

## Estimating Capacity Utilization Using a SVAR Model: An Application to the US and Canadian Economies

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### *Abstract*

This paper develops a method for the estimation of the rate of capacity utilization based on standard growth theory and the Structural VAR estimating technique. The measures of capacity utilization we derive for the US and Canadian economies are compared with widely-used survey measures. We show that the degree of association and synchronization of the two measures for both economies is pretty high, a result that encourages the use of our method as a reliable alternative.

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## 1. Introduction

The rate of capacity utilization (CU), measuring the extent to which actual output differs from normal or capacity output, is one of the central variables in economic analysis. It is especially useful in the study of inflation, where many authors aim to establish the rate beyond which CU generates inflationary pressures – (the non-accelerating inflation capacity utilization) NAICU (e.g. McElhattan, 1978 and 1985; Bauer, 1990; Emery and Chang, 1997, *inter alia*). Although capacity is an important macroeconomic variable, crucial for the implementation of economic policy (Cotis, *et al.*, 2005); it is not directly observable. Efforts to provide an approximate measure of the rate of CU have focused on the use of survey responses, the estimation of production functions or the use of filtering techniques<sup>1</sup>. Of these approaches, the use of surveys to reveal businesses' view of the economy has been the most popular (at least in the US and Canada<sup>2</sup>); regrettably, only a few countries' survey data span a sufficiently long period for the measure to be used in macroeconometric models for policy analysis. This paper presents a new approach -- using existing profit and investment data -- that allows the generation of a time series for CU highly correlated with series constructed with survey data from the US and Canada.

The model of CU that we propose is based on the close connection between gross profits (*i.e.*, profit before accounting for reinvestment in the firm and paying of dividends) and investment. The underlying idea is that investment spending is financed mainly from gross profits and that investment is the variable whose variations have a long-lasting effect on the economy's production capacity. If investment increases and exerts pressure on profits, the economy expands quickly; the CU rate follows suit<sup>3</sup>, producing a rise in the price level. If, by contrast, the amount of investment falls short of the amount of profits and, therefore, investment does not exert pressure on profits it follows that capacity is under utilized, implying a fall in the rate of inflation. Thus, over (under) utilization of capacity is accompanied by an acceleration (deceleration) of the inflation rate. Consequently, the rate of CU at which there is neither acceleration nor deceleration of investment activity is the equilibrium one and is identified as the steady state of the economy, that is, the state at which the growth rate of profits equals to the growth rate of investment.

At any particular time, actual investment differs from its steady-state level. Profits constitute the major source of investment activity, which is primarily responsible for economic fluctuations<sup>4</sup>. However, the effects of profit shocks on investment are short-lived and the rationale for this is that if, for example, the current profits growth rises, it will lead to more investment growth in the short run, which will lead to more investment. The rate of return on these investments is lower, on average, than the rates of return on investments previously undertaken—hence the standard assumption of a declining marginal efficiency of investment schedule—so the effects of profit shocks are exhausted in the long-run. Hence, short-run profits

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<sup>1</sup> Christiano (1981) and Chagny and Döpke (2001) present some widely used methods.

<sup>2</sup> For a relevant discussion see Morin and Stevens (2004).

<sup>3</sup> Rising investment affects both production and productive capacity positively. The impact on the CU rate depends on the growth of production and capacity: there may be an increasing, decreasing or constant CU rate. But in fact, investment takes time-to-built (Kydland and Prescott, 1982); as a result the capacity to produce grows slower than production itself, implying an increasing CU rate as a result of the increased investment.

<sup>4</sup> Profits become the direct source of investment funds and also constitute the basis for the flow of credit from financial institutions.

matter for investment decisions. Once these decisions are made, the profit shock soon disappears, while the effect of investment on capacity persists<sup>5</sup>.

Consequently, it is important to disentangle temporary demand (profit) shocks from permanent supply (investment) shocks. This allows us to isolate deviations of actual investment spending from what is required to maintain the steady-state condition. The ideal method to achieve such a separation uses the SVAR technique, imposing a single long-run restriction (for details of our bivariate model, see the appendix). The ratio of actual investment to the normal or equilibrium-maintaining investment defines the rate of CU. Our definition of CU is different from a survey-based measure, since we use investment to define normal capacity.

The remainder of the paper is structured as follows: Section 2 discusses the econometric analysis of the model. Section 3 continues with a comparison of our estimates of CU with the corresponding surveys for the US and Canadian economies. Section 4 concludes and makes some remarks about future research efforts.

## 2. Empirical Application

As we argue, CU is a theoretical variable and cannot be directly measured. The estimates of the rate of CU that we derive using a bivariate SVAR model are compared with the survey estimates of the US and Canadian economies for the periods 1964:1 to 2005:4 and 1963:1 to 2005:3, respectively<sup>6</sup>.

Prior to the estimation of the reduced form of VAR equations, we need to ensure that both variables are stationary. In fact, we cannot reject the hypothesis of  $I(1)$  either for real profits or for investment; however by taking their difference in logarithms we convert them both to stationary variables. Augmented Dickey-Fuller tests allow us to reject the hypothesis that the growth rates of profits and investment are  $I(1)$ ; similarly KPSS tests do not allow us to reject hypothesis that both growth rates are  $I(0)$ <sup>7</sup>. The results of the statistical analysis for both countries are displayed in Table 1 below:

(Insert Table 1, here)

A correctly-specified SVAR model requires not only that its variables be of the same order of integration, but also that there does not exist an equilibrium relationship among them. In other words, the variables in a correctly specified SVAR model should not be cointegrated. For this reason, we apply the Johansen (1988) cointegration test assuming the presence of a linear deterministic trend. Table 2 below illustrates the results of the analysis, which suggest that the two variables (real investment and real profits) for both countries are not cointegrated. Consequently, we can safely proceed with our estimation of the SVAR system of equations.

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<sup>5</sup> The dividend discount model (from corporate finance theory) uses the same idea of short-run profits. This idea remains the basis for more sophisticated versions of the model, where some of its restrictive assumptions are relaxed, such as the exogenous discount rate or the constant growth rate of profits (see Elton and Gruber, 1995, ch. 18).

<sup>6</sup> All data-series are from OECD Statistical Compendium, where profits are approximated by the operating surplus deflated by the GDP deflator and investment is approximated by business investment deflated by the corresponding deflator. The survey measure of CU for the US is from the Federal Reserve Board's website ([www.federalreserve.gov](http://www.federalreserve.gov)) and for Canada from the Statistics of Canada ([www.statcan.ca](http://www.statcan.ca)).

<sup>7</sup> See Dickey and Fuller (1979) and (1981), Kwiatkowski, *et al.* (1992) for discussion of the tests applied.

(Insert Table 2, here)

Another issue related to the specification of the bivariate reduced form of the SVAR model is the determination of the optimal lag length. For this reason, we employed several statistics like the Likelihood Ratio test, Hannan-Quinn, Schwarz and Akaike information criteria for the two countries (see Table 3). Optimal lag length selection is crucial, since a sufficient number of lags prevents the possibility of serial correlation and leads to unbiased estimates of structural components. The results of the tests suggest that the optimal lag length for the US and Canada is three and one, respectively.

(Insert Table 3, here)

The structural residuals are obtained by imposing the restriction that the long-run effect of profit growth on investment growth is zero. Furthermore, in our specification, aggregate demand shocks are due to profit changes, in the sense that higher profits lead to higher investment spending, and so aggregate supply shocks are caused by investment changes. Once the structural residuals are extracted, the changes in the growth of investment caused by aggregate demand shocks (*i.e.*, the investment gap) can be distinguished by merely accumulating the series of the structural demand residuals.

Figures 1 and 2, below, display the two alternative measures of CU for the US and Canadian economies, respectively. The two measures of CU display similar patterns for both countries. In the Canadian case, we observe that in the 1990s the survey measure of CU, as depicted by the dotted line, is above the threshold CU of 82 percent<sup>8</sup>. By contrast, the CU measure derived by our SVAR model (solid line) is higher than the threshold level of 82 percent most of the time in the 1970s and until the middle of 1980s and falls thereafter. This result is consistent with the acceleration of inflation in the 1970s and early 1980s and its deceleration in the 1990s. In the US case, our measure of CU (solid line) strongly resembles the FRB measure of CU (dotted line). Both measures of CU are consistent with their predictions with respect to the growth rate of inflation.

(Insert Figures 1 and 2, here)

### 3. Cyclical Association and Synchronization

A visual inspection of Figures 1 and 2 is not enough to confirm the strength of association between the two measures. To derive more rigorous conclusions about the degree of association for the alternative measures of CU we offer the results of some statistical tests. We estimate the simple or Pearson correlation coefficient for linear association, the Spearman's rank correlation coefficient and the concordance statistic. The rank correlation has the advantage over the simple correlation in that it accounts for non-linear associations. The results of the tests are displayed in Table 4.

(Insert Table 4, here)

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<sup>8</sup> This percentage is estimated econometrically; it is the steady-state level of capacity.

From the three statistics of Table 4, the concordance statistic<sup>9</sup> is the most relevant, because we are interested in identifying not only an association between the variables, but also the proportion of time that the two variables spend in the same state (overutilization or underutilization). Two variables may be closely associated, yet the phase of one variable is frequently opposite to that of the other. Both the Pearson's and the Spearman's correlation coefficients suggest that the strength of association between the two variables is positive and strong. More specifically, in the US economy the two statistics are somewhat higher than those of Canada; however, in both countries the degree of association is considered high and statistically significant. The two measures are in phase about 70 per cent of the time (quite substantial for volatile, quarterly data). Moreover, the two estimated concordance statistics are statistically significant at the one percent significance level.

The detection of a cyclical association is not a strong piece of evidence for cyclical synchronization. Three approaches are used to identify the degree of synchronization. First, we apply the concordance statistic to the growth rates of CU measures to confirm whether a positive change at time  $t$  in one measure of CU is also associated with a positive change at time  $t$  for the other measure and *vice versa*. Figure 3 below portrays the growth rates for each CU measure; a visual inspection of graphs reveals similar patterns for both countries. The concordance statistic demonstrates the high degree of synchronization of CU measures (see Table 5).

(Insert Figure 3, here)  
(Insert Table 5, here)

The second measure of cyclical synchronization is the maximum correlation, which is obtained from the Pearson correlation coefficient estimated for leading and lagging periods (+4 to -4). The degree of synchronization is specified by the period at which correlation is maximum. Perfect synchronization implies maximum contemporaneous correlation (see Table 6).

Finally, the third measure for synchronization is the multiple correlation coefficient (Belo, 2001). For the estimation of the multiple correlation coefficient, we assume that the relationship between the SVAR measure ( $CU^s$ ) and the survey measure ( $CU^o$ ) for each country is given by the following specification:

$$CU_t^s = c + \sum_{i=0}^4 \delta_i CU_{t-i}^o + \sum_{j=1}^4 \theta_j CU_{t+j}^o + \varepsilon_t, \quad (1)$$

where  $\delta_i$  and  $\theta_i$  are parameters to be estimated and  $t$  is time. The multiple correlation coefficient between the two measures is given by the square root of the coefficient of

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<sup>9</sup>The concordance statistic ( $C_{ij}$ ) is estimated from the following formula:

$C_{ij} = T^{-1} \{ \sum_{t=1}^T (S_{i,t} \cdot S_{j,t}) + \sum_{t=1}^T (1-S_{i,t}) \cdot (1-S_{j,t}) \}$ , where  $T$  is the sample size,  $S_{it}$  and  $S_{jt}$  are binary variables taking

the value of one when each of the CU measures is greater than the threshold level of 82% (approximately) and zero otherwise. The significance of the concordance statistic is conferred from the  $a$  coefficient whose value is obtained by the following moment condition:  $E((S_{it} - \bar{S}_i) \cdot (S_{jt} - \bar{S}_j) - a) = 0$ . In the

GMM estimation, we use the Bartlett kernel option with a fixed bandwidth of 3. For further discussion on the concordance statistic, see McDermott and Scott (1999), Harding and Pagan (2002), Hall and McDermott (2004).

determination derived from the estimated equation (1). The merit of this approach is that it provides an exact measure of linear association of the leading and lagging values between the measures. A simple way to specify the degree of synchronization is to detect among the estimated parameters ( $\delta_i$  and  $\theta_i$ ) the one with the greatest impact on the dependent variable. The results of maximum and multiple correlations presented in Table 6 suggest that synchronization between the measures of CU is achieved contemporaneously, which lends additional support to the conclusions derived from the concordance statistic about the contemporaneous synchronization of the measures of CU.

(Insert Table 6, here)

#### 4. Conclusions

This paper develops a simple, alternative definition of CU -- the ratio of actual investment to the equilibrium-maintaining investment. This definition is based on standard economic theory and a SVAR estimating technique. It is important to stress that, although our definition is based on a single factor of production, capital, it nevertheless yields a data series highly correlated, both contemporaneously and dynamically, with the survey measures created by the Federal Reserve Board and Statistics Canada, derived from a combination of all factors of production. Moreover, the proposed measure can be used to create CU series that predate the survey measures.

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**Table 1. Unit Root tests**

Variables	ADF* test statistic		KPSS** LM-Stat.	
	United States	Canada	United States	Canada
Investment Growth	-4.667567	-9.4228	0.316936	0.14058
Profit Growth	-12.05373	-11.6823	0.141783	0.04723
Critical values for different levels of significance for ADF and KPSS test				
1% critical value	-3.469214		0.739000	
5% critical value	-2.878515		0.463000	
10% critical value	-2.575899		0.347000	
* The Schwartz information criterion used for the lag selection on Augmented Dickey-Fuller test and the maximum lag length was set to nine.				
** The Bartlett Kernel spectral estimation method was selected for Kwiatkowski-Phillips-Schmidt-Shin test.				

**Table 2. Johansen Cointegration Test with a Deterministic Trend**

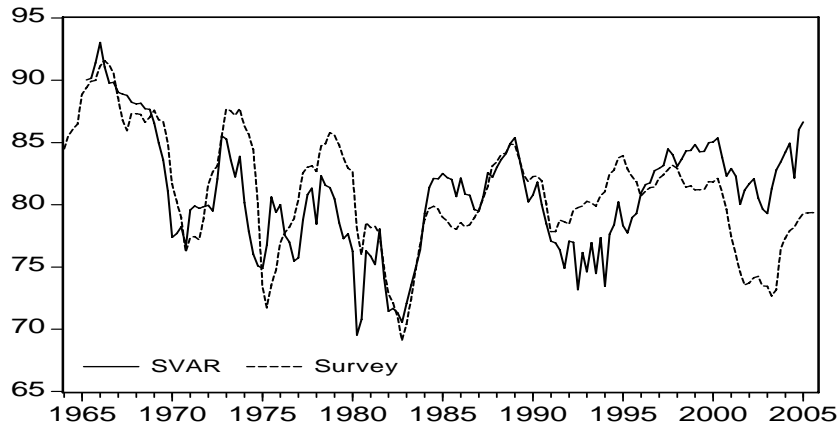
Country	Trace Statistic	Max.-Eigen Statistic	5% Critical Values		1% Critical Values	
			Trace	Max.-Eigen	Trace	Max.-Eigen
U.S.	6.30462	5.35814	15.41	14.07	19.93	18.52
Canada	4.31246	4.25194	15.41	14.07	19.93	18.52
The null hypothesis suggests that there are zero cointegrating vectors ( $H_0: r = 0$ )						
Trace test indicates no cointegration at both 5% and 1% level.						
Max-Eigenvalue test indicates no cointegration at both 5% and 1% level.						

**Table 3. VAR Lag Order Selection Criteria**

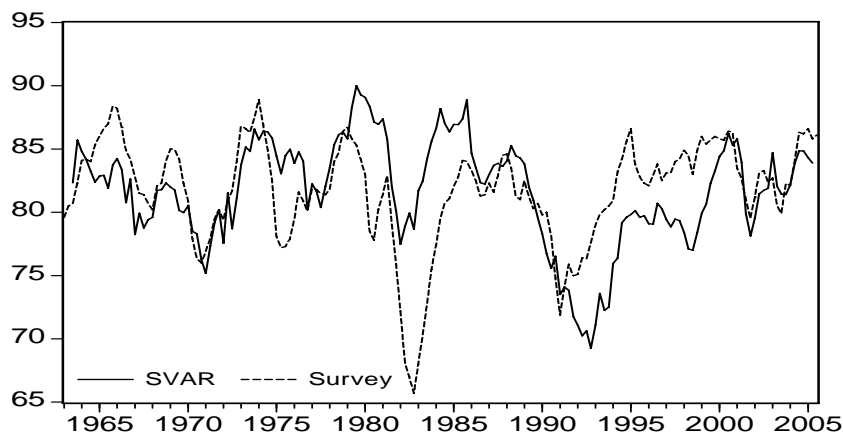
Lag	LR		AIC		SC		HQ	
	USA / Canada	USA / Canada	USA / Canada	USA / Canada	USA / Canada	USA / Canada	USA / Canada	USA / Canada
1	83.947	26.23*	-9.506	-9.07*	-9.388	-8.95*	-9.458	-9.02*
2	20.946	4.092	-9.593	-9.049	-9.39*	-8.858	-9.51*	-8.971
3	11.54*	3.688	-9.61*	-9.023	-9.346	-8.756	-9.508	-8.915
4	2.353	6.186	-9.584	-9.014	-9.232	-8.670	-9.441	-8.875
5	9.347	4.564	-9.597	-8.995	-9.167	-8.574	-9.422	-8.824
6	6.375	3.781	-9.590	-8.971	-9.082	-8.473	-9.384	-8.769
7	1.700	7.758	-9.551	-8.975	-8.965	-8.400	-9.313	-8.741
8	6.664	3.121	-9.548	-8.946	-8.883	-8.296	-9.278	-8.682

\* indicates lag order selected by the criterion  
 LR: sequential modified LR test statistic (each test at 5% level), AIC: Akaike information criterion,  
 SC: Schwarz information criterion, HQ: Hannan-Quinn information criterion.

**Figure 1. SVAR vs. Survey Measure of CU, US Economy**



**Figure 2. SVAR vs. Survey Measure of CU, Canadian Economy**



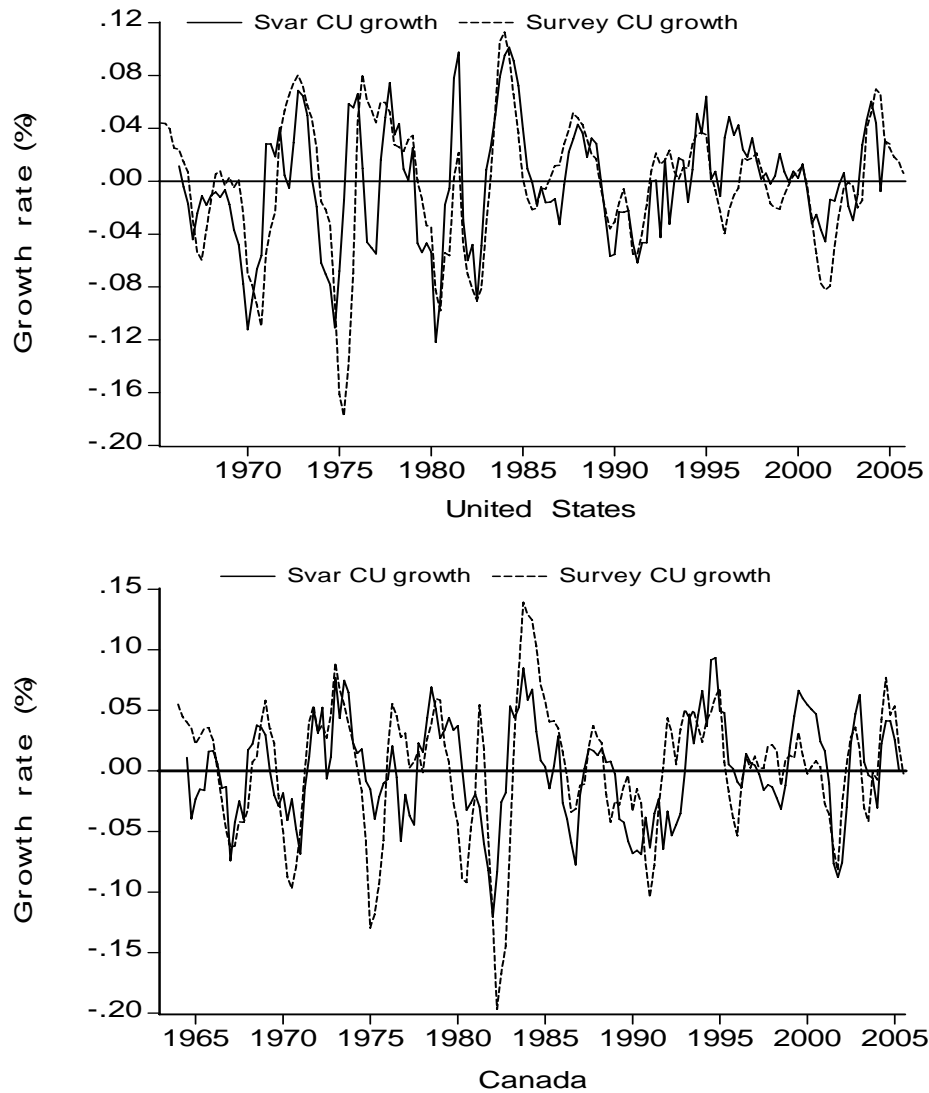


**Table 4. Statistics for Association**

Pearson Correlation			
Country	Period	Statistic Value	p-value
United States	1965:2 – 2005:3	0.6500	0.000*
Canada	1963:3 – 2005:2	0.4175	0.000*
Spearman's rank Correlation			
United States	1965:2 – 2005:3	0.5620	0.000*
Canada	1963:3 – 2005:2	0.4120	0.000*
Concordance Statistic			
United States	1965:2 – 2005:3	0.7185	0.003*
Canada	1963:3 – 2005:2	0.6783	0.008*

\* The statistic is significant at the 1 percent level.

**Figure 3. Growth Rates of CU for the US and Canadian Economies**



**Table 5. Concordance Statistic for Synchronization<sup>10</sup> (CU growth rates)**

Country	Period	Statistic Value	p-value
United States	1966:2 – 2005:1	0.7435	0.000*
Canada	1964:3 – 2005:2	0.7317	0.000*

\* The concordance statistic is significant at the 1 percent level.

**Table 6. Multiple and Maximum Correlations**

Country	Multiple Correlation		Maximum Correlation	
	United States	1965:2 – 2005:2	$t^{(*)}$	1965:2 – 2004:4
0.6878		0	0.65005	0
Canada	1964:1 – 2004:3	$t^{(*)}$	1963:3 – 2005:3	$t^{(**)}$
	0.4353	0	0.41759	0

\* Denotes the time value where the effect on the dependent variable (equation1) from a unit change in the independent variables is maximum.  
\*\*Denotes the time value where the correlation is maximum ( $t=-4,-3,-2,-1,0, +1,+2,+3,+4$ ). A positive (negative or zero) value for  $t$  suggests lead (lag or contemporaneous) cycle of the Svar measure with respect to survey measure.

**Appendix: The Structural Var Model**

For the estimation of the rate of CU we use the SVAR methodology with long-run restrictions. This methodology was initially advanced by Blanchard and Quah (1989) and it has since been used in a number of applications. The variables that we use in our econometric specification are the growth rate of investment and the growth rate of profits. We suppose that these two variables are affected by the same structural shocks, which come either from the demand or from the supply side of the economy. The structural model can be written in terms of a moving average representation of current and past structural residuals.

$$DI_t = \sum_{n=0}^{\infty} a_{11}(n)u_{1t-n} + \sum_{n=0}^{\infty} a_{12}(n)u_{2t-n} \quad (A1)$$

$$DP_t = \sum_{n=0}^{\infty} a_{21}(n)u_{1t-n} + \sum_{n=0}^{\infty} a_{22}(n)u_{2t-n} \quad (A2)$$

where  $DI_t$  and  $DP_t$  denote the growth rates of investment and profits, while  $a_{ij}(n)$  stand for the individual coefficients. Finally,  $u_{1t}$  and  $u_{2t}$  represent the structural residuals, which are assumed to be serially uncorrelated, with their covariance matrix equal to the identity matrix.

$$E(uu^T) = I_2 = \Sigma_u \quad (A3)$$

<sup>10</sup> The threshold level for CU growth rates equals zero in this case.

In matrix representation, equations (A1) and (A2) can be written as follows:

$$DY_t = A(L)u_t \quad (A4)$$

$DY_t$  is the vector of endogenous variables,  $u_t$  is the vector of structural residuals and  $A(L)$  is a (2x2) matrix, where its elements  $A_{ij}$  are polynomials in the lag operator. In order to identify the structural residuals, we estimate an unrestricted VAR model of the form:

$$DY_t = \Theta(L)DY_{t-1} + \varepsilon_t \quad (A5)$$

Given that the stochastic process is stationary, (A5) can be rewritten an infinite-order moving average process:

$$DY_t = C(L)\varepsilon_t \quad (A6)$$

Where  $C(L) = (I_2 - \Theta(L)L)^{-1}$ . Thus, through relations (A4) and (A6) the structural residuals are connected to the estimated residuals in the following way:

$$\varepsilon_t = A(0)u_t \quad (A7)$$

Equation (A5) implies that the covariance matrix of the estimated residuals will be:

$$E(\varepsilon_t \varepsilon_t^T) = E[A(0)u_t (A(0)u_t)^T] = A(0)E(u_t u_t^T)A(0)^T = A(0)A(0)^T = \Sigma_\varepsilon \quad (A8)$$

Our purpose is to estimate the four elements of  $A(0)$  matrix, so a system of at least four equations is required. From the symmetric, positive definite covariance matrix of the estimated residuals  $\Sigma_\varepsilon$ , the next three equations could be derived.

$$a_{11}(0)^2 + a_{12}(0)^2 = \text{var}(\varepsilon_{1t}) \quad (A9)$$

$$a_{21}(0)^2 + a_{22}(0)^2 = \text{var}(\varepsilon_{2t}) \quad (A10)$$

$$a_{21}(0)a_{11}(0) + a_{22}(0)a_{12}(0) = \text{cov}(\varepsilon_{2t}, \varepsilon_{1t}) \quad (A11)$$

The above three equations contain four unknown parameters, that is the elements of the desired  $A(0)$  matrix; consequently, this system of equations cannot be solved. For the determination of the  $A(0)$  matrix, one more equation is needed. Combining equations (A4), (A6), and (A7), we have:

$$\begin{aligned} A(L)u_t &= C(L)\varepsilon_t \\ A(L)u_t &= C(L)A(0)u_t \\ A(L) &= C(L)A(0) \end{aligned} \quad (A12)$$

A restriction should be imposed to the elements of the matrix  $A(L)$  in order to specify one more equation. By transforming  $A(L)$  to a lower triangular matrix, we eliminate long-run effects of demand shocks on investment.

$$C_{11}(L)a_{12}(0) + C_{12}(L)a_{22}(0) = 0 \quad (\text{A13})$$

It is now possible to estimate all the elements of the  $A(0)$  matrix and thus to recover the structural residuals from the estimated residuals. In the moving-average representation, the change in investment can be expressed as a linear combination of current and past structural shocks:

$$DY_t = A_{11}(L)u_{1t} + A_{12}(L)u_{2t} \quad (\text{A14})$$

The change in investment growth due to demand shocks is defined as the investment gap and is equal to the second part of the right-hand side of equation (A14). In particular, the investment gap is given by:

$$A_{12}(L)u_{2t} \quad (\text{A15})$$

Based on the above estimated investment gap we can extract the equilibrium maintaining investment spending.