E C O N O M I C S B U L L E T I N

The stability of intergenerational cooperation in altruistic families

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Abstract

This paper analyses the stability of bargaining solutions in a family consisting of two parents and one adult child, by developing a non-cooperative family game. Assuming different bargaining powers between parents and the child, we find that the greater bargaining power of the parents allows them to take greater gains from the cooperation, and reduce the incentives to deviate from the cooperative agreement. The presence of altruism between the players will significantly reduce the probability that there will be incentives to break the cooperative agreement. A higher level of altruism increases the stability of cooperation, and will overcome the contrary effect of other factors.

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1. Introduction

The application of bilateral bargaining models represents an important advance in the study of family decision-making. One of the essential features of these bargaining models is that family demand does not depend solely on total family resources, but also on those controlled by each member individually. This implies that the results achieved depend on whatever is the threat point or status quo of the bargaining process, for example, the divorce situation. In this way, the family bargaining models reflected in the literature consider the decisions made by individuals to be the result of an explicitly defined bargaining solution (Manser and Brown, 1980; McElroy and Horney, 1981; Chen and Woolley, 2001; Andaluz and Molina, 2007a, 2007b).

Nevertheless, divorce does not represent the only possible threat point in a process of this nature. In this sense, a non-cooperative equilibrium could equally be the threat point in the bargaining process, in such a way that the repeated interaction between the agents over time can tacitly lead to efficient results (Lundberg and Pollak, 1993, 1994). More specifically, and in accordance with the folk theorem, a Pareto-efficient solution can be derived as a Nash equilibrium in a repeated game, always provided that there is some strategy which penalizes all deviations from the efficient solution. Therefore, Pareto-optimum results can arise as repeated game solutions. However, it has also recently been shown that the achievement of private gains on the part of each spouse, combined with the limitations in compromising the future behavior of both spouses, can give rise to decisions that are no longer Pareto-efficient (Lundberg and Pollak, 2003).

In this context, the objective of this paper is to analyze the stability of bargaining solutions by developing a supergame where the players are two spouses and an adult child. Under a particular specification of the individual preferences, and given the existence of a family public good, we analyze the effects of altruism and bargaining power on the maintenance of the agreement derived from the Nash bargaining solution.

2. The model

We develop a repeated game in which the three members of a family can contribute voluntarily to the supply of one household public good. After assuming that we do not know the moment at which the dissolution of the family takes place, the objective of each agent is to maximize the discounted value of a flow of utilities $\sum_{t=1}^{\infty}\delta^{t-1}W_j(U_1,U_2,U_3);\ (j=1,2,3)$, where δ denotes the discount factor, common to all agents, and $W_j(U_1,U_2,U_3)$ indicates the welfare function of agent j, which itself depends on the agent's utility level, U_j and on that of the other agents. Formally, each player has a welfare function of the type $W_j = U_j + s\,(U_k+U_h)$ with $s \in [0, 1]$ denoting the degree of altruism of the players, which it assumes, for simplification purposes, to be common to all agents ("caring preferences"). The utility of each agent is represented by a function of the type $U_j = U_j(C_j, l_j, Q, z);\ j=1,2,3$, where C_j represents a Hicksian composite good, whose price is unitary, l_j denotes the quantity of leisure consumed by agent j, Q is a family public good whose technology is represented by a Cobb-Douglas production function, whose inputs are the hours dedicated to production by each agent, and z is a vector of subjective variables which determine the way in which each agent values his/her consumption. Specifically, the utilities are given by $U_j = C_j + Lnl_j + \alpha Lnh_j + (1-\alpha)L(h_h h_k)\ \forall j \neq h \neq k;\ j,h,k = 1,2,3$, and the production function is $Q = h_1 h_2 h_3$, with h_j being the time dedicated to production of the family good by agent j, and α being a parameter indicating the subjective valuation of the contribution of agent j to the given good.

In the development of the non-cooperative equilibrium, each agent decides, given the decisions made by the other players, both the consumption of the private good and the contribution to the household public good. In this case, the solution of the one-shot game is given by the Cournot-Nash equilibrium:

$$\underset{(C_i,l_i,h_i)}{Max} \quad W_i = u_i + s(u_j + u_k) = C_i + \ln l_i + \alpha \ln h_i + (1-\alpha)\ln h_j^I h_k^I +$$
$$+ s\left(C_j + \ln l_j + \alpha \ln h_j + (1-\alpha)\ln h_i^I h_k^I + C_k + \ln l_k + \alpha \ln h_k + (1-\alpha)\ln h_i^I h_j^I\right)$$

s.t. *i)* $C_i + \omega_i(l_i + h_i) \le Y_i$

ii) $C_i, l_i, h_i \ge 0$

iii) $Y_i = y_i + \omega_i T$

with $i,j,k=(1,2,3)$, $i\neq j\neq k$, and T being the total time available for leisure, family production and work outside the home, y_i being the non-labour income and w_i being the individual hourly wage. From the first order conditions, we can obtain both the Hicksian consumption demands, leisure time, and the individual contribution to the family public good, with the indirect utility functions being:

$$W_1^I = Y_1 + s(Y_2 + Y_3) + \alpha(2s-1)\ln\omega_1 + \alpha\ln\omega_2\omega_3 +$$
$$+ (1+2s)\left((2-\alpha)\ln[\alpha + 2s(1-\alpha)] - 1 - \alpha - 2s(1-\alpha) - \ln\omega_1\omega_2\omega_3\right)$$

$$W_2^I = Y_2 + s(Y_1 + Y_3) + \alpha(2s-1)\ln\omega_2 + \alpha\ln\omega_1\omega_3 +$$
$$+ (1+2s)\left((2-\alpha)\ln[\alpha + 2s(1-\alpha)] - 1 - \alpha - 2s(1-\alpha) - \ln\omega_1\omega_2\omega_3\right)$$

We should note that the repetition of the game gives rise to multiple equilibria, some of which must represent Pareto-efficient solutions. Indeed, all agents may implicitly create some strategy that avoids all possible deviation from an optimal solution, and which guarantees the achievement of Pareto-efficiency as a Nash equilibrium in the one-shot game. One of these possible strategies consists of penalizing the agent who unilaterally deviates from the agreement. More specifically, we adopt a relatively simple, but nevertheless commonly employed, punishment scheme, namely the trigger strategy, according to which the quantities of private and public good revert forever to non-cooperative levels, following a deviation from the efficient solution on the part of one of the agents. The threat of punishment, through the return to the non-cooperative solution, is credible and guarantees the stability of solutions which are more efficient than the Cournot-Nash equilibrium.

Furthermore, and again for the sake of simplicity, we consider the case of stationary trajectories, arguing that a stationary trajectory is sustainable in a sub-game perfect equilibrium if, for all *j,* the following conditions are satisfied $W_j^C - W_j^N \ge 0$ and $\frac{W_j^C}{(1-\delta)} \ge W_j^{NC} + \delta\frac{W_j^N}{(1-\delta)}$, where W_j^C and W_j^{NC} denote the levels of welfare obtained by agent j in the Pareto-efficient solution derived from the Nash-bargaining agreement, and in the deviation equilibrium, respectively. Note that the second of these restrictions can be expressed in the following form $\delta \ge \frac{W_j^{NC} - W_j^C}{W_j^{NC} - W_j^N} \equiv \bar{\delta}_j$, where $\bar{\delta}_j$ is the critical discount factor of individual j. Thus, the stability of the optimal solution requires that the discount factor, common to all individuals, is greater than or equal to the corresponding critical factor. In other words, the higher the value of the critical factor, the lower the stability of the Pareto-efficient equilibrium, given that the number of discount factors which guarantee the stability of the agreement will be less.

Let us suppose that there is a bargaining process according to which the agents choose the generalized Nash-bargaining solution. That is to say, they choose the stationary trajectory of amounts that maximize the product of the utilities normalized by the levels associated with the non-cooperative equilibrium. Formally:

$$\underset{(C_1,C_2,C_3,l_1,l_2,l_3,h_1,h_2,h_3)}{\text{Max}} \quad N=\left[u_1+s(u_2+u_3)-W_1^I\right]^{\beta}\cdot\left[u_2+s(u_1+u_3)-W_2^I\right]^{\beta}\cdot\left[u_3+s(u_1+u_2)-W_3^I\right]^{(1\text{-}\beta)}$$

$$\text{s.a.} \quad \text{i) } C_1+C_2+C_3+\omega_1(l_1+h_1)+\omega_2(l_2+h_2)+\omega_3(l_3+h_3)\le Y_1+Y_2+Y_3$$

$$\text{ii) } C_1,C_2,C_3,l_1,l_2,l_3,h_1,h_2,h_3\ge 0$$

$$\text{iii) } Y_i=y_i+\omega_i T \ ,\ i=(1,2,3)$$

where $\beta \in$ *[0,1]* is a parameter which represents the bargaining power of the parents and thus, *1-*β indicates the bargaining power of the child. The resolution gives rise to levels of consumption of leisure and time dedicated to the family good and private consumption.

Under symmetric preferences, the behaviour of the agent who deviates is the same for all players, with the problem of maximization being:

$$\underset{(C_i,l_i,h_i)}{Max} \quad W_i=C_i+\ln l_i+\alpha\ln h_i+(1-\alpha)\ln h_j^C h_k^C +$$

$$+s\left(C_j^C+\ln l_j^C+\alpha\ln h_j^C+(1-\alpha)\ln h_i h_k^C+C_k^C+\ln l_k^C+\alpha\ln h_k^C+(1-\alpha)\ln h_i h_j^C\right)$$

$$s.t. \quad i)\ C_i+C_j^C+C_k^C+\omega_i(l_i+h_i)+\omega_j\left(l_j^C+h_j^C\right)+\omega_3\left(l_3^C+h_3^C\right)\le Y_i+Y_j+Y_3$$

$$ii)\ C_i,C_j^C,C_3^C,l_i,l_j^C,l_3^C,h_i,h_j^C,h_3^C\ge 0$$

$$iii)\ Y_i=y_i+\omega_i T$$

The resolution gives rise to levels of private consumption, and provision of the family public good and leisure time.

3. Stability of cooperation

In accordance with the trigger strategy, the condition that the discount factor will be less or equal to the corresponding critical factor, implies that, for each member of the family, the surplus associated with deviation does not compensate for the fact that it will be impossible to return to cooperation in the future. The gains of cooperation for the parents and the child are given as:

$$W_i^C-W_i^I=C_i^C+s\left(C_j^C+C_k^C\right)-Y_i-s\left(Y_j+Y_k\right)+\left(1+2s\right)\left(1+\alpha+2s\left(1-\alpha\right)\right)+$$

$$+\left(1+2s\right)\left(2-\alpha\right)\left(ln\left[2-\alpha\right]-ln\left[\alpha+2s\left(1-\alpha\right)\right]\right)i\ne j\ne k;i,j,k=1,2,3$$

In Figures 1 and 2, we can observe how a greater bargaining power of the parents (a greater β) allows them to take a greater profit from cooperation for all values of α and s . By contrast, for the case of the adult child, Figures 3 and 4 show that a greater bargaining power of the parents leads to a lesser bargaining power of the child, and consequently a lesser profit for the child from the agreement.

We now proceed with the evolution of the critical discount factor with respect to bargaining power. In order to calculate the threshold discount factor, the numerator is:

$$W_i^{NC}-W_i^C=2(1-\alpha)(1-s)-(\alpha+2s(1-\alpha))(\ln[2-\alpha]-\ln[\alpha+2s(1-\alpha)])$$

whereas the denominator is given by the difference between the welfare associated with equilibrium in the deviation, and the welfare derived from the non-cooperative equilibrium, for both parents and child, respectively:

$$W_i^{NC} - W_i^{I} = \frac{2(1-\alpha)(1-s)(1-\beta(2+6s))}{(1+\beta)} - $$
$$- 2s\,\frac{(\alpha - 6\beta + 4\alpha\beta + 2(1-5\beta-\alpha(1-2\beta)))}{(1+\beta)}\left(\ln[2-\alpha] - \ln[\alpha + 2s(1-\alpha)]\right), i = 1,2$$

$$W_3^{NC} - W_3^{I} = 2\,\frac{2(1-\alpha)(1-s)(2\beta - 3s(1-\beta)-1)}{(1+\beta)} + $$
$$+ 2s\,\frac{((3+5s)(1-\beta) - 2\beta s + \alpha((\beta-2)(1+s)+3\beta s))}{(1+\beta)}\left(\ln[2-\alpha] - \ln[\alpha + 2s(1-\alpha)]\right)$$

Figure 1 ($\beta = 0.2, (i = 1,2)$)

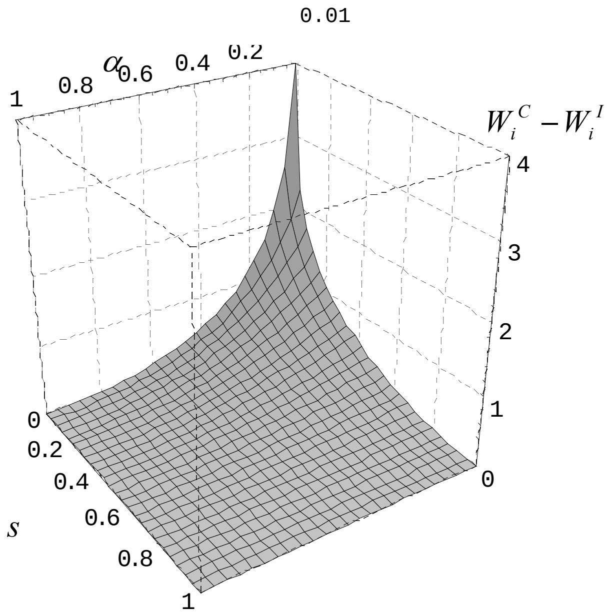


Figure 2 ($\beta = 0.9, (i = 1,2)$)

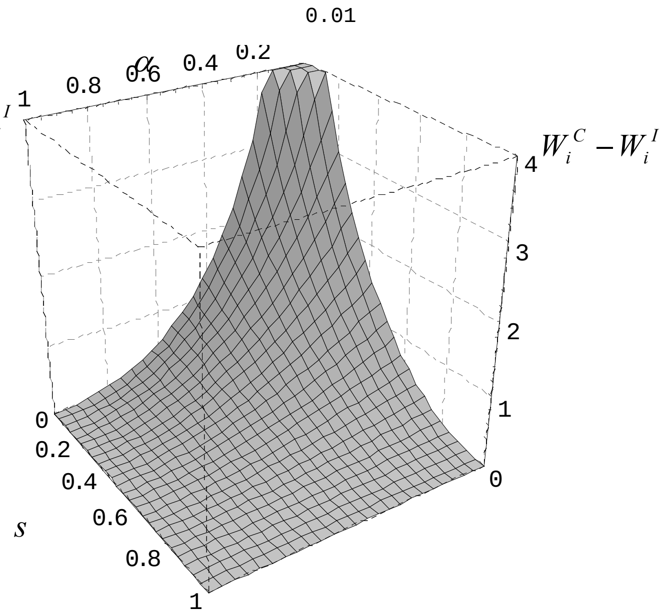


Figure 3 ($\beta = 0.2$)

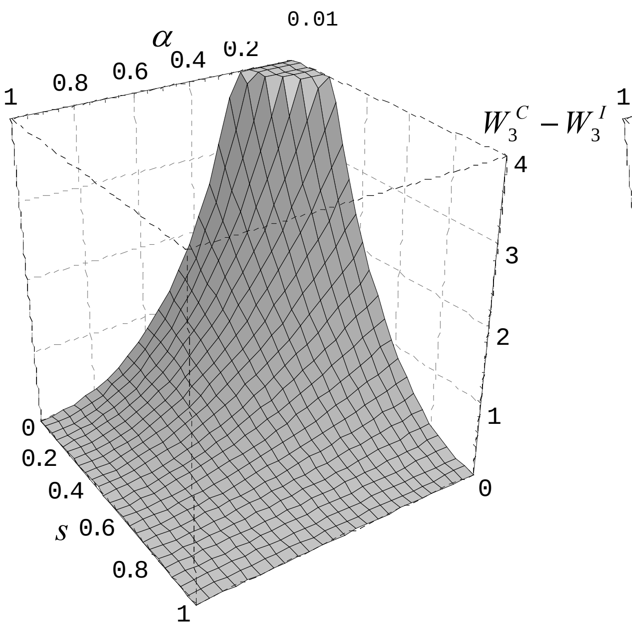
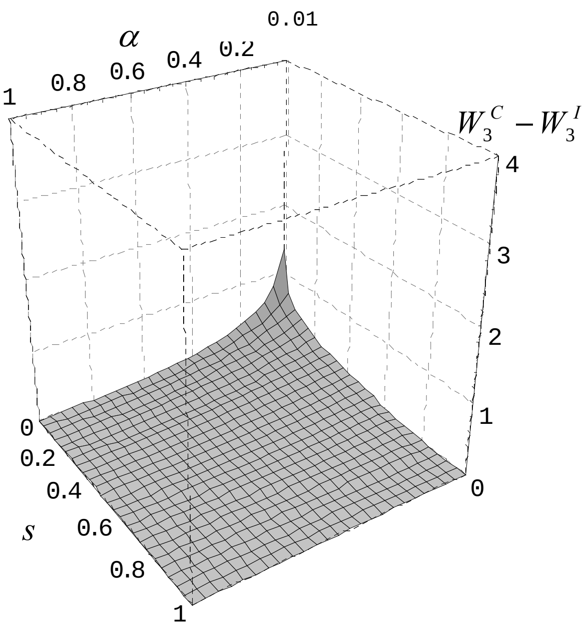


Figure 4 ($\beta = 0.9$)



In Figures 5 and 6 we can see that a greater bargaining power significantly reduces the critical discount factor of the parents, for the same values of α and s. Clearly, there is a difference between the maximum values, around 0.7 for $\beta = 0.2$, and 0.42 for $\beta = 0$. It is also observable that the slope of the surface in the case of the greater bargaining power of the parents is more pronounced, and thus, the maximum value is less and decreases faster as s increases, with this being especially clear for higher values of α.

Figure 5 ($\beta = 0.2, (i = 1,2)$)

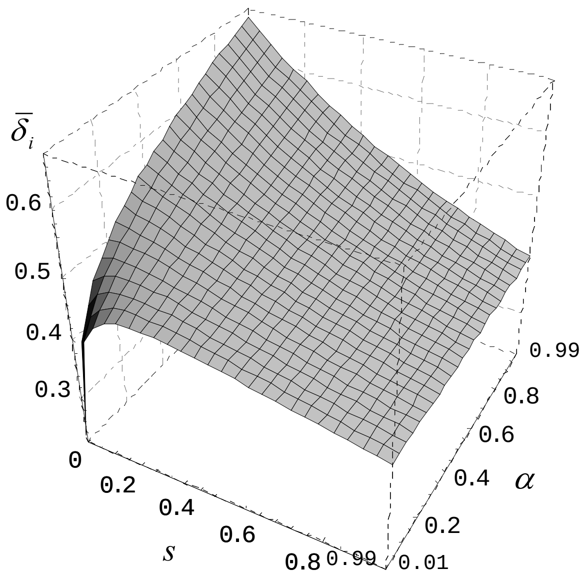
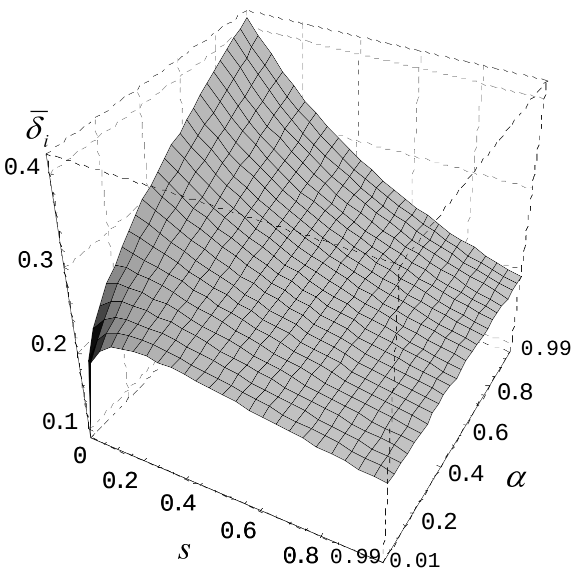


Figure 6 ($\beta = 0.9, (i = 1,2)$)



For the adult child, the opposite holds. Figures 7 and 8 show that the greater bargaining power of the parents implies a greater critical discount factor for the same values of α and s. Further, in our example, $\beta = 0.9$, the minimum value of the critical discount factor of the parents is much greater than the maximum value of that factor for the adult child, $\beta = 0.2$. Hence, a lesser sensitivity with respect to the altruism variable, s, for high values of β in the case of the parents, moves the surface and produces a gentler slope as α increases.

In summary, the greater bargaining power of the parents allows them to take greater gains from the cooperation, and reduce the incentives to deviate from the cooperative agreement. In the presence of altruism between the players, and given the maximization of individual wellbeing, a change in bargaining power in either direction will significantly affect the gains of cooperation, as well as the incentives for the adult child to deviate. In general, the existence of altruism, even if one of the agents has the greater bargaining power, will significantly reduce the values of the individual critical discount factors, thus reducing the probability that there will be incentives to break the cooperative agreement. A higher level of altruism increases the stability of cooperation, and will overcome the contrary effect of other factors.

Figure 7 ($\beta = 0.2$)

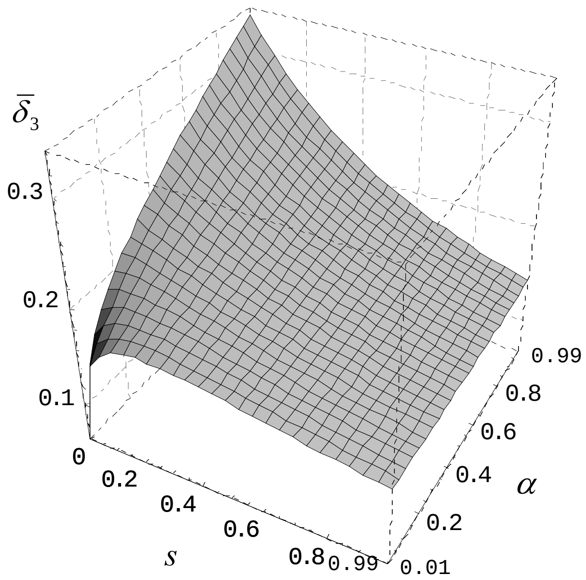
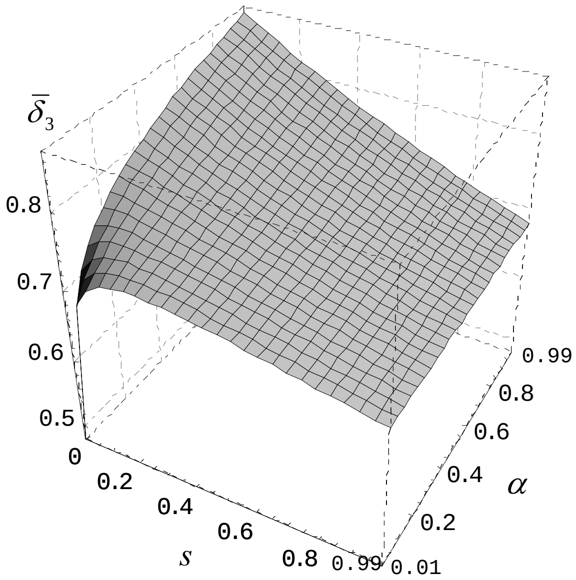


Figure 8 ($\beta = 0.9$)



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