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Luca Lambertini
Department of Economics, University of Bologna

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Platform stickiness in a spatial voting model*

Luca Lambertini
Dipartimento di Scienze Economiche, Università di Bologna
Strada Maggiore 45, 40125 Bologna, Italy
fax +39-051-2092664, phone +39-051-2092600; lamberti@spbo.unibo.it
ENCORE, University of Amsterdam
Roetersstraat 11, WB1018 Amsterdam, The Netherlands
RCEA (Rimini Centre for Economic Analysis)
Via Patara 3, 47900 Rimini, Italy

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Abstract

The spatial voting approach is extended to account for the existence of a loyalty effect driving the choice of parties' platforms during elections. There emerges a non-linear relationship between these variable, whereby a party sticking to its historical heritage may lose to a rival more keen to approach the position of the median voter, whose pivotal role is also investigated.

Keywords: elections, politics, political platforms
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1 Introduction

After Downs's (1957) pioneering work, a very large stream of literature has investigated spatial voting models to study parties' strategic behaviour and predict the outcome of elections. The spatial approach has been extended in several directions, to account for, e.g., stochastic voting (Anderson *et al.*, 1994; Adams, 1999; Patty, 2005; Schofield, 2006); parties' (or candidates') incomplete information or bounded rationality (Kollman *et al.*, 1992; Page *et al.*, 1993; Kollman *et al.*, 1997); multidimensional platforms with valence

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issues (Ansolabehere and Snyder, 2000); the existence of majority-rule equilibria in spatial voting games (Bartholdi *et al.*, 1991).[1] A major subset of this literature stems from Wittman's (1977, 1983) papers, where electoral behaviour is shaped under the assumption that candidates have policy preferences as well as an interest in winning elections *per se*. More specifically, in Wittman's analysis a candidate maximises a function resulting from the sum of the expected utility of implementing the set of preferred policies if s/he wins and the expected utility generated by the opponent's preferred policies in the opposite case.

The aim of the present paper is to extend the spatial approach in order to investigate the bearings of parties' traditional platforms, as inherited from history, on their strategies and ultimately on the outcome of electoral competition. When I refer to the presence of traditional platforms, I mean what follows. Any given party may take from its past history some essential features conditioning its views on relevant policy issues, such as monetary and fiscal policy, welfare, foreign policy, etc., so much so that they end up shaping to a large extent the choice of such a party's platform during the elections. This aspect, close in spirit but not equivalent to Wittman's approach (and its follow-ups),[2] to the best of my knowledge has been overlooked thus far and may in fact play a relevant role, as it can be easily ascertained on the basis of casual observation.

For instance, this clearly appears to be the case if one looks at the political elections held in Italy in April 2006, with particular regard to behaviour of the center-left during the last two weeks of the electoral campaign, when Mr Romano Prodi's public speeches and declarations were increasingly stressing the need to increase taxation (or introduce new taxes) on large patrimonies and high incomes so as to make the distribution of income in Italy less unfair. How much *large* and *high* these were supposed to be, remained a vague concept until the new government produced the new fiscal law. Yet, these declaration of intents produced the effect (predictable but clearly - and quite strangely indeed - unforeseen by the center-left itself) of decreasing their margin of consensus over the center-right coalition to such an extent that the outcome of the elections was pretty tight, being determined by a few thousand votes only. It is a widely accepted interpretation that such declarations were dictated not by a risk-loving attitude but rather from the will (or need) to satisfy some essential (or tradition-driven) requirements of

[1] Relevant contributions adopting a spatial approach to investigate other aspects of electoral competition are virtually uncountable. See, e.g., Weber (1997), Adams (1999), McKelvey and Patty (2006) and Huck *et al.* (2006).

[2] See Calvert (1985) and Roemer (1994), *inter alia*.

the communist components of the coalition led by Mr Prodi.

To model this issue, I propose an extension of the standard two-party spatial voting approach to explicitly account for the presence of a *loyalty effect* in the objective function of each party, in addition to the extent of electoral consensus. The main results of the ensuing analysis can be summarised as follows. If the loyalty effect is low enough, both parties choose the median voter's preferred platform; as a consequence, the outcome of the elections is indeterminate. Otherwise, the outcome of the elections is determined by a non-trivial interplay between the loyalty effect and the parties' locations around the median voter. This interplay also entails that, whenever at least one party locates apart from the median voter, the latter will be pivotal, unlike what happens in the usual approach where both parties locate in correspondence of the median voter's preferred platform.

2 The model

Examine the following two-party electoral competition game. Parties 1 and 2 choose their respective electoral (or political) platforms in $[0,1]$, with $x_1 \in [0,1/2]$ and $x_2 \in [1/2,1]$. The reason for this assumption is to avoid *perverse* cases with the left choosing a platform on the right hand side of the preference spectrum (and conversely for the right).[3] The unit interval defines the support of the distribution of voters' electoral preferences, which I assume to be uniform. for the sake of simplicity I also assume that all voters indeed vote (either for party 1 or for party 2), so that the total amount of votes is equal to one. The generic voter, located at point $m \in [0,1]$, votes for the party (or the candidate) whose political platform x_i maximises $U = s - t(m - x_i)^2$, where $s > 0$ is the gross value that any individual associates with the fact itself of voting, while parameter $t > 0$ measures the disutility of voting for a party whose platform differs form the voter's ideal one.[4] The voter who is indifferent between candidate 1 and candidate 2 is identified by the equation $s - t(\widetilde{m} - x_1)^2 = s - t(\widetilde{m} - x_2)^2$, which can be solved to find $\widetilde{m} = (x_1 + x_2)/2$, entailing that the amount of voters located in (x_1, x_2) will split evenly. Using the above expression, one may define the amount of votes v_i accruing to each party, as follows:

[3]Credibility issues being absent, the unrestricted case where $x_i \in [0,1]$, $i = 1,2$, could be considered. The consequences of allowing unrestricted platform choices on the electoral outcome will be discussed in the remainder.

[4]The use of a linear disutility function would not entail any significant change in the qualitative conclusions of the model.

$$v_1 = \frac{x_1 + x_2}{2} \, ; \, v_2 = 1 - \frac{x_1 + x_2}{2} \, . \tag{1}$$

So far, the setup closely replicate Down's (1957). Should v_i define the objective function of party i, the voting paradox would obtain. Instead, I will pose that party (or candidate) i's objective function is defined by $O_i = v_i - b_i \left(\mu_i - x_i\right)^2$, where:

♣ $\mu_i \in [0,1]$, with $\mu_1 < \mu_2$, $\mu_1 \in [0, 1/2]$ and $\mu_2 \in [1/2, 1]$, is the platform that ideally party i would adopt if there were no incentive at all to capture the preferences of voters located far away from μ_i. An intuitive interpretation of μ_i is that it may represent a traditional platform that party i has inherited from its previous history;

♠ $b_i \left(\mu_i - x_i\right)^2$, with $b_i \geq 0$, is the cost (either real or psychological) associated with departing from the ideal/historical platform μ_i.[5] This cost component describes the *loyalty effect* felt by party i to the its historical heritage, such loyalty becoming stronger as parameter b_i increases.

The noncooperative one-shot game takes place under imperfect, symmetric and complete information. Party i must choose its electoral platform x_i in order to maximise O_i. Using $\widetilde{m}$, the objective functions can be rewritten as follows:

$$O_1 = \frac{x_1 + x_2}{2} - b_1 \left(\mu_1 - x_1\right)^2 \, ; \, O_2 = 1 - \frac{x_1 + x_2}{2} - b_2 \left(\mu_2 - x_2\right)^2 , \tag{2}$$

and the associated first order conditions are:

$$\begin{aligned} \frac{\partial O_1}{\partial x_1} &= \frac{1 + 4b_1 \left(\mu_1 - x_1\right)}{2} = 0; \\ \frac{\partial O_2}{\partial x_2} &= \frac{4b_2 \left(\mu_2 - x_2\right) - 1}{2} = 0, \end{aligned} \tag{3}$$

yielding the Nash equilibrium platforms:

$$x_1^* = \mu_1 + \frac{1}{4b_1} \, ; \, x_2^* = \mu_2 - \frac{1}{4b_2} \, . \tag{4}$$

Given that $x_1^* \leq 1/2 \leq x_2^*$ by assumption, this requires:

$$b_1 \geq \frac{1}{2\left(1 - 2\mu_1\right)} \triangleq \overline{b}_1 \, ; \, b_2 \geq \frac{1}{2\left(2\mu_2 - 1\right)} \triangleq \overline{b}_2 \, . \tag{5}$$

[5] A similar cost function has been used in the economic model describing product location or differentiation in the Hotelling (1929) vein, to design a taxation rule that a policy maker could adopt as a remedy to the excess differentiation caused by firms' profits incentives. To this regard, see Lambertini (1997).

It is easily checked that conditions $x_1^* > 0$ and $x_2^* < 1$ are met for all $\mu_1 \in [0, 1/2]$ and $\mu_2 \in [1/2, 1]$. By looking at (4), one can immediately state:

Lemma 1 $x_i^* \in [\mu_1, \mu_2]$, *with* $\partial x_1^* / \partial b_1 < 0$ *and* $\partial x_2^* / \partial b_2 > 0$.

In words, the above Lemma states that both parties find it optimal to abandon their traditional platforms and relocate towards the median voter, the more so the lower is the (psychological or real) cost associated to relocation. Consequently, the degree of differentiation between the two electoral platforms is decreasing in both b_i's. Lemma 1 implies a relevant corollary:

Corollary 2 *If* $b_i \leq \overline{b}_i$, $i = 1, 2$, *then* $x_i^* = 1/2$.

This means that if party i's cost of relocating away from the traditional is sufficiently low, then this party will indeed choose the platform preferred by the median voter. Given that, in general, b_i is not symmetric across parties/candidates, this may well hold for one but not for the other.

Given that if $b_i \leq \overline{b}_i$ for both i, then both parties choose the platform preferred by the median voter. Corollary 2 has the following straightforward implication:

Proposition 3 *The condition* $\left\{ b_1 \leq \overline{b}_1, b_2 \leq \overline{b}_2 \right\}$ *is sufficient to make the outcome of the elections undetermined.*

Note, however, that indeterminacy may also arise for $\left\{ b_1 > \overline{b}_1, b_2 > \overline{b}_2 \right\}$, in correspondence of any symmetric platform pair $x_1^* = 1 - x_2^*$. Using (4), the following claim can be easily checked:

Proposition 4 *In the region where* $\left\{ b_1 > \overline{b}_1, b_2 > \overline{b}_2 \right\}$, *the electoral outcome is undetermined for* $b_1 = b_2 / \left[1 + 4 b_2 \left(1 - \mu_1 - \mu_2 \right) \right]$, *as along this locus* $x_1^* = 1 - x_2^*$.

Now take $\left\{ b_1 > \overline{b}_1, b_2 > \overline{b}_2 \right\}$. Plugging expressions (4) into (1) one obtains the explicit electoral outcome in term of vote shares, for all $x_1^* < 1/2 < x_2^*$:

$$v_1^* = \frac{b_2 \left[1 + 4 b_1 \left(\mu_1 + \mu_2 \right) \right] - b_1}{8 b_1 b_2}; \quad (6)$$

$$v_2^* = \frac{b_1 - b_2 \left[1 + 4 b_1 \left(\mu_1 + \mu_2 - 2 \right) \right]}{8 b_1 b_2}. \quad (7)$$

To begin with, one has to look at the non-negativity requirements for expressions (6-7). First, consider $v_1^* \geq 0$:

$$v_1^* \geq 0 \Leftrightarrow b_2 + b_1 \left[4b_2 \left(\mu_1 + \mu_2\right) - 1\right] \geq 0. \tag{8}$$

This is surely true for all $b_2 \geq 1/\left[4\left(\mu_1 + \mu_2\right)\right]$, with $1/\left[4\left(\mu_1 + \mu_2\right)\right] < \overline{b}_2$ for all admissible values of μ_1 and μ_2. Secondly, take $v_2^* \geq 0$:

$$v_2^* \geq 0 \Leftrightarrow b_1 - b_2 \left[1 + 4b_1 \left(\mu_1 + \mu_2 - 2\right)\right] \geq 0 \tag{9}$$

which is surely true for all $b_1 \geq 1/\left[4\left(2 - \mu_1 - \mu_2\right)\right]$, with $1/\left[4\left(2 - \mu_1 - \mu_2\right)\right] < \overline{b}_1$ for all admissible values of μ_1 and μ_2. Hence, $v_i^* > 0$ and consequently $v_j^* < 1$ for all $b_j < \overline{b}_j$, $i \neq j$, $i, j = 1, 2$. Accordingly:

Lemma 5 $\left\{b_1 > \overline{b}_1, b_2 > \overline{b}_2\right\}$ *suffices to ensure* $\left\{v_1^* \in (0,1), v_2^* \in (0,1)\right\}$.

It is worth noting that the above Lemma essentially rules out unanimity (in favour of either party) as long as $x_i^* \neq 1/2$, $i = 1, 2$. The next step consists in establishing the condition(s) on the basis of which party 1 wins (i.e., $v_1^* > 1/2$ or, equivalently, $v_1^* > v_2^*$) iff:

$$b_2 - b_1 \left[1 + 4b_2 \left(1 - \mu_1 - \mu_2\right)\right] > 0. \tag{10}$$

Therefore, we have three alternative cases:
Case I: If $\mu_2 = 1 - \mu_1$, then we have sign$\{v_1^* - 1/2\}$ =sign$\{v_1^* - v_2^*\}$ = sign$\{b_2 - b_1\}$. This amounts to saying that, if historical platforms are symmetric around the median voter, then party 1 (respectively, 2) wins for all $b_1 < b_2$ (resp., $b_2 < b_1$).
Case II: If $\mu_2 > 1 - \mu_1$ and $b_2 \geq 1/\left[4\left(\mu_1 + \mu_2 - 1\right)\right]$, then $v_1^* > 1/2$ for all admissible values of b_1.
Case III: If either (i) $1 < \mu_1 + \mu_2$ and $b_2 < 1/\left[4\left(\mu_1 + \mu_2 - 1\right)\right]$ (which is necessary to ensure that $1 + 4b_2\left(1 - \mu_1 - \mu_2\right) > 0$), or (ii) $1 \geq \mu_1 + \mu_2$ (which suffices to ensure that $1 + 4b_2\left(1 - \mu_1 - \mu_2\right) > 0$ for all $b_2 > 0$), then $v_1^* > 1/2$ for all

$$b_1 < \frac{b_2}{1 + 4b_2\left(1 - \mu_1 - \mu_2\right)} \triangleq \widehat{b}_1 . \tag{11}$$

and conversely if the opposite inequality holds. Note that (11) is non linear in b_2 for all $\mu_2 \neq 1 - \mu_1$. In the special case where $\mu_2 = 1 - \mu_1$, (11) trivially reduces to $b_1 < b_2$.

Of course, in view of the symmetry of the model, similar conclusions would hold, *mutatis mutandis*, if one examined the condition $v_2^* > 1/2$.

Observe that Case II describes a situation where $\mu_1 > 1-\mu_2$, which amounts to saying that party 2's historical platform is closer to the extreme right than party 1's one is to the extreme left. In light of this, intuitively, it appears that there exists a critical threshold of b_2 above which party 1 is the winner irrespective of how painful may be for party 1 itself to choose any $x_1^* \neq \mu_1$.

In Case III, it is necessary to check whether inequality (11) is compatible with (5), i.e., the sign of $\widehat{b}_1 - \overline{b}_1$, as long as $1 + 4b_2(1-\mu_1-\mu_2) > 0$:

$$\widehat{b}_1 - \overline{b}_1 = \frac{-1 + 2b_2(2\mu_2 - 1)}{2(1-2\mu_1)[1 + 4b_2(1-\mu_1-\mu_2)]} \gtreqless 0 \text{ for all } b_2 \gtreqless \overline{b}_2. \qquad (12)$$

This immediately entails:

Lemma 6 *The sign of $b_1 - \widehat{b}_1$ reveals the outcome of the elections outside the region $\left\{b_1 \leq \overline{b}_1, b_2 \leq \overline{b}_2\right\}$.*

Having established this, I may now proceed to characterise the electoral outcome in the non trivial cases where at least one party's platform differs from the median voter's. When $1+4b_2(1-\mu_1-\mu_2) > 0$, the overall picture of the equilibrium outcome is represented by Figure 1, where:
♦ in regions A and F, platforms are $x_1^* = x_2^* = 1/2$ everywhere; therefore, the outcome of the elections is undetermined;
♦ in region B, party 1 wins, with platforms $x_1^* = 1/2$, $x_2^* > 1/2$;
♦ in region C, party 1 wins with platforms $x_1^* < 1/2 < x_2^*$;
♦ in region D, party 2 wins, with platforms $x_1^* < 1/2 < x_2^*$;
♦ in region E, party 2 wins, with platforms $x_1^* < 1/2$, $x_2^* = 1/2$.

Figure 1 is drawn for $\mu_2 \neq 1-\mu_1$. If instead $\mu_2 = 1-\mu_1$, (11) coincides with the 45° line and therefore the graph becomes fully symmetric. Accordingly, I may sum up the forgoing discussion in:[6]

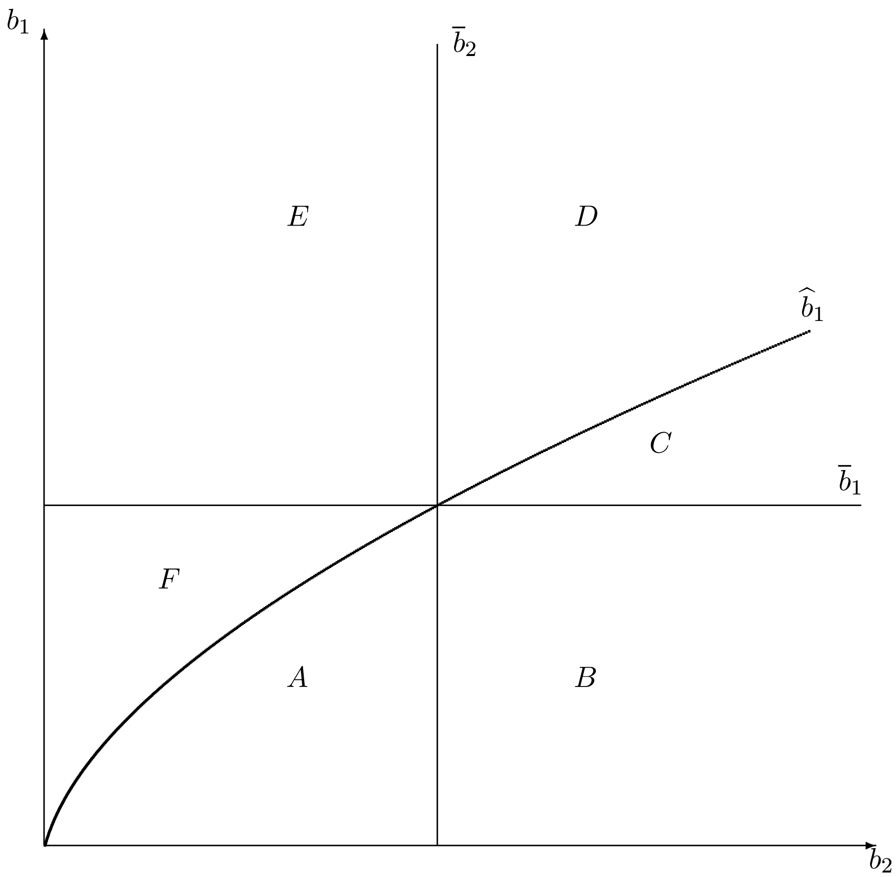
Theorem 7 *Consider the range $\left\{b_1 > \overline{b}_1, b_2 > \overline{b}_2\right\}$. In this parameter region, the outcome of the elections can be characterised as follows:*

i) Take $1 + 4b_2(1-\mu_1-\mu_2) < 0$. If so, then party 1 wins the elections for all $b_1 \geq \overline{b}_1$.

ii) Take $1+4b_2(1-\mu_1-\mu_2) > 0$. For all $b_1 \in \left[\overline{b}_1, \widehat{b}_1\right)$, party 1 wins the elections; for all $b_1 > \widehat{b}_1$, party 2 wins; for $b_1 = \widehat{b}_1$, the indifferent (median) voter is pivotal.

[6] With unrestricted platform choices $x_i \in [0,1]$, $i = 1,2$, the region where the electoral outcome is undetermined restricts to the locus $\widehat{b}_1$, whereby party 1 (resp. 2) wins for all $b_1 < \widehat{b}_1$ (resp. $b_1 > \widehat{b}_1$). The sign of $b_i - \overline{b}_i$ determines whether party i locates to the left or to the right of the median voter.

Figure 1: the parameter space



To complete the picture, it is worth discussing briefly the relationship between b_2 and $\widehat{b}_1$, in the range where $1+4b_2(1-\mu_1-\mu_2)>0$. This is done by observing that $b_2 > \widehat{b}_1$ is equivalent to

$$4b_2(1-\mu_1-\mu_2) > 0, \tag{13}$$

which is always true in the relevant parameter range. To conclude the analysis, the particular case where $b_1 = b_2$ can be investigated closely. Here, $\text{sign}\{v_1^* - 1/2\} = \text{sign}\{v_1^* - v_2^*\} = \text{sign}\{\mu_1 + \mu_2 - 1\}$, implying $v_1^* > v_2^* \Longleftrightarrow \mu_1 > 1 - \mu_2$ and conversely. Hence, we have:

Corollary 8 *If $b_1 = b_2$, then the party whose traditional platform is closest to the median voter wins the elections.*

3 Concluding remarks

I have investigated the role of political parties' loyalty to their historical heritage in determining the choice of their political platforms and ultimately the outcome of elections. The foregoing analysis has highlighted that, unless the weight attached to history is low enough, the outcome of elections is determinate and driven by a party's capability of adjusting its own electoral platform to the political preferences of the voters located around the median one. Being characterised by either (i) a less compelling historical heritage, or (ii) a traditional platform closer to the position of the median voter than the rival, entails *per se* no warranty of victory. Depending on the relative weight of these two factors, the winner may well be a party that, judging from its history, is farther away from the median voter but can approach him/her during the elections at a relatively lower cost than the rival, or, conversely, a party that suffers from a relatively higher stickiness with respect to its tradition but is lucky enough to find itself closer to the median voter precisely thanks to such historical tradition.

References

[1] Adams, J. (1999), "Policy Divergence in Multicandidate Probabilistic Spatial Voting", *Public Choice*, **100**, 103-22.

[2] Adams, J. (2000), "Multicandidate Equilibrium in American Elections", *Public Choice*, **103**, 297-325.

[3] Anderson, S.P., A. Kats and J.-F. Thisse (1994), "Probabilistic Voting and Platform Selection in Multi-party Elections", *Social Choice and Welfare*, **11**, 305-22.

[4] Ansolabehere, S. and J.M. Snyder, Jr. (2000), "Valence Politics and Equilibrium in Spatial Election Models", *Public Choice*, **103**, 327-36.

[5] Bartholdi J.J., L.S. Narasimhan and C.A. Tovey (1991), "Recognizing Majority-Rule Equilibrium in Spatial Voting Games", *Social Choice and Welfare*, **8**, 183-97.

[6] Calvert, R.L. (1985), "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence", *American Journal of Political Science*, **29**, 69-95.

[7] Downs, A. (1957), *An Economic Theory of Democracy*, New York, Harper & Row.

[8] Hotelling, H. (1929), "Stability in Competition", *Economic Journal*, **39**, 41-57.

[9] Huck, S., V. Knoblauch and W. Muller (2006), "Spatial Voting with Endogenous Timing", *Journal of Institutional and Theoretical Economics*, **162**, 557-70.

[10] Kollman, K., J.H. Miller and S.E. Page (1992), "Adaptive Parties in Spatial Elections", *American Political Science Review*, **86**, 929-37.

[11] Kollman, K., J.H. Miller and S.E. Page (1997), "Landscape Formation in a Spatial Voting Model", *Economics Letters*, **55**, 121-30.

[12] Lambertini, L. (1997), "Optimal Fiscal Regime in a Spatial Duopoly", *Journal of Urban Economics*, **41**, 407-20.

[13] McKelvey, R.D. and J.W. Patty (2006), "A Theory of Voting in Large Elections", *Games and Economic Behavior*, **57**, 155-80.

[14] Page, S.E., K. Kollman and J.H. Miller (1993), "Adaptive Parties and Spatial Voting Theory", in Grofman, B. (ed.), *Information, Participation, and Choice: An Economic Theory of Democracy in Perspective*, Ann Arbor, University of Michigan Press, 161-72.

[15] Patty, J.W. (2005), "Local Equilibrium Equivalence in Probabilistic Voting Models", *Games and Economic Behavior*, **51**, 523-36.

[16] Roemer, J.E. (1994), "A Theory of Policy Differentiation in Single Issue Electoral Policy", *Social Choice and Welfare*, **11**, 355-80.

[17] Schofield, N. (2006), "Equilibria in the Spatial Stochastic Model of Voting with Party Activists", *Review of Economic Design*, **10**, 183-203.

[18] Weber, S. (1997), "Entry Deterrence in Electoral Spatial Competition", *Social Choice and Welfare*, **15**, 31-56.

[19] Wittman, D. (1977), "Candidates with Policy Preferences: A Dynamic Model", *Journal of Economic Theory*, **14**, 180-89.

[20] Wittman, D. (1983), "Candidate Motivation: A Synthesis of Alternative Theories", *American Political Science Review*, **77**, 142-57.