E C O N O M I C S B U L L E T I N

Volatility Clustering in High-Frequency Data: A self-fulfilling prophecy?

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Abstract

Clustering volatility is shown to appear in a simple market model with noise trading simply because agents use volatility forecasting models. At the core of the argument lies a feed-back mechanism linking past observed volatility to present observed volatility. Its stability properties are critical as to what kind of volatility will ultimately be observed.

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1 Introduction

Financial data exhibits certain typical properties, such as (markedly) non-normal return distributions or strong intermittent fluctuations. Regarding return variations, Mandelbrot observed in 1963 that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes", or that stock volatility tends to cluster.

However, almost 20 years passed until a time-series model exhibiting such time-changing variance (as a proxy for volatility and risk) was developed: the ARCH model of Engle (1982). Engle applied his model to the UK inflation, but the model was quickly adopted for use with other financial data.

Ever since, many time-series or econometric models for conditionally heteroscedastic time series have been put forward. Virtually all of these are applied with financial data, with the intent of modeling and especially forecasting time-varying volatility. The GARCH model of Bollerslev (1986) allows for more parsimonious modeling, the E-GARCH model of Nelson (1991) captures the so-called leverage effect, first noted by Black (1976), according to which changes in stock price correlate negatively with changes in stock volatility, and the ARCH-M model of Engle et al. (1987) accounts for risk premia associated in standard investment theory with risky assets. There also are multivariate generalizations for use in portfolio optimization, due to Baba et al. (1990), Engle and Kroner (1995), or, more recently, Engle (2002). See Poon and Granger (2003) for a comprehensive survey, with special attention to the volatility forecasting aspect.

With macroeconomic, low-frequency data, volatility clusters can be explained by variations in economy-wide uncertainty. This also appears to be true for low-frequency financial data. Using monthly volatility estimates based on daily returns of the S&P and Dow Jones composite portfolios, Schwert (1989) finds evidence in favor of this explanation, showing that volatility increases in recession times and drops in boom periods.

However, this argument is not applicable to high-frequency data, such as intra-day returns of stock prices, exchange rates or stock indices. One approach to explaining volatility clustering in this kind of data is to examine the market microstructure, see O'Hara (1995) for an introduction to the market microstructure approach. Within this framework, Brock and LeBaron (1996), Cabrales and Hoshi (1996) or Timmermann (2001), among others, have carried out research concerning volatility clustering. As a common feature, their models all assume some degree of information asymmetry across traders. Granger and Machina (2006) take a different approach and analyze structural, time-invariant economic systems with non-heteroscedastic random shocks, which, under suitable assumptions, also lead to volatility clustering. All these models can explain the behavior of high-frequency data, even if they were not specifically designed for it. This problem has attracted the attention of physicists as well, who argue that the complexity of the interactions on the market is similar to that of many physical systems. While it does exhibit some microstructure aspects, this approach is not usually categorized as such. Typical examples are Giardina et al. (2001) or Wagner (2003).

In this note, we draw attention to yet another possible explanation of this problem. We analyze a simple model of a financial market with noise trading (i.e., the respective traders do not have any special information, but buy/sell for exogenous reasons, like portfolio adjustment), where traders actively forecast volatility. We focus on an order-driven market, motivated both by the actual propagation of this form of exchange, and by theoretical arguments (cf. Glosten, 1994). Informational asymmetry is not excluded, but, at the same time, is not a necessary assumption. Our contribution is to show how volatility clusters can appear as a consequence of the volatility forecasting activity itself.

The remainder of this contribution is structured as follows. We begin by describing the market equilibrium model we work with. Then, a heuristic study of its dynamic properties is given and our statement is derived. The final section concludes.

2 The model

Our starting point is the market clearing condition, where supply of the traded asset satisfies the demand. Let us consider how the supply and demand curves are established. Each trader believes the asset has a certain value. Since these values differ across traders, it is more appropriate to talk about perceived values. These would be scattered around a middle value. Many traders use statistical forecasting models based on past prices and/or fundamental data, but there also are traders using technical analysis to estimate their perceived value and derive trading rules.

Traders would only be likely to sell (buy) if the price is higher (lower) than their perceived value adjusted for the risk of the security: The higher this risk, the higher the price correction. Its sign depends on the risk attitude of the trader. A risk-averse buyer would ask for a price reduction to compensate for the risk, while a risk lover would be willing to pay a higher price than his/her perceived value. This is because the risk lover sees the chances the value variability has to offer, while the risk averse is more perceptive of possible losses. When acting on the market, traders place limit orders which express their limit prices and the desired amount.

This risk is by no means absolute. It is also a perceived risk and may differ from trader to trader. Often, it is proxied by the volatility of the asset, but each trader may have supplementary criteria to decide upon risk. For individual stocks, for instance, supplementary knowledge, e.g. about the management, may induce additional risk to the one inferred from volatility of the stock alone. Similar to the perceived values, these perceived risks would be centered around a middle value, let us call it perceived market risk. This stands for the "majority opinion" about risk of the stock. If there is no reason why the risk-adjustment should depend on the perceived value, the scatter of the risk adjustments is simply added to the dispersion of the perceived values.

Assume noise traders need immediate execution of their orders. Since higher market risk implies a higher scatter of the limit orders, in terms of price, it also implies a higher price impact of market orders, these being matched against limit orders. This effect is especially interesting, because of two reasons.

On the one hand, a higher price impact of market orders implies a higher realized volatility of the changes in the stock price, the price impact actually being this change and its absolute value a measure of realized volatility. Note that, with only noise trading taking place, price impacts alone lead to price changes. Of course, the size of incoming orders also influences the price impact, but, with noise trading, this size is not expected to vary much.

On the other hand, traders use past data in order to evaluate current volatility of the stock. They may use different models, or methods, in addition to ARCH-type ones, such as historical estimates or the so-called implied volatility (the volatility implied by prices of options on the same asset). Still, it is very unlikely that these methods do not agree at all. Assume, for our purposes, that the bulk of the market uses a model belonging to the ARCH/GARCH family. Thus, an increase of recently observed volatility leads to higher estimates of current volatility and thus higher perceived market risk. Correspondingly, a decrease of recently observed volatility leads to lower perceived market risk.

In turn, changing perceived market risk eventually leads to changing price impacts. Hence, present and past volatility estimates are linked in a feed-back loop. This loop is independent of trends in returns, since volatility models allow for changing mean values.

As a remark, note the relationship to the liquidity of the respective financial market. Indeed, liquidity is often measured by price impacts (Aitken and Comerton-Forde, 2003): the higher the price impact of a fixed-size order, the lower the liquidity. However, Gouriéroux et al. (1999) or Sandås (2001), among others, find that liquidity follows certain patterns during the day, tending to be higher around opening, rather than clustering the way volatility does, although higher volatility around opening and closing times is sometimes observed.

Note that the described situation is a case of positive feed-back, since increasing perceived market risk leads to increased price impacts, or higher observed volatility, which increases volatility estimates, thus inducing the belief that market risk would have increased, and so on. This feed-back could be broken, if traders had sticky beliefs. That is, if traders are relatively reluctant to accept short-term volatility fluctuations as proof of modified risk, they will not adjust their perceived values and perceived risks, or only adjust them marginally.

On the other hand, if traders do react to such short-term fluctuations, they ultimately modify the observed volatility (by the mechanism described before), which enforces the initial change in perceived risk.

We argue that traders *will* react to changes in volatility, and this because the GARCH-type models used by them to forecast volatility are especially designed to react to changes in observed volatility. Hence, it is a plausible scenario that one observes volatility clusters simply because everybody expects them.

The existence of active noise trading is a key assumption, without which the feed-back link is broken. If there isn't some market activity, one cannot observe modified price impacts and adjust his/her volatility forecast.

With this model, three different possibilities may arise. First, the volatility process could be stable. This would induce clusters in observed volatility. Second, volatility may "wander about“, and exhibiting no decay of serial dependence or its serial dependence decays very slowly. Indeed, there is a whole literature on I-GARCH, see Engle and Bollerslev (1986), and FIGARCH, see Bollerslev and Mikkelsen (1996) or Baillie et al. (1996). These contributions also contain some empirical evidence in favor of the IGARCH and FIGARCH models. Third, volatility may have explosive phases, but this is likely to only happen in bubbles.

3 Concluding remarks

We present a simple market model which explains how volatility clustering arises in the presence of noise trading in an order-driven market as a consequence of volatility forecasting itself.

However, we do not suggest that volatility clustering is entirely self-induced. We rather believe that its importance may be overrated due to over-modeling observed volatility variations. This may happen when traders tend to overlook the fact that observed volatility is not the same thing as "true" volatility and fail to examine data with a critical eye.

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