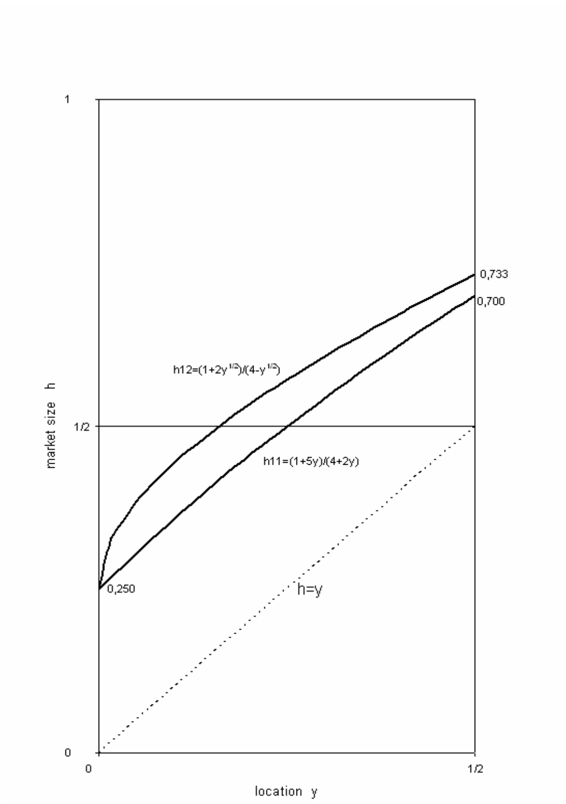

Figure 3

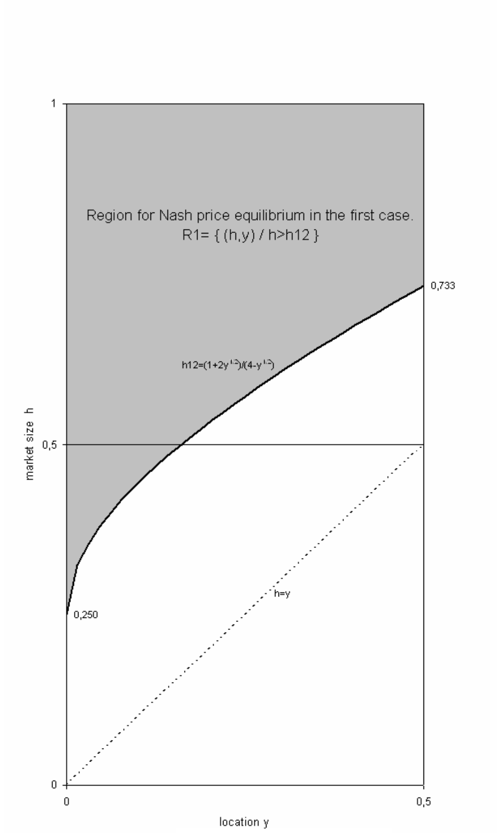

Figure 4

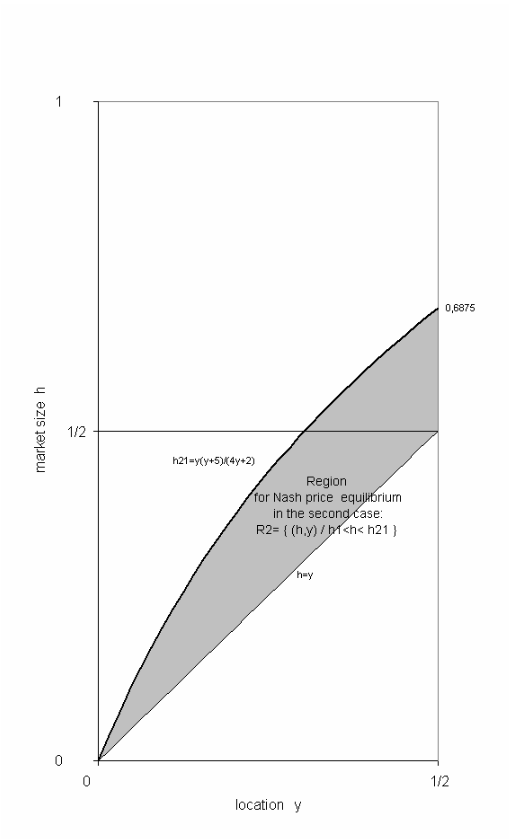

Figure 5

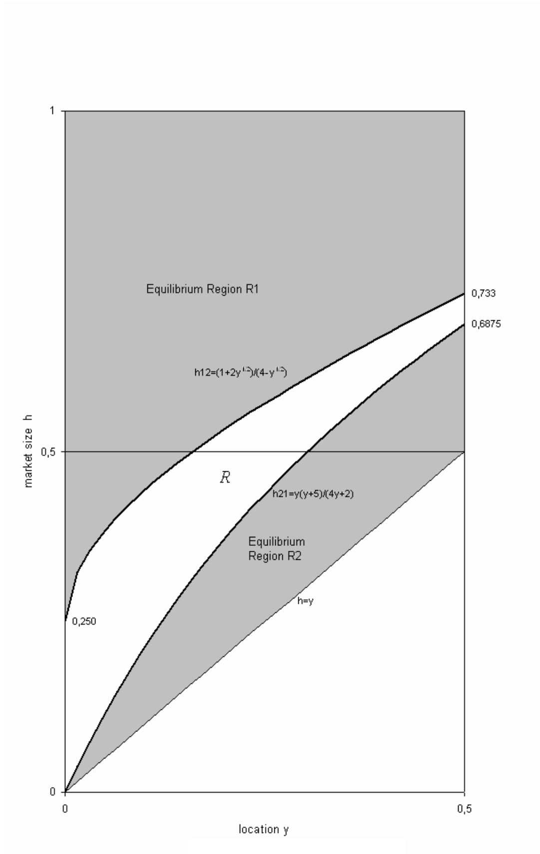

Figure 6

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