E C O N O M I C S   B U L L E T I N

Habit Formation, Parents' Education Spending, and Growth

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Abstract

This paper investigates the impact of habits on economic growth in an overlapping generations (OLG) economy with physical and human capital in which altruistic parents finance the education of their children. Habit formation interacts with the role of human capital as an engine of growth by reducing education spending in the short run and by increasing the wage rate and decreasing the interest rate in the long run. When relative risk aversion (RRA) lies around unity, or when the RRA is no less than one and production is physical capital intensive and the level of the total production factor is large or the strength of habits are large, the effect of increasing the wage rate dominates the other effects and, therefore, the desired level of human capital investment increases in the long run, with habits. As a result, compared with a case with time-separable utility, the stationary growth rate implied by a model with habits is higher.

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1 Introduction

Since Abel (1990) and Constantinides (1990) showed that the risk premium puzzle can be resolved using habit formation, many studies have shown that habits are statistically significant in explaining consumers' behavior (e.g., Naik and Moore 1996), asset markets dynamics and the business cycle (Boldrin et al. 2001). Habit persistence also brings a new dimension to policy studies (e.g., Fuhrer 2000). A large number of contributions related to habits exist; however, these have been limited to the analysis without human capital.

This paper explores the implications of habit formation in overlapping generations (OLG) economies with human and physical capital[1]. In particular, we investigate the impact of habits on human capital investment and the growth rate. Parents derive utility from the human capital of their children and hence invest during their productive period in their children's education. When parents retire, the labor income of their children is not transferred to their parents and therefore parental spending on children's human capital is not motivated by the amount of the transfer anticipated from children during old age [2]. This framework shows that a rise in the strength of habits lowers educational spending to smooth the utility over an agent's lifetime when the relative risk aversion (RRA) is large and, when the RRA is low, there is no impact. Further, an increase in the strength of habits always lowers the human–physical capital ratio, and this in turn increases the wage rate and decreases the interest rate in the balanced growth path.

It is well known that with habits the desired stock of physical capital increases[3]. Therefore, the growth rate decreases with the standard neoclassical production function because of the diminishing marginal returns on capital. In our model, which introduces human capital, when the RRA lies around one, or the RRA is no less than one and the share of physical capital is large enough and the strength of habits is strong enough, or the RRA is no less than one and the share of physical capital is large enough and the level of total production factor is no less than one, the effect of the increasing wage rate dominates the other effects and long-run human capital investment increases with habits. Consequently, and contrary to

[1] Lahiri and Puhakka (1998) and Wendner (2001) also construct the OLG model with habits, but they do not introduce human capital.

[2] This setting appears valid in industrialized economies where advanced social security systems exist. According to a questionnaire survey exploring the Japanese lifestyle carried out by the Japanese Economic Planning Agency, 49.6% of respondents think that they should financially support their children while they are students. Furthermore, 5.9% of respondents believe that they should financially support their children until they marry. In spite of such long-term assistance, approximately 90% of respondents do not desire an economic payoff from their children (Economic Planning Agency 1994).

[3] See, for instance, Wendner (2002).

well-known facts, the stationary growth rate with habits is higher than that in the case with time-separable utility.

2 An OLG Economy with Education Spending and Habits

We consider an overlapping generations economy where individuals live for two periods and are considered to be "young" or "old". We assume that each individual works and only consumes when old. Members of each generation are identical and inelastically supply one unit of labor when they are young. There is no population growth. The preferences of a generation, t, are represented by the additively separable and constant RRA-type utility function:

$$\frac{(c_t^1)^{1-\sigma}-1}{1-\sigma}+\beta\left\{\frac{(c_{t+1}^2-\gamma c_t^1)^{1-\sigma}-1}{1-\sigma}+\frac{(h_{t+1})^{1-\sigma}-1}{1-\sigma}\right\}, \tag{1}$$

where σ is the relative risk aversion (RRA), c_t^1 is consumption of the young at period t, c_{t+1}^2 is the consumption of the old at period $t+1$, and $\beta \in (0,1)$ is the subjective discount factor. Following Lahiri and Puhakka (1998), instantaneous utility in the second period of life is derived from the difference between current consumption and a fraction of past consumption. Parameter $\gamma \in [0,1]$ indexes the strength of habits or the importance of past consumption in the instantaneous utility function. An individual of generation t spends on their children's education an amount e_t in period t. This results in a per capita level of children's human capital:

$$h_{t+1} = Ae_t, \tag{2}$$

where $A > 0$ and h_{t+1} is the human capital of an individual in generation $t+1$; hence, h_{t+1} stands for the effective labor supply [4]. Young agents gain the wage rate w_t, which is a real value for one unit of labor supply, h_t, and spend it for consumption c_t^1 and education spending e_t, or savings s_t. They can buy $R_{t+1}s_t$ units of consumption goods in the second period, where R_{t+1} is the gross interest rate at period $t+1$. The budget constraints of agents are then:

$$c_t^1 + e_t + s_t = w_t h_t \tag{3}$$

and

$$c_{t+1}^2 = R_{t+1}s_t. \tag{4}$$

[4] As in Kaganovich and Zilcha (1999), to simplify the analysis in the sequel, we disregard the effect of parent's human capital on the offspring's human capital level.

In generation t an individual maximizes the utility (1) subject to the budget constraints (3) and (4), and (2) (arguments are c_t^1, c_{t+1}^2, and e_t). From the first-order condition, we obtain:

$$s_t = \frac{(AR_{t+1}^{-1})^{\frac{\sigma-1}{\sigma}}[\beta^{\frac{1}{\sigma}} + \gamma(R_{t+1}+\gamma)^{-\frac{1}{\sigma}}]}{(AR_{t+1}^{-1})^{\frac{\sigma-1}{\sigma}}[\beta^{\frac{1}{\sigma}} + (R_{t+1}+\gamma)^{\frac{\sigma-1}{\sigma}}] + \beta^{\frac{1}{\sigma}}} w_t h_t \quad (5)$$

$$e_t = \frac{\beta^{\frac{1}{\sigma}}}{(AR_{t+1}^{-1})^{\frac{\sigma-1}{\sigma}}[\beta^{\frac{1}{\sigma}} + (R_{t+1}+\gamma)^{\frac{\sigma-1}{\sigma}}] + \beta^{\frac{1}{\sigma}}} w_t h_t. \quad (6)$$

We assume that the value of RRA is as follows:

Assumption 1
$\sigma \geq 1$.

Estimates of RRA usually lie around or above unity. Therefore, it can be said that this range is empirically plausible. Here, we have a result that is similar to Lahiri and Puhakka (1998) and Wendner (2001). Namely, for the value of RRA assumed in Assumption 1, the higher the strength of the habit formation, the higher are the savings in the short run[5]. In addition to this, we can see how habit formation affects educational spending from equation (6).

Lemma 1
Habit formation affects educational spending as follows in the short run. When the relative risk aversion is one, it has no effect on educational spending. When the relative risk aversion is above one, the higher the strength of the habit formation, the lower is educational spending.

When RRA is low, utility does not decrease as much if the agent reduces consumption[6]. Therefore, agents greatly reduce first-period consumption in order to prevent the second-period marginal utility from increasing because of an increase in the strength of habits. On the other hand, they do not need to increase savings as much because the increase in the marginal utility in the second period (caused by a rise in habits) is not as large. Agents then spend the remaining income on their children's education. When RRA is high, on the other hand, utility decreases dramatically by reducing consumption. Therefore, agents do not reduce first-period consumption as much. Instead, they reduce educational spending to increase savings.

[5]From equation (5), we have $\frac{ds_t}{d\gamma} = \frac{\frac{1}{\sigma}\beta^{\frac{1}{\sigma}}(R_{t+1}+\gamma)^{\frac{1}{\sigma}}[(A^{-1}R_{t+1})^{\frac{\sigma-1}{\sigma}}(\sigma-\gamma(R_{t+1}+\gamma)^{-1})+R_{t+1}(R_{t+1}+\gamma)^{-1}]+R_{t+1}}{\{\beta^{\frac{1}{\sigma}}[1+(A^{-1}R_{t+1})^{\frac{\sigma-1}{\sigma}}](R_{t+1}+\gamma)^{\frac{1}{\sigma}}+R_{t+1}+\gamma\}^2} w_t h_t$. $\sigma - \gamma(R_{t+1}+\gamma)^{-1} > 0$ for all $R_{t+1} > 0$ from Assumption 1. Therefore, $\frac{ds_t}{d\gamma} > 0$ for all t under Assumption 1.

[6]When RRA is below one (although this range is excluded by Assumption 1), the greater the strength of habit formation, the higher is educational spending.

3 Endogenous Growth and Habits

In this section, we analyze the effect of habit formation on growth. We take the economy's production function to be homogenous, of degree one, and production factors are physical capital and human capital (equivalent to 'efficient labor'). The output, in per capita terms, can be written as:

$$q_t = Bk_t^{\alpha}h_t^{1-\alpha}, \tag{7}$$

where $B > 0$ is the level of total production factor (TPF), $\alpha \in (0, 1)$ is the share parameter of physical capital, and k_t is the per capita stock of physical capital in period t. The firms behave perfectly and competitively to maximize profits. Capital depreciates at rate one. In this setting, we have:

$$w_t = (1-\alpha)Bk_t^{\alpha}h_t^{-\alpha} \tag{8}$$

$$R_{t+1} = \alpha Bk_{t+1}^{\alpha-1}h_{t+1}^{1-\alpha}. \tag{9}$$

As we assume no population growth, the market clearing condition becomes:

$$k_{t+1} = s_t. \tag{10}$$

From (5), (8), (9), and (10), the growth rate of physical capital is:

$$\frac{k_{t+1}}{k_t} \equiv \theta_{k,t} = \frac{(A^{-1}\alpha B\chi_{t+1}^{1-\alpha})^{\frac{1-\sigma}{\sigma}}[\beta^{\frac{1}{\sigma}} + \gamma(\alpha B\chi_{t+1}^{1-\alpha} + \gamma)^{-\frac{1}{\sigma}}](1-\alpha)B\chi_t^{1-\alpha}}{(A^{-1}\alpha B\chi_{t+1}^{1-\alpha})^{\frac{1-\sigma}{\sigma}}[\beta^{\frac{1}{\sigma}} + (\alpha B\chi_{t+1}^{1-\alpha} + \gamma)^{\frac{\sigma-1}{\sigma}}] + \beta^{\frac{1}{\sigma}}}. \tag{11}$$

From (2), (6), (8), and (9), on the other hand, the growth rate of human capital is:

$$\frac{h_{t+1}}{h_t} \equiv \theta_{h,t} = \frac{A\beta^{\frac{1}{\sigma}}(1-\alpha)B\chi_t^{-\alpha}}{(A^{-1}\alpha B\chi_{t+1}^{1-\alpha})^{\frac{1-\sigma}{\sigma}}[\beta^{\frac{1}{\sigma}} + (\alpha B\chi_{t+1}^{1-\alpha} + \gamma)^{\frac{\sigma-1}{\sigma}}] + \beta^{\frac{1}{\sigma}}}, \tag{12}$$

where $\chi_t \equiv \frac{h_t}{k_t}$. Equations (11) and (12) yield the evolution of the human–physical capital ratio:

$$\chi_{t+1} = \frac{[A\beta(\alpha B\chi_{t+1}^{1-\alpha} + \gamma)]^{\frac{1}{\sigma}}(\alpha B\chi_{t+1}^{1-\alpha})^{\frac{\sigma-1}{\sigma}}}{[\beta(\alpha B\chi_{t+1}^{1-\alpha} + \gamma)]^{\frac{1}{\sigma}} + \gamma}. \tag{13}$$

In the Appendix A.1, we show that there exists a unique positive stable steady-state ratio $\chi = \chi_t = \chi_{t+1}$ by solving (13), and we can derive the following from (13):

$$\frac{d\chi}{d\gamma} = \frac{\chi[-(\sigma-1)\gamma - \sigma R]}{(1-\alpha)[\beta^{\frac{1}{\sigma}}(R+\gamma)^{\frac{\sigma+1}{\sigma}} + \gamma^2] + \alpha\sigma[\beta^{\frac{1}{\sigma}}(R+\gamma)^{\frac{1}{\sigma}} + \gamma]}. \tag{14}$$

Lemma 2

Under Assumption 1, a rise in the strength of habits formation decreases the steady-state human–physical capital ratio.

Habit formation affects the human–physical capital ratio by affecting savings, and by affecting education spending as given in Lemma 1. An increase in habits increases savings and, consequently, it increases the amount of physical capital. On the other hand, whether it decreases or does not affect education spending depends on the value of RRA and, therefore, it decreases or does not affect the level of human capital. Consequently, the human–physical capital ratio decreases as the strength of habits becomes greater.

Using the steady-state ratio χ in either (11) or (12), we obtain the stable steady-state growth rate as:

$$\theta = \frac{A\beta^{\frac{1}{\sigma}}(1-\alpha)B\chi^{-\alpha}}{(A^{-1}\alpha B\chi^{1-\alpha})^{\frac{1-\sigma}{\sigma}}[\beta^{\frac{1}{\sigma}} + (\alpha B\chi^{1-\alpha}+\gamma)^{\frac{\sigma-1}{\sigma}}] + \beta^{\frac{1}{\sigma}}}. \tag{15}$$

Differentiating (15) with γ, we can see that habit formation affects the growth rate as follows:

$$\frac{d\theta}{d\gamma} \lesseqgtr 0 \Leftrightarrow \underbrace{-\frac{\sigma-1}{\sigma}\Gamma}_{Direct Effect} + \underbrace{\underbrace{\frac{d\chi}{d\gamma}\chi^{-1}(-\alpha\Lambda)}_{Wage Effect} + \underbrace{\frac{d\chi}{d\gamma}\chi^{-1}(1-\alpha)\frac{\sigma-1}{\sigma}\Psi}_{Interest Rate Effect}}_{Indirect Effect} \lesseqgtr 0, \tag{16}$$

where $\Gamma = R^{-\frac{\sigma-1}{\sigma}}(R+\gamma)^{-\frac{1}{\sigma}} > 0$, $\Lambda = R^{-\frac{\sigma-1}{\sigma}}[\beta^{\frac{1}{\sigma}} + (R+\gamma)^{\frac{\sigma-1}{\sigma}}] + A^{-\frac{\sigma-1}{\sigma}}\beta^{\frac{1}{\sigma}} > 0$, and $\Psi = R^{-\frac{\sigma-1}{\sigma}}[\beta^{\frac{1}{\sigma}} + \gamma(R+\gamma)^{-\frac{1}{\sigma}}] > 0$. The Direct Effect on the growth rate is that habit formation directly affects the growth rate by affecting education spending. This effect, whether it decreases or does not affect the growth rate, depends on the value of RRA through affecting education spending as in Lemma 1. The Indirect Effect on the growth rate is that habit formation indirectly affects the growth rate by affecting the human–physical capital ratio. An increase in the strength of habit formation decreases the ratio under Assumption 1 as given in Lemma 2. However, how falls in this ratio affect the growth rate is ambiguous. It increases the wage rate from (8). That is, human capital becomes scarce relative to physical capital by the force of habit formation and therefore the wage rate per unit of effective labor supply increases. Furthermore, an increase in the wage rate increases education spending from (6). In turn, this increases the growth rate (Wage Effect). On the other hand, it decreases the gross interest rate from (9), and whether it decreases or does not affect education spending depends on the value of RRA. Furthermore, a decrease in the human–physical capital ratio lowers the growth rate when the value of RRA is above one (Interest Rate Effect). Whether the Wage Effect or Interest Rate Effect dominates depends on the parameter values. Here, we obtain the following sufficient, but not necessary, condition for an increase in habit persistence to increase the growth rate within the range of $\sigma \geq 1$ (see Appendix A.2 for the proof).

Proposition 1

Consider a stable steady-state growth rate in an OLG economy with human capital and physical capital. If the relative risk aversion (σ) lies around 1, or the relative risk aversion is no less than one and the share of physical capital (α) is large enough and the strength of habits (γ) is great enough, or the relative risk aversion is no less than one and the share of physical capital is large enough and the level of total production factor (B) is no less than one, the greater the strength of habits, the higher the steady-state growth rate (θ).

As RRA approaches one, the Interest Rate Effect approaches zero and therefore the Indirect Effect becomes positive (because the Wage Effect is always positive) and, furthermore, the Direct Effect approaches zero. Consequently, an increase in habits increases the steady-state growth rate. When RRA is larger than one, on the other hand, the Direct Effect is negative. However, when the share of physical capital is large enough, the Interest Rate Effect becomes weaker and the Wage Effect dominates. Therefore, the Indirect Effect is positive. Moreover, when the level of TPF is large enough, the interest rate is also great enough from (9). As the interest rate or the strength of habits becomes larger, the Direct Effect becomes smaller (note that the Direct Effect contains $R^{-\frac{\sigma-1}{\sigma}}(R+\gamma)^{-\frac{1}{\sigma}}$). Namely, a high interest rate allows agents higher consumption for a unit of savings in the second period and therefore an increase in habits do not reduce educational spending as much. Furthermore, the degree of habit formation in second-period utility decreases with the strength of habits because of the strong concavity of the utility function. Consequently, when the value of TPF or the strength of habits is large, the Indirect Effect (which is positive in this case) dominates the Direct Effect (which is negative in this case); therefore, an increase in habits increases the steady-state growth rate when the share of physical capital is large enough and the level of TPF or the strength of habits is great enough.

To examine more explicitly how habit formation affects the growth rate, we construct a simple numerical example. The combination of parameter values is: $\beta = 0.5$, $A = B = 3.5$. We vary the values of RRA and the share of physical capital (these are key parameter values in our analysis) within a range that is consistent with the empirical evidence as: σ =1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0, and α =0.28 and α =0.33[7]. We examine how the effect of habit formation on the

[7] Epstein and Zin (1989) found values of relative risk aversion clustering around unity, consistent with the widely used logarithmic utility function. Constantinides et al. (2002) presented alternative evidence to suggest that the coefficient of relative risk aversion lies plausibly within the range 2–5 by using habit formation. Considering these empirical studies, we set the range of relative risk aversion as 1–4. Moreover, according to Greenwood et al. (1993, p.6), the share of physical capital in market production deducted from the US national accounts could be anywhere between 0.25 and 0.43, depending on various details, such as the treatment of proprietor's

σ			1.0	1.5	2.0	2.5	3.0	3.5	4.0
α =0.33	varying γ								
	γ=0	χ	3.50	3.10	2.83	2.66	2.50	2.37	2.30
		θ	1.37	1.29	1.20	1.17	1.13	1.13	1.12
	γ=0.3	χ	2.91	2.31	1.99	1.76	1.56	1.44	1.39
		θ	1.46	1.34	1.29	1.27	1.20	1.18	1.14
α =0.28	varying γ								
	γ=0	χ	3.50	2.94	2.60	2.34	2.15	1.99	1.89
		θ	1.56	1.41	1.36	1.32	1.29	1.26	1.26
	γ=0.3	χ	2.85	2.13	1.75	1.50	1.29	1.13	1.07
		θ	1.66	1.50	1.41	1.34	1.29	1.25	1.21

Table 1: The value of χ and θ when $\beta = 0.5$ and $A = B = 3.5$

growth rate changes depending on these values by varying the strength of habits from $\gamma = 0$ to $\gamma = 0.3$. Table 1 shows that, for each value of σ and α, the human–physical capital ratio (χ) in the case of $\gamma = 0.3$ is lower than that of $\gamma = 0$, which is consistent to Lemma 2. Moreover, in most of the cases excluding $(\alpha, \sigma) = (0.28, 3.0), (0.28, 3.5)$, and $(0.28, 4.0)$ (these are cases where RRA is departed from one and the production share of human capital is not large enough and therefore do not satisfy the conditions of Proposition 1), the growth rate (θ) in the case of $\gamma = 0.3$ is higher than that of $\gamma = 0$.

4 Conclusions

This paper explores the impact of habits on the growth rate in OLG economies where altruistic parents finance their children's education costs. Compared with the case without habits (with a time-separable utility function), the stationary growth rate is higher when the relative risk aversion lies around one, or the relative risk aversion is no less than one and the share of physical capital is large enough and the strength of habits is great enough, or the relative risk aversion is no less than one and the share of physical capital is large enough and the level of total production factor is no less than one.

income. We vary the values of the share of physical capital from 0.33 (which is most widely used in the literature) to 0.28.

A Appendix

A.1 The Unique Existence of Equilibrium χ

In this Appendix, we prove that there exists a unique χ that satisfies (13). Substituting $\chi = \chi_t = \chi_{t+1}$ into (13), and using $\chi_t = \frac{h_t}{k_t}$ and (9), we obtain:

$$\left(\frac{R}{\alpha B}\right)^{\frac{1}{1-\alpha}} \left[\beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}} + \gamma \left(\frac{R}{R+\gamma}\right)^{\frac{1}{\sigma}}\right] = (A\beta)^{\frac{1}{\sigma}} R, \tag{A.1}$$

where R is the steady-state interest rate. We define the left-hand side of (A.1) as $f(R)$, and the right-hand side as $g(R)$. Then, we can derive $f'(R) = \left(\frac{1}{\alpha B}\right)^{\frac{1}{1-\alpha}} (\phi(R) + \psi(R)) > 0$, where $\phi(R) = \frac{1}{1-\alpha} R^{\frac{\alpha}{1-\alpha}} \left[\beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}} + \gamma \left(\frac{R}{R+\gamma}\right)^{\frac{1}{\sigma}}\right]$ and $\psi(R) = \frac{1}{\sigma} \beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma} + \frac{\alpha}{1-\alpha}} + \frac{\gamma^2}{\sigma} \left(\frac{R}{R+\gamma}\right)^{\frac{1}{\sigma} + \frac{1}{1-\alpha}} (R+\gamma)^{\frac{\alpha}{1-\alpha}}$. For $\frac{R}{R+\gamma}$ is increasing in R or constant for $\gamma \in [0,1]$, it is obvious that $f''(R) > 0$ for all $R > 0$. We can also derive the relations $g'(R) > 0$ for all $R > 0$, $f'(0) = 0 < (A\beta)^{\frac{1}{\sigma}} = g'(0)$, and $f'(\infty) = \infty > (A\beta)^{\frac{1}{\sigma}} = g'(\infty)$. Therefore, given $A > 0$, $B > 0$, $\alpha \in (0,1)$, $\beta \in (0,1)$, $\gamma \in [0,1]$, and $\sigma > 0$, there exists a unique positive R that satisfies (A.1). Then, there exists unique positive χ that satisfies (13) because $\chi = \left(\frac{R}{\alpha B}\right)^{\alpha-1}$.

A.2 Proof of Proposition 1

We can rewrite the relation (16) as:

$$\frac{d\theta}{d\gamma} \lesseqgtr 0 \Leftrightarrow -\frac{\sigma-1}{\sigma}\Gamma' - [(\sigma-1)\gamma + \sigma R]\left[-\alpha\Lambda' + (1-\alpha)\frac{\sigma-1}{\sigma}\right] \lesseqgtr 0, \tag{A.2}$$

where $\Gamma' = (1-\alpha)\left[\frac{\beta^{\frac{1}{\sigma}}(R+\gamma)^{\frac{\sigma+1}{\sigma}} + \gamma^2}{[\beta(R+\gamma)]^{\frac{1}{\sigma}} + \gamma}\right] + \sigma\alpha > 0$ and $\Lambda' = 1 + \frac{(A^{-1}R)^{\frac{\sigma-1}{\sigma}}[\beta(R+\gamma)]^{\frac{1}{\sigma}} + R}{[\beta(R+\gamma)]^{\frac{1}{\sigma}} + \gamma} > 0$. First, it is obvious that $sign\left(\frac{d\theta}{d\gamma}\right) > 0$ for all $A > 0$, $B > 0$, $\alpha \in (0,1)$, $\beta \in (0,1)$, and $\gamma \in [0,1]$ when σ approaches 1 from (A.2). Next, let us consider the case with $\sigma \geq 1$ and α approaches 1. From (9), we obtain $\lim_{\alpha \to 1} R = B$. Then, as $\alpha \to 1$, the relation (A.2) becomes $sign\left(\frac{d\theta}{d\gamma}\right) = sign\left[(\sigma-1)(\gamma\Phi + \gamma - 1) + \sigma B(1+\Phi)\right]$ where $\Phi = \frac{(A^{-1}B)^{\frac{\sigma-1}{\sigma}}[\beta(B+\gamma)]^{\frac{1}{\sigma}} + B}{[\beta(B+\gamma)]^{\frac{1}{\sigma}} + \gamma} > 0$. Therefore, when α approaches 1 and also γ approaches 1, $sign\left(\frac{d\theta}{d\gamma}\right) > 0$ for all $A > 0$, $B > 0$, $\beta \in (0,1)$, and $\sigma \geq 1$. Mreover, $(\sigma-1)(\gamma\Phi + \gamma - 1) + \sigma B(1+\Phi) = (\sigma-1)\gamma(1+\Phi) + \sigma(B - 1 + B\Phi) + 1$. Therefore, when α approaches 1 and B is no less than 1, $sign\left(\frac{d\theta}{d\gamma}\right) > 0$ for all $A > 0$, $\beta \in (0,1)$, $\gamma \in [0,1]$, and $\sigma \geq 1$.

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