E C O N O M I C S   B U L L E T I N

The Transfer Paradox In a Two-Sector Overlapping Generatoins Model: The Duality Approach

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Abstract

In this paper, we apply the duality approach, which is generally used in a static framework, to a two-sector overlapping generations model. Applying the duality approach enables one to determine clearly the welfare effects of a transfer and to explain how the transfer paradox might occur. Especially, we showed that whether the transfer paradox occurs depends on two effects: the dynamic terms-of-trade effect and the capital accumulation effect.

I am truly grateful to my adviser, Assistant Professor Mitsuyoshi Yanagihara, for helpful suggestions and advice. Responsibility for any remaining errors is mine.
Citation: Shinozaki, Tsuyoshi, (2007) "The Transfer Paradox In a Two-Sector Overlapping Generatoins Model: The Duality Approach." *Economics Bulletin,* Vol. 6, No. 10 pp. 1-9
Submitted: October 25, 2006. **Accepted:** March 26, 2007.
URL: http://economicsbulletin.vanderbilt.edu/2007/volume6/EB-06F00007A.pdf

1. Introduction

In this paper, we apply the duality approach, which is generally used in a static framework, to a two-sector overlapping generations model. Our objective is to clearly determine the welfare effects of international transfers and to demonstrate how the transfer paradox can occur.

The first classic analytical work on the transfer paradox was by Samuelson (1952, 1954). He showed that in a perfectly competitive, two-good, two-country, distortion-free and Walrasian-stable world, the transfer paradox cannot occur. That is, international transfers cannot result in enrichment of the donor and impoverishment of the recipient. This is because the direct income transfer effect always dominates the indirect terms-of-trade effect. Subsequently, many researchers have investigated the transfer paradox, having relaxed the assumptions made by Samuelson. Most studies based on static frameworks have applied the duality approach. This is because this approach makes it easier to analyze the welfare effects of international transfers. In particular, Kemp (1992) revised the work of Samuelson (1952, 1954) by applying the duality approach.

The seminal paper that deals with the transfer paradox in a dynamic framework is by Galor and Polemarchakis (1987). They used a two-country, one-sector overlapping generations model. They showed that the transfer paradox may occur; that is, in a steady state that does not comply with the golden rule, the direct transfer effect may be dominated by the (indirect) capital-accumulation effect. Based on this work, to clarify how the transfer paradox may occur in a dynamic framework, Haaparanta (1989) incorporated government bonds and Yanagihara (1998) incorporated public goods. Shinozaki and Yanagihara (2006) used a two-sector overlapping generations model to show how paradoxical results can arise.

However, no study has yet applied the duality approach in a dynamic framework. Therefore, in this paper, we reconsider the results obtained by Shinozaki and Yanagihara (2006) by applying the duality approach. This approach has been used by Ihori (1996) in a one-sector overlapping generations model.

2. The Duality Approach

Our formulation is a straightforward extension of Galor and Lin’s (1997) two-country, two-sector overlapping generations model, which is based on Galor’s (1992) pioneering work based on a one-country, two-sector overlapping generations model.

Consider a world consisting of two countries, the donor and the recipient, denoted by D and R respectively, which are identical in all respects except for their rates of time preference. Economic activity takes place over an infinite discrete-time period and is conducted under perfect competition and certainty. Each country produces a pure (perishable) consumption good, X, and a pure (nonperishable and nonconsumable) capital good, Y, traded in every period, t. Both goods are produced from two factors, capital and labor. Because it is assumed that there is no population growth, the supply of labor in each country is $L_t^D = L_t^R = L_0^D = L_0^R$, for all $t \geq 1$, where L_t^i is the total labor supply (or population) in country i in period t. Labor is fully employed in either production sector, so that $1 = l_t^{i,x} + l_t^{i,y}$, where $l_t^{i,j} \equiv L_t^{i,j} / L_t^i$ $\left(0 < l_t^{i,j} < 1\right)$ is the aggregate labor ratio of sector j in country i in period t. Assuming that capital depreciates fully during production, the stock of capital in period $t+1$, K_{t+1}^i, is equal to the quantity of the capital good owned by country i in period t, $\hat{Y}_t^i$. Thus, given that $k_{t+1}^i \equiv K_{t+1}^i / L_{t+1}^i$ and $\hat{y}_t^i \equiv \hat{Y}_t^i / L_t^i$, in per worker terms, it follows that $k_{t+1}^i = \hat{y}_t^i$, where k_0^i is exogenously given.

The production of both goods takes place under constant returns to scale in capital and labor. In country i in period t, the output of the consumption good, X_t^i, and the capital good produced, Y_t^i, are $X_t^i = F_x\left(K_t^{i,x}, L_t^{i,x}\right) \equiv L_t^{i,x} f_x\left(k_t^{i,x}\right)$ and $Y_t^i = F_y\left(K_t^{i,y}, L_t^{i,y}\right) \equiv L_t^{i,y} f_y\left(k_t^{i,y}\right)$ respectively, where $k_t^{i,j}$ is the capital-labor ratio in sector j of country i. Suppose that labor and capital are perfectly mobile between sectors but are immobile internationally, and that the capital good is capital intensive. The factor payments of each sector in country i are therefore characterized by the first-order conditions for profit maximization by each firm,

$$r_t = p_t f_{k^x}^x\left(k_t^{i,x}\right) = f_{k^y}^y\left(k_t^{i,y}\right), \tag{1}$$

$$w_t = p_t\left[f^x\left(k_t^{i,x}\right) - f_{k^x}^x\left(k_t^{i,x}\right)k_t^{i,x}\right] = f^y\left(k_t^{i,y}\right) - f_{k^y}^y\left(k_t^{i,y}\right)k_t^{i,y}, \tag{2}$$

where r_t, w_t and p_t are the returns to capital, the wage rate and the relative price of the consumption good in terms of the capital good in period t, respectively. In this context, the subscripts in a function represent the partial derivatives of that function with respect to the corresponding variables; that is, $f^j_{k^j}\left(k^{i,j}_t\right) \equiv \partial f^i\left(k^{i,j}_t\right) / \partial k^{i,j}_t$. From (1) and (2), the factor prices, r_t and w_t, can be expressed as functions of p_t; that is, $w_t = w(p_t)$ and $r_t = r(p_t)$. Therefore, given k^i_t and p_t, for country i in period t, in per capita terms, the quantity of the consumption good, x^i_t, and the output of the capital good, y^i_t, are

$$x^i_t = x\left(p_t, k^i_t\right), \qquad y^i_t = y\left(p_t, k^i_t\right). \tag{3 - H, F}$$

Note that, under free trade, which implies that $\hat{y}^i_t = y^i_t \pm M^i_t$ (where M^i_t denotes the imports or exports of the capital good in country i in period t), factor prices and relative prices are equal in both countries.

We assume that, in each period, there are two types of individuals; those from the young generation and those from the old generation. In each country, individuals, who live for two periods, are identical within and between generations. During the first period of their lifetimes, individuals supply one unit of labor inelastically in exchange for wage income of w_t. Consider a permanent transfer from country D to country R. The net income of country i, denoted by τ_t (with $w_t - \tau_t$ and $w_t + \tau_t$ being the net incomes of the young in countries D and R, respectively) is split between consumption, $c^{i,1}_t$, and savings, s^i_t. Savings earn the given gross rate of return, r^i_{t+1}, in the following period and enable individuals to consume during retirement. Thus, individual consumption in the second period is $\frac{r_{t+1}s^i_t}{p_{t+1}} \equiv c^{i,2}_{t+1}$. Therefore, the lifetime budget constraints of individuals in countries D and R are $p_t c^{D,1}_t + \frac{p_{t+1}}{r_{t+1}} c^{D,2}_{t+1} = w_t - \tau_t$ and $p_t c^{R,1}_t + \frac{p_{t+1}}{r_{t+1}} c^{R,2}_{t+1} = w_t + \tau$, respectively.

The preferences of all agents of generation $t \geq 0$ in country i are represented by the utility function $U_t^i = u^i\left(c_t^{i,1}, c_{t+1}^{i,2}; \rho^i\right)$, where $u(\cdot,\cdot;\cdot)$ is twice continuously differentiable and strictly quasi concave, with $\rho^i > 0$ representing the rate of time preference; we assume that $\rho^R > \rho^D$.

For determining the equilibrium of the economy, it is convenient to specify the individual's expenditure minimization problem. This problem is

$$\min_{c_t^{i,1}, c_{t+1}^{i,2}} \left\{ p_t c_t^{i.1} + \tfrac{p_{t+1}}{r_{t+1}} c_{t+1}^{i,2} - w_t \middle| u^i\left(c_t^{i,1}, c_{t+1}^{i,2}; \rho^i\right) \geq U_t^i \right\}.$$

Given profit maximization by firms, the minimum expenditure required to achieve a certain level of utility, u_t^i, given p_t, p_{t+1}, $r\left(p_{t+1}\right)$ and $w\left(p_t\right)$, is obtained as the following solution:

$$E^i\left[p_t, p_{t+1}, r\left(p_{t+1}\right), w\left(p_t\right), u_t^i; \rho^i\right] = p_t c^1\left(p_t, p_{t+1}, r\left(p_{t+1}\right), u_t^i\right) + \frac{p_{t+1}}{r\left(p_{t+t}\right)} c^1\left(p_t, p_{t+1}, r\left(p_{t+1}\right), u_t^i\right) - w\left(p_t\right).$$

(4 - H, F)

Simultaneously, the following savings function is obtained:

$$s_t^i = s^i\left(p_t, p_{t+1}, u_t^i; \rho^i\right). \qquad \text{(5 - H, F)}$$

Consequently, the equilibrium of the economy is described by the following equations:

$$s^D\left(p_t, p_{t+1}, u_t^D; \rho^D\right) = k_{t+1}^D, \qquad \text{(6 - D)}$$

$$s^R\left(p_t, p_{t+1}, u_t^R; \rho^R\right) = k_{t+1}^R, \qquad \text{(6 - R)}$$

$$E^D\left[p_t, p_{t+1}, r\left(p_{t+1}\right), w\left(p_t\right), u_t^D; \rho^D\right] = -\tau_t, \qquad \text{(7 - D)}$$

$$E^R\left[p_t, p_{t+1}, r\left(p_{t+1}\right), w\left(p_t\right), u_t^R; \rho^R\right] = \tau_t, \qquad \text{(7 - R)}$$

$$s^D\left(p_t, p_{t+1}, u_t^D; \rho^D\right) + s^R\left(p_t, p_{t+1}, u_t^R; \rho^R\right) = y^D\left(p_t, k_t^D\right) + y^R\left(p_t, k_t^R\right). \qquad \text{(8)}$$

Equations (6 - D, R), (7 - D, R) and (8) represent the evolution of the capital stock, the individual's budget constraint in each country i in period t and the world market equilibrium, respectively. These five equations contain five variables, p_{t+1}, u_t^D, u_t^R, k_{t+1}^D and k_{t+1}^R, the parameter τ_t and the given variables p_t and k_t. In what follows, we omit time subscripts because we concentrate on steady-state properties.

As shown by Galor and Lin (1997), if the capital good is capital intensive, the country with the low rate of time preference (the donor country) exports the capital good (the capital-intensive good). For analytical convenience, we assume that the capital good is capital intensive, and, in addition, we assume that the economy is dynamically efficient.

3. The Effects of Transfers

To evaluate the effect of a transfer on p, u^D and u^R in the steady state, we totally differentiate equations (6 - D, R) to (8), and obtain

$$\begin{bmatrix} E_p^D & 1 & 0 \\ E_p^R & 0 & 1 \\ W_p & s_{u^D}^D\left(1-y_k\right) & s_{u^R}^R\left(1-y_k\right) \end{bmatrix}\begin{bmatrix} dp \\ du^D \\ du^R \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} d\tau$$

where $W_p \equiv \left(s_p^D + s_p^R\right)\left(1-y_k\right) - \left(y_p^D + y_p^R\right) > 0$, and $s_{u^i}^i > 0$ represents the marginal propensity to save. Note that because we have normalized $E_{u^i}^i = 1$, it follows that $1 - y_k < 0$ represents Rybczynski's magnification effect. Hence, we obtain

$$\frac{dp}{d\tau} = \frac{1}{\Delta}\left(s_{u^R}^R - s_{u^D}^D\right)\left(y_k - 1\right) < 0, \tag{9}$$

$$\frac{du^D}{d\tau} = \frac{1}{r}\left[\left(-r_p\right)\left(r-1\right) + \left\{x^D - \left(c^{D,1} + \frac{1}{r}c^{D,2}\right)\right\}\right]\frac{dp}{d\tau} - 1, \tag{10}$$

$$\frac{du^R}{d\tau}=\frac{1}{r}\left[\left(-r_p\right)\left(r-1\right)+\left\{x^R-\left(c^{R,1}+\frac{1}{r}c^{R,2}\right)\right\}\right]\frac{dp}{d\tau}+1 \tag{11}$$

where $r_p<0$ represents the Stolper–Samuelson effect, and $\Delta = s_p^D + s_p^R - \left\{\left(y_p^D + y_p^R\right) + \left(E_p^D s_{u^R}^D + E_p^R s_{u^R}^R\right)\right\}\left(1-y_k\right)$ is the determinant of the above matrix. It can be shown that $\Delta > 0$ is not only a sufficient but also a necessary condition for dynamic stability.

Given (9), a permanent transfer from country D to country R increases savings in country R by s_u^R units on the one hand, but on the other hand, it reduces savings by s_u^D units in country D. In a static model, only these two terms would appear, which implies that relative prices rise by $\left(-\frac{1}{\Delta}\right)\left(s_{u^R}^R - s_{u^D}^D\right)$. The novel feature of the present model is the inclusion of the magnification effect, which is represented by the term in the final bracket, $\left(\frac{y_k}{\Delta}\right)\left(s_{u^R}^R - s_{u^D}^D\right)$. Because the magnification effect is greater than the static effect, $dp/d\tau$ is negative.

Applying these findings to (10) and (11) reveals that the welfare effects in both countries can be divided into three distinct effects: (i) the capital accumulation effect; (ii) the dynamic terms-of-trade effect; and (iii) the direct transfer effect. The first term in the square brackets of (10) and (11), $\left(-r_p\right)\left(r-1\right)\frac{dp}{d\tau}$, corresponds to the capital accumulation effect, which, because we assume dynamic efficiency, is negative in both countries. The second term in the square brackets, $\left\{x^i-\left(c^{i,1}+\frac{1}{r}c^{i,2}\right)\right\}\frac{dp}{d\tau}$, corresponds to the dynamic terms-of-trade effect, which is negative in one country and positive in the other; this is because of the direction of trade. A similar terms-of-trade effect arises in the static framework: the only difference is that in the dynamic framework, consumption demand is the sum of consumption by the young and old. Effects (i) and (ii) are the indirect effects of the transfer. The third term, which is $+1$ for country R and -1 for country D, represents the direct transfer effect.

The transfer paradox can occur if and only if the dynamic terms-of-trade effect dominates the other two effects in country D, and if the capital accumulation effect and the dynamic terms-of-trade effect dominate the direct transfer effect in country R (as indicated by equations (10) and (11), respectively).

Proposition *If the rate of time preference in a donor country is higher than that in a recipient country, a permanent transfer may give rise to the transfer paradox.*

Note that in this overlapping generations framework, there is a capital accumulation effect. *If the economy is dynamically inefficient, this effect is positive for welfare in both countries. This is because world demand for the capital good falls by the amount of the permanent transfer.* In addition, if the economy follows the golden rule, this effect is negligible for welfare in both countries.

Next, we evaluate the effect of a transfer on world welfare.

Lemma *If the rate of time preference in a donor country is higher than that in a recipient country when there is dynamic inefficiency, in which case,* $r < n = 1$ *(with dynamic efficiency requiring* $r > n = 1$*), a permanent transfer increases (decreases) world welfare.*

Proof Summing (10) and (11) in $r < n = 1$ ($r > n = 1$) yields the condition under which world welfare increases (decreases). ■

4. Conclusion

Applying the duality approach enables one to determine clearly the welfare effects of a transfer and to explain how the transfer paradox might occur. In this paper, we showed that whether the transfer paradox occurs depends on two effects: the dynamic terms-of-trade effect and the capital accumulation effect.

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