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Hurst exponents, power laws, and efficiency in the Brazilian foreign exchange market

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Abstract

We find evidence of weak informational efficiency in the Brazilian daily foreign exchange market using Hurst exponents (Hurst 1951, 1955, Feder 1988), which offer an alternative (from statistical physics) to traditional econometric gauges. We show that a trend toward efficiency has been reverted since the crisis of 1999. We also find power laws (Mantegna and Stanley 2000) in means, volatilities, the Hurst exponents, autocorrelation times, and complexity indices of returns for varying time lags.

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1. Introduction

Previous work has found evidence of weak informational efficiency in the Brazilian daily foreign exchange market using standard econometric techniques (Laurini and Portugal 2002, 2004). This note replicates such a result but employs Hurst exponents (Hurst 1951, 1955, Feder 1988), which offer an alternative gauge of informational efficiency (Cajueiro and Tabak 2004) from the perspective of statistical physics. The standard deviation in independent, normally distributed series behaves as $\sigma(t) \sim t^H$, where H = ½ and t is time (Gnedenko and Kolmogorov 1968). The exponent of this scaling relationship between the standard deviation of a time series and the time increments used is the Hurst exponent.

So an exponent H = ½ gives indication of a Brownian motion (random walk), i.e. a random process with no long range memory. The efficient market hypothesis thus assumes H = ½. Therefore values different from ½ suggest long range memory and then that data points are not pairwise independent. Values ranging from ½ to one are indicative of a persistent, trend-reinforcing series (positive long range dependence). And positive values that are shorter than ½ suggest antipersistence, i.e. that past trends tend to reverse in future (negative long range dependence).

Yet we go further and show pervasive regularities in the *real*-dollar returns $r(t) \equiv e(t+\Delta t) - e(t)$ for varying Δt (where e is the exchange rate in levels). Studying returns by extracting several subsets of non-overlapping price changes $r(t)$ by varying Δt from 1 to n periods is common in the realm of "econophysics" (Mantegna and Stanley 2000, chapter 9). Plots of periods 1 to n against Δt usually show straight lines on a log-log scale (power laws) until some finite date. Thereafter, scaling breaks down. Power laws are suggestive of lack of a typical scale in a series range (Mantegna and Stanley 2000, chapters 1, 4). Symmetry in big and small scales is meant, for instance, that daily changes are essentially similar to changes in an intraday frequency (fractality).

Section 2 presents data and power laws in the first two statistical moments of returns. Section 3 reckons Hurst exponents and assesses informational efficiency. Section 4 shows power laws in the autocorrelation time and in a measure of complexity of the series. And Section 5 concludes.

2. Data and Power Laws in Statistical Moments

The series covers the period from 2 January 1995 to 31 August 2006. The set is obtained from the Federal Reserve website. The series has a unit root in levels but gets stationary in first differences. This is already known in literature with the help of Perron's test for series with structural breaks (Moura and Da Silva 2005). Thus daily returns are stationary.

The series presents a structural break at the naked eye in 13 January 1999, when a currency crisis struck. A previous fixed exchange rate regime of "exchange rate anchor" made way for a floating rate. Yet Table 1 shows the mean to be similar in both regimes. Despite the fact that the volatility in the floating regime is about ten times bigger, this does not seem to interfere with the stationarity of the returns' mean.

We detected regularities in these returns $r(t) \equiv e(t+\Delta t) - e(t)$ as Δt was let to vary from 1 to 1000. Not surprisingly, both the means and volatilities grow as Δt is raised. Yet it is remarkable that power laws govern the changes. Figure 1 shows these findings. The statistical moments can thus be expressed as $\omega(\Delta t)^{\beta}$, where the effect of ω on the moments is larger the greater Δt is (Gleria *et al.* 2002).

Scaling symmetry in moments is dubbed "structure function analysis" and can be exploited for forecasting (Richards 2004), although it is far from being obvious that profits can be made from this after adjusting for transaction costs and risk. Scaling is also related to the degree of multi-fractality (Schmitt *et al.* 2000) of a series and can inform the type of the underlying distribution.

To evaluating the hypothesis of fractality we were able to replicate all the findings in this paper to an intraday frequency dataset of the *real*-dollar returns (not shown). The 15-minute spaced set comprises data points from 9:30AM of 19 July 2001 to 4:30PM of 14 January 2003 (source: Agora Senior Consultants). Self-similarity in price changes of both the daily and intraday frequencies could not be dismissed, but this result may be blurred by aspects of the market microstructure (Campbell *et al.* 1997).

3. Informational efficiency and Hurst exponent

For the entire sample of single returns ($\Delta t = 1$) of the daily *real*-dollar rate, we reckoned a Hurst exponent $H = 0.54$. The exponent is also similar for portions of the dataset (Table 1). The figures are compatible with the finding of weak efficiency in the *real*-dollar market, i.e. they are slightly different from ½. Yet there is also room for autocorrelation in the series.

Although the Hurst tracks long range dependence, our focus in this paper is its use as a measure of informational efficiency. Estimates of H can be misleading when the series has either a jump in the mean or a slow trend (Teverovsky and Taqqu 1997). Our series has a jump, but the means are similar in both regimes (Table 1). Despite that we still think that the issue of long range dependence versus short memory with regime switching calls for more research along the lines suggested by, for instance, Teverovsky and Taqqu (1997) and Shimotsu (2006).

Anyway as Δt is raised in the definition of returns, the Hurst exponents are expected to grow (because aggregation is heightened). Yet, surprisingly, there is a power law governing the growth pace of the exponents (Figure 2).

The exponents above were calculated using *Chaos Data Analyzer* (Sprott and Rowlands 1995), whose program does not rely on rescaled range (*R*/*S*) analysis (Hurst 1951). Since the value of the variable on average moves away from its initial position by an amount proportional to the square root of time (in which case H = ½, as observed), the program plots the root-mean-square displacement versus time, using each point in the time series as an initial condition. The slope of this curve is the Hurst exponent. (More details on this technique can be found in Sprott (2003).)

We also reckoned the exponents using *R*/*S* analysis. Given that the variable displacement scales as the square root of time, Hurst expressed the absolute displacement in terms of rescaled cumulative deviations from the mean (R_n/S_n) and defined time as the number of data points (n) used. The scaling exponent of the relationship $R_n/S_n = cn^H$ (where c is a constant) is now the Hurst exponent. If data are independent, the distance traveled will increase with the square root of time and H = ½. Our calculation with *R*/*S* analysis showed an even bigger exponent, thereby reinforcing the case for slight departure from efficiency. In the best fit to straight line $\ln[R(n)/S(n)] = -0.536 + 0.62886\ln(n)$, the Hurst exponent $H = 0.63$ is implied.

Most studies in literature finding $H \neq ½$ fail to provide an accompanying significance test (Couillard and Davison 2005). Thus we carried out Couillard and Davison's suggested test for the above finding (Table 2). We found the exponent to be statistically significant with *p*–value < 0.001.

R/*S* analysis has been criticized for not properly distinguishing between short and long range memory (Lo 1991). Suggested modifications (Lo 1991), however, present a bias against the hypothesis of long range dependence (Teverovsky *et al.* 1999, Willinger *et al.* 1999). More recently, it has been suggested to filter *R*/*S* analysis by an AR(1)–GARCH(1, 1) process (Cajueiro and Tabak 2004). (We will adopt this suggestion below.)

We also examined time-varying Hurst exponents (reckoned by *R*/*S* analysis) to evaluating whether the series gets more or less efficient as time goes by (Cajueiro and Tabak 2004). When examining the time evolution of the Hurst of daily *real*-dollar returns (Figure 3), we considered a moving time window of four years (1008 observations at a time). Then we checked the respective histogram to examine whether the exponents are normally distributed, in which case variations should be ascribed to measurement errors. Data were filtered by an AR(1)–GARCH(1, 1) process given by $r_1(t) = a + \psi r_1(t-1) + \varepsilon(t)$, $\varepsilon(t) = s(t)\sqrt{h(t)}$, $h(t) = b + \Theta_1\varepsilon^2(t-1) + \Theta_2 h(t-1) + \Psi D(t)$, where a, b, ψ, Ψ, Θ_1, Θ_2 are estimated parameters, $h(t)$ is conditional variance of the residuals, and $s(t)$ is assumed to be normally distributed and independent of $s(t')$, for $t \neq t'$.

Figure 3 shows the Hurst approaching ½ by nearly observation 1010 (December 1998), after a previous overshooting. This means that the market gets more efficient. Yet from December 1998 on the Hurst moves away from ½. Figure 3 also shows the 95 percent confidence bounds using Couillard and Davison's test, i.e. 0.4811 and 0.6277 respectively (under the null hypothesis that the time series is both independent and Gaussian).

Our finding makes sense. Until December 1998 the Brazilian central bank had devalued the currency at nearly 0.003 per cent on a daily basis. Since market participants could easily take advantage of such a piece of information, it is not so surprising the market to become more efficient. After the currency crisis of 13 January 1999, the *real*-dollar rate was let to float. Several shocks, ranging from domestic macroeconomic and political problems to contagion of foreign currency crises, have made the processing of new information hitting the market more difficult. And this might explain why the foreign exchange market has become less efficient since then.

4. Autocorrelation time and complexity

Because the Hurst exponents calculated are compatible with the presence of autocorrelation, we examined the behavior of the autocorrelation time, which measures how much a current observation depends on the previous ones. The autocorrelation time is expected to increase with Δt. Yet that a power law governs its growth rate is surprising (Figure 4).

Related to both the Hurst exponent and autocorrelation time is the index of Lempel-Ziv (*LZ*) complexity relative to Gaussian white noise (Lempel and Ziv 1976, Kaspar and Schuster 1987). An *LZ* index of zero is associated with perfect predictability, and an index of about one gives piece of evidence of genuine randomness (maximum complexity). To reckon the algorithmic complexity of a series, a data point is converted into a binary digit and then compared to the median of the entire series.

For single returns ($\Delta t = 1$) of the daily *real*-dollar rate we found LZ = 1.04. Such a figure is consistent with both weak efficiency and the Hurst exponents above. As Δt is raised, heightened aggregation introduces more structure in the series, these get more predictable, and thus the LZ index tends to decay toward zero. Yet it is still remarkable that a power law governs the decay (Figure 5).

5. Conclusion

We find Hurst exponents that are not at odds with the usual result in econometric studies that the daily *real*-dollar market is weakly efficient. Time-varying Hurst exponents also show that a trend toward informational efficiency has been reverted since the crisis of 1999. Central bank intervention turned the market more predictable, more informationally efficient, but might also have precipitated the crisis.

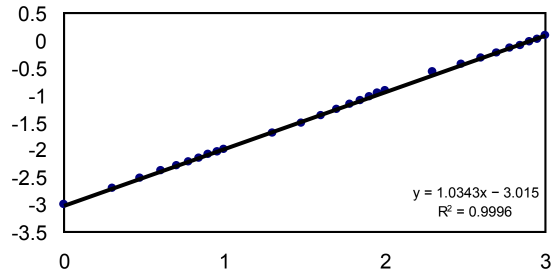
Allowing the time lag to rise in the definition of returns, we also find power laws in mean, volatility, Hurst exponent, autocorrelation time, and a complexity index.

Table 1 Daily *Real*-Dollar Returns' Descriptive Statistics and Hurst Exponent

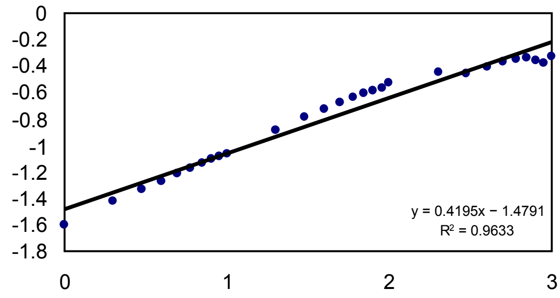
Time Period	2 Jan 95–12 Jan 99	13 Jan 99–31 Aug 06	2 Jan 95–31 Aug 06
Data Points	1009	1921	2930
Mean	0.00036	0.00043	0.00044
Standard Deviation	0.002	0.029	0.024
Skewness	0.73	0.02	0.06
Kurtosis	32.82	8.96	15.2
Hurst Exponent	0.55	0.51	0.54

Table 2 Couillard and Davison's Significance Test for the Hurst Exponent Calculated by *R*/*S* Analysis

Time Period	2 Jan 95–31 Aug 06
Data Points	2930
Hurst Exponent	0.63
$\|t$ statistic$\|$	3.26
p–value	< 0.0006

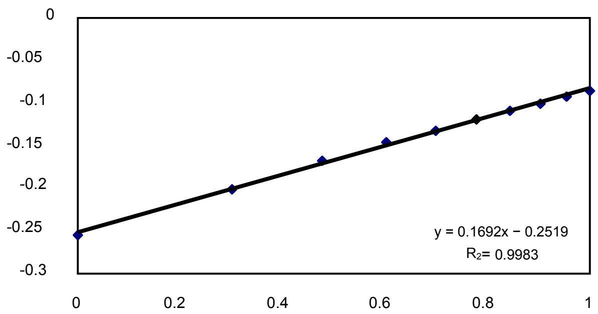


(a)

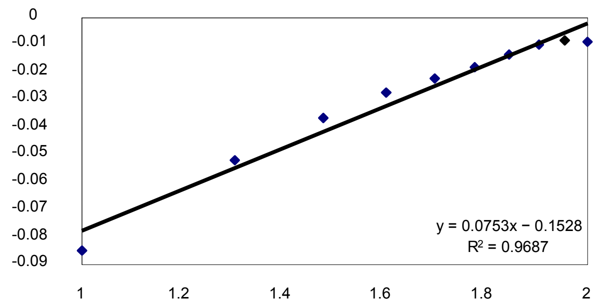


(b)

Figure 1. Log-log plots of means (a) and standard deviations (b) (vertical axes) versus $\Delta t = 1, \ldots, 1000$ (horizontal axes). Power laws (straight lines) emerge for increasing lags of the *real*-dollar returns.



(a)



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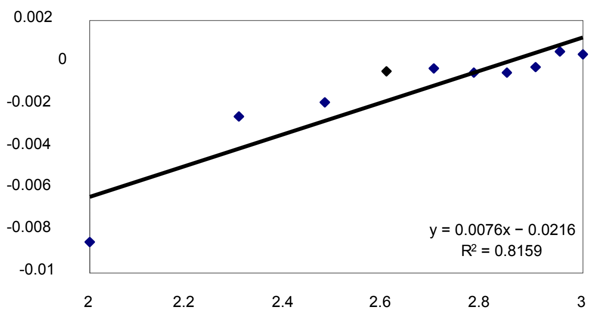
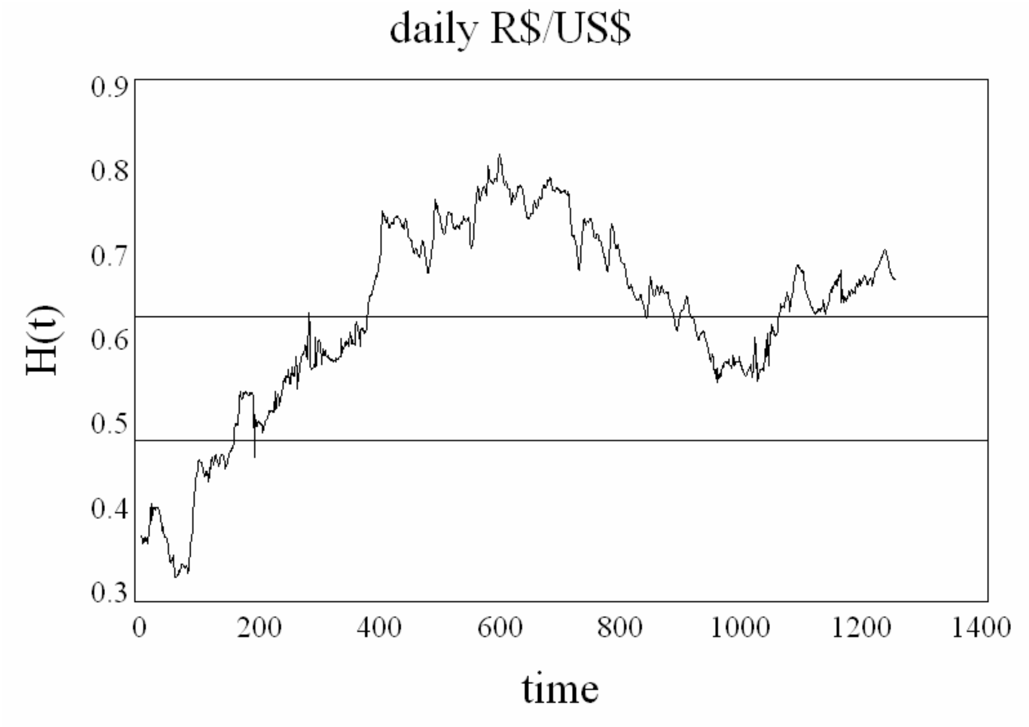


(c)

Figure 2. Power law in the Hurst exponent (logs in the vertical axes) for the daily *real*-dollar returns when the time lag is raised in the definition of returns (logs of Δt in the horizontal axes).



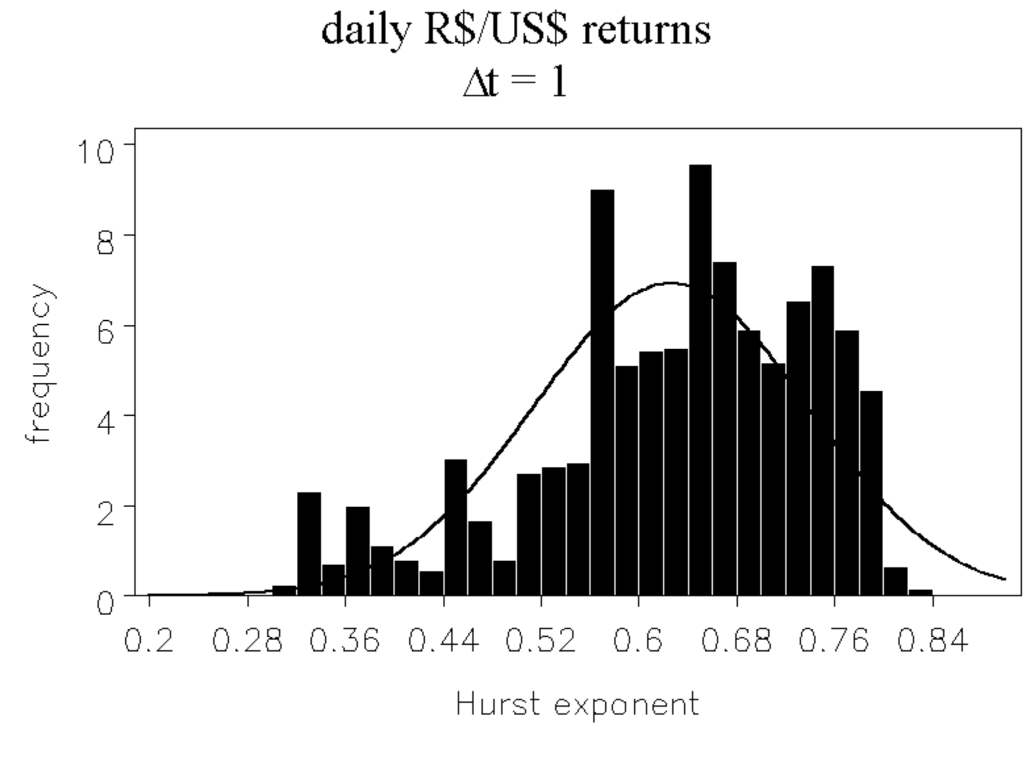


Figure 3. Time varying Hurst exponents for the daily *real*-dollar rate filtered by an AR(1)–GARCH(1, 1) (top), and their histogram (bottom). Horizontal lines are upper and lower 95 percent confidence bounds under the null hypothesis that the time series is both independent and Gaussian.

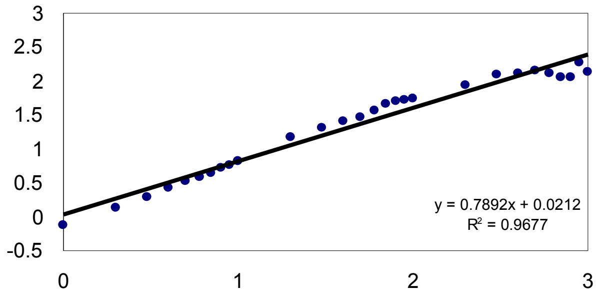


Figure 4. Power law in the autocorrelation time (logs in the vertical axis) for increasing lags of the daily *real*-dollar returns (logs of Δt in the horizontal axis).

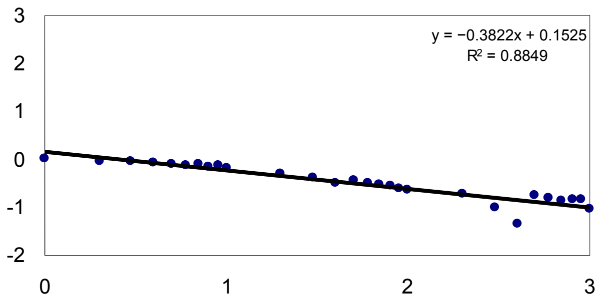


Figure 5. Power law in relative *LZ* complexity (logs in the vertical axis) for increasing lags of the daily *real*-dollar returns (logs of Δt in the horizontal axis).

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