

A note on the link between public expenditures and distortionary taxation

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Abstract

This note deals with the optimal provision of a public good in the context of the Ramsey tax model. It is shown that the second-best level of public good provision is inefficiently low relative to a situation where additional expenditures can be financed by lump-sum taxation.

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1. Introduction

The appropriate size of government expenditures is a prominent issue in economic debate. This topic is usually discussed by comparing marginal costs and marginal benefits of public spending. With respect to the cost side of such a comparison, the excess burden of taxation has gained considerable attention in the literature. This issue goes back to a hypothesis first proposed by Pigou (1947) who argued that distortionary taxation inflicts indirect damage on the representative citizen such that tax financed provision of a public good should not be carried out as far as would be optimal in a fictitious world with lump-sum taxation.

In terms of formal analysis, this hypothesis has been interpreted in two different ways. First, it was translated to the claim that the sum of the marginal rates of substitution $\sum \text{MRS}_{Gk}$ between a public good G and a private reference commodity k exceeds the marginal rate of transformation MRT between the public good and the private commodity in the second-best optimum with distortionary taxation. Second, it was suggested that the optimal level of public good provision G^S in the presence of distortionary taxation is below the first-best provision level G^F , where lump-sum taxes are used for financing public expenditures (see e.g. Atkinson and Stiglitz 1980, Ch. 16 and Myles 1995, Ch. 9 for a discussion of these hypotheses). However, none of these statements is correct: First, using Ramsey's tax model, Atkinson and Stern (1974) have shown that the claim $\sum \text{MRS}_{Gk} > \text{MRT}$ holds only in specific circumstances, which depend on the choice of the reference commodity k . Second, some recent analyses have pointed out that counterexamples to the claim $G^S < G^F$ can be constructed as well (de Bartolomé 1998, Gaube 2000, Gronberg and Liu 2001). Taken together, these findings raise the question whether the intuitive claim that a less efficient tax system should lead to a lower level of public expenditures can be justified by a robust formal property of the second-best optimum at all.

In this note, such a justification is provided: I show that a local increase in public good provision is desirable in second best as long as additional expenditures can be financed by means of lump-sum taxation. This finding is in the spirit of another local result derived by Atkinson and Stern (1974). They consider the first-best optimum and show that a marginal reduction of the lump-sum tax decreases the optimal level of public good provision for the case of additively separable preferences.¹ The present analysis refers to the more interesting second-best allocation and relies only on the assumption that preferences are strictly quasiconcave. In contrast to earlier analyses, the finding is established by analyzing the second-best problem in quantity space. In this way, I also provide a geometric characterization of the second-best provision rule, which shows that Samuelson's condition holds in the optimum with distortionary taxation as long as it is taken into account that a change in tax rates affects all private commodities simultaneously.

¹Since the (fictitious) first-best optimum is of limited interest in the present context, the local approach proposed by Atkinson and Stern (1974) received little attention in the literature. For an exception, see Mirrlees (1994).

2. The Model

Consider an economy with H identical individuals² whose preferences are described by a strictly quasiconcave utility function $U(x, l, G)$, where x, l , and G stand for private consumption, leisure, and a public good respectively. Each individual has an endowment of $e > 0$ units of leisure which can be used to produce the commodities x and G by means of a linear technology. Normalizing the marginal rates of transformation MRT to unity, the per-capita production constraint

$$e - l - x - \frac{G}{H} \geq 0 \tag{1}$$

is obtained. It is assumed that the public good is provided for free by the government, which finances its expenditures either by linear taxes t_l, t_x imposed on labor supply $e - l$ and consumption x , or by means of a lump-sum tax T . The producer prices of x, l , and G are normalized to unity. Hence, the consumer prices of x and l equal $q = 1 + t_x$ and $w = 1 - t_l$. Accordingly, the budget constraint of the individuals can be written in the form $we - wl - qx - T \geq 0$. Maximizing utility $U(x, l, G)$ subject to this constraint, the demand functions $x(q, w, T, G), l(q, w, T, G)$ and the indirect utility function $V(q, w, T, G)$ are obtained.

The benevolent government maximizes welfare of the representative individual subject to the budget constraint

$$(q - 1)x + (1 - w)(e - l) + T - \frac{G}{H} \geq 0. \tag{2}$$

As long as the lump-sum tax T is available, no distortionary taxes t_l, t_x are imposed in the optimum and a first-best allocation (x^F, l^F, G^F) is implemented. In this case, the Samuelson conditions $\sum \text{MRS}_{Gx} = 1$ and $\sum \text{MRS}_{Gl} = 1$ are satisfied, where $\text{MRS}_{Gx} := U_G/U_x$ and $\text{MRS}_{Gl} := U_G/U_l$ denote the agents' marginal rates of substitution between the public good and the two private commodities x and l , respectively.

Consider now the second-best framework, where lump-sum taxation is ruled out by assumption. The common way of investigating this model proceeds as follows: First, one of the tax rates t_x, t_l is normalized to zero such that the corresponding commodity x or l serves as the unit of account. Then the indirect utility $V(\cdot)$ is maximized with respect to G and the remaining tax rate subject to the government's budget constraint (2). Based on the first-order conditions of this problem, a modified Samuelson rule $\sum \text{MRS}_{Gk} = \text{MCF}$ is derived, where $k \in \{x, l\}$ is the unit of account and MCF is the marginal cost of public funds. The interpretation of this rule, however, is difficult because the MCF - in contrast to the second-best allocation - is different for the cases $t_x = 0$ and $t_l = 0$. In particular, the MCF can be above or below unity depending on whether x or l is chosen as the numéraire (see e.g. Mayshar 1990, Håkonsen 1998, and Gaube 2000 for a discussion). Therefore, I will complement these analyses by investigating the second-best

²In the present context, only the efficiency effects of distortionary taxation and public good provision are of interest. Clearly, distributional effects may play an important role as well (see e.g. Wilson 1991, Mirrlees 1994, Sandmo 1998, and Gaube 2000, 2005).

problem in commodity space. In this context nominal prices play no role and $t_l = 0$ can be chosen just for convenience. Since lump-sum taxation is ruled out by assumption, the government's budget constraint (2) then reduces to

$$B(q, G) := (q - 1)x(q, 1, 0, G) - \frac{G}{H} \geq 0.$$

Using this constraint, we can define

$$OC := \{l, x \mid \exists q, G : l = l(q, 1, 0, G), x = x(q, 1, 0, G), B(q, G) = 0\}.$$

The set OC contains all pairs (l, x) that can be implemented in an equilibrium with distortionary taxation provided that the government's budget is balanced. Note that this set is equivalent to the agent's offer curve as long as the demand functions $x(\cdot)$ and $l(\cdot)$ are independent of G .³ In general, however, OC represents a modified offer curve because the effect of public good provision on private demand $x(\cdot), l(\cdot)$ is taken into account as well.

Since the elements of OC fulfil all restrictions that follow from distortionary taxation, any vector (x, l, G) that satisfies OC and the production constraint (1) can be implemented. Accordingly, the second-best allocation can be defined as follows:

$$(l^S, x^S, G^S) := \operatorname{argmax}_{l, x, G} \left\{ U(l, x, G) \mid (l, x) \in OC, e - l - x - \frac{G}{H} \geq 0 \right\}. \quad (3)$$

In the subsequent analysis, I will establish two properties of this allocation. First, it is pointed out that a specific version of Samuelson's rule holds in the optimum (3). Based on this finding, I will then show that a marginal increase in public good provision is welfare improving, provided that these additional expenditures can be financed by means of lump-sum taxation.

Before investigating the optimum (3) one should note that the first-best allocation can be derived in a similar way: With a lump-sum tax, the government's budget is balanced as long as $T - G/H = 0$. Since T affects private consumption only via an income effect, all pairs (l, x) that lie on the Engel curve⁴

$$EC := \{l, x \mid \exists T, G : l = l(1, 1, T, G), x = x(1, 1, T, G), T - \frac{G}{H} = 0\}$$

can be implemented. In the first-best optimum (l^F, x^F, G^F) , utility $U(l, x, G)$ is thus maximized subject to the constraints (1) and $(l, x) \in EC$.

3. Provision Rule and Welfare Improvements

³This property holds if and only if $U(x, l, G)$ is weakly separable between (x, l) and G . In the literature, weak separability has often been used as a benchmark for analyzing the link between public good provision and distortionary taxation. Some work (see e.g. Chang 2000) relies on another benchmark where compensated demand is assumed to be independent of G .

⁴In analogy to the definition of OC , the set EC coincides with the individuals' Engel curves only if the demand functions $x(\cdot), l(\cdot)$ are independent of G .

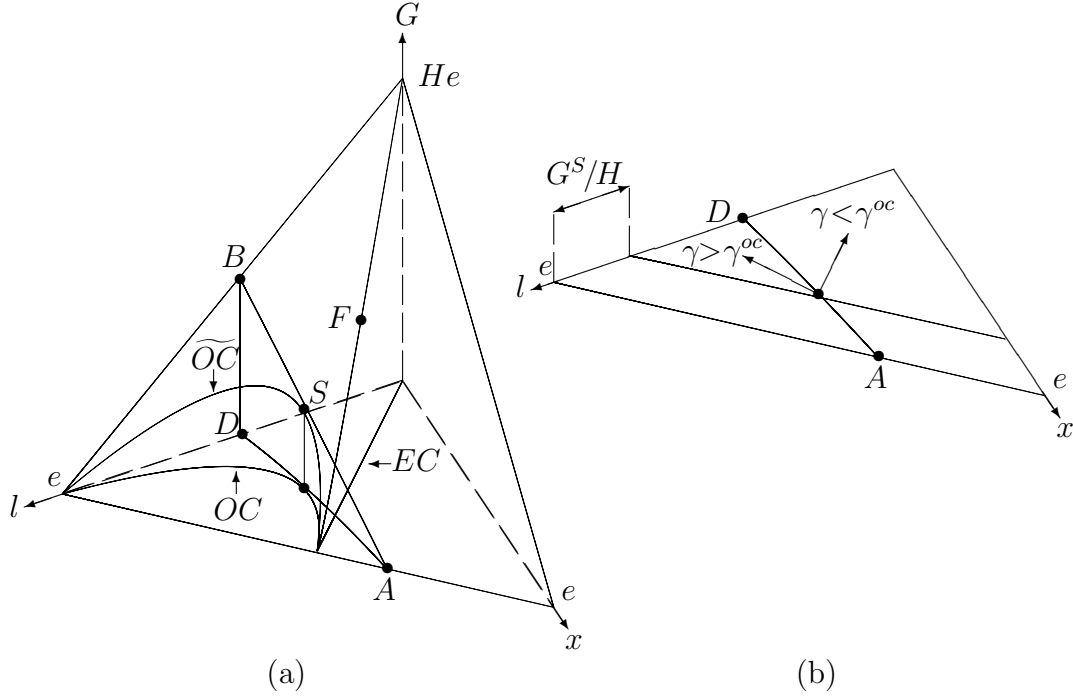


FIGURE 1. The second-best optimum

The maximization problem (3) is illustrated in Figure 1a for a situation where $U(x, l, G)$ is weakly separable between (x, l) and G and homogeneous in (x, l) . Separability implies that the demand functions $x(\cdot)$ and $l(\cdot)$ do not depend on the level of public good provision G . Due to homogeneity, we thus obtain a linear Engel curve EC which hits the offer curve OC only for the trivial case $(q - 1) = T = 0$ where the government's tax revenue equals zero.⁵

The triangle $e - e - He$ in Figure 1a represents the per-capita production frontier, i.e. those combinations of x , l and G which satisfy the constraint (1) with equality. Since the curve EC describes those combinations of x and l that can be achieved if lump-sum taxation is employed for financing public expenditures, the first-best optimum F must lie above of EC on the production frontier. With distortionary taxation, however, only allocations above of OC can be implemented. The second-best optimum (x^S, l^S, G^S) is thus obtained by maximizing utility $U(x, l, G)$ subject to the line \widetilde{OC} , i.e. the projection of OC on the production frontier. A point $S \in \widetilde{OC}$ can be second best only if the agent's indifference surface (in $x - l - G$ -space) is tangent to \widetilde{OC} in S . This implies that the indifference surface must also be tangent to the line AB on the production frontier. Therefore, Samuelson's condition holds in second best provided that the amount of

⁵Due to the substitution effect, OC must lie on the left of EC for any allocation with $x + l < e$. The specific shape of OC depends on whether an increase in q increases or decreases l , that is whether the substitution effect dominates the income effect, or vice versa.

private-goods consumption is measured by means of a composite commodity $\gamma x + (1 - \gamma)l$ whose weights γ and $(1 - \gamma)$ are proportional to the slope of the offer curve OC .

Before exploring the second-best Samuelson condition in more detail, it should be noted that an example of this rule is well-known from the literature: With Cobb-Douglas preferences $U(x, l, G) = \tilde{U}(x^\alpha l^{1-\alpha}, G)$, the offer curve OC is linear and parallel to the x -axis. Hence, AB is also parallel to the x -axis and a specific version of Samuelson's rule, namely $\sum MRS_{Gx} = 1$ must hold in second best. Accordingly, one obtains $MCF = 1$ as long as x serves as the unit of account. Due to the latter property, the Cobb-Douglas example has often been used for illustrating that $MCF \leq 1$ is possible even though the marginal excess burden is positive.⁶

For an analytical exposition of the graphical argument in Figure 1a, let define one unit of private consumption by means of a commodity bundle $\gamma x + (1 - \gamma)l$ that consists of $\gamma \in \mathbb{R}$ units of x and $(1 - \gamma) \in \mathbb{R}$ units of l . This definition leads to the marginal utility of private consumption $U'(\gamma) := \gamma U_x + (1 - \gamma)U_l$ and the corresponding marginal rate of substitution

$$MRS(\gamma) := \frac{U_G}{U'(\gamma)}.$$

Consider next the offer curve OC . Totally differentiating the budget constraint $B(q, G) = 0$, one obtains $dq/dG := -(\partial B/\partial G)/(\partial B/\partial q)$, i.e. the effect of a change in public good provision on the consumer price q . Taking this indirect effect into account, the total effect of an increase in G on the individual's demand for private consumption x and leisure l can be written in the form

$$x_G^{oc} := \frac{\partial x(\cdot)}{\partial q} \frac{dq}{dG} + \frac{\partial x(\cdot)}{\partial G} \quad \text{and} \quad l_G^{oc} := \frac{\partial l(\cdot)}{\partial q} \frac{dq}{dG} + \frac{\partial l(\cdot)}{\partial G}.$$

Note that the ratio between the derivatives x_G^{oc} and l_G^{oc} determines the slope of the offer curve OC . Hence, according to the graphical argument presented above, Samuelson's rule $\sum MRS(\gamma) = 1$ holds in second best as long as the weights γ and $1 - \gamma$ of the composite commodity are proportional to x_G^{oc} and l_G^{oc} . The latter property holds if we choose

$$\gamma^{oc} := \frac{x_G^{oc}}{x_G^{oc} + l_G^{oc}},$$

which implies $1 - \gamma^{oc} = l_G^{oc}/(x_G^{oc} + l_G^{oc})$. Based on this definition, one obtains

Proposition 1: *In the second-best optimum (3), we have $\sum MRS(\gamma^{oc}) = 1$.*

Proof: Totally differentiating the budget constraint $e - qx(\cdot) - l(\cdot) = 0$ with respect to G and q , one obtains $q(\partial x(\cdot)/\partial G) + \partial l(\cdot)/\partial G = 0$ and $q(\partial x(\cdot)/\partial q) + x + \partial l(\cdot)/\partial q = 0$ respectively. Using these equations, the property $x_G^{oc} + l_G^{oc} = -1/H$ can easily be established. This implies $\gamma^{oc} = -Hx_G^{oc}$ and $1 - \gamma^{oc} = -Hl_G^{oc}$. Consider now an allocation (x, l, G) where the constraints contained in (3) are satisfied. If the level of public good

⁶See Mayshar 1990, Triest 1990, Ballard and Fullerton 1992, and Håkonsen 1998 for a discussion.

provision G is changed without violating the constraint $(x, l) \in \text{OC}$, the total effect on the agents' utility $U(x, l, G)$ can be written in the form $dU/dG = U_G + U_x x_G^{oc} + U_l l_G^{oc}$. Because of $x_G^{oc} + l_G^{oc} = -1/H$ the constraint (1) then remains satisfied as well. Hence, in the second-best optimum (3), we must have $U_G + U_x x_G^{oc} + U_l l_G^{oc} = 0$. Using $\gamma^{oc} = -H x_G^{oc}$ and $1 - \gamma^{oc} = -H l_G^{oc}$, this condition immediately leads to $\sum \text{MRS}(\gamma^{oc}) = 1$. \square

In the first-best optimum F , the agents' indifference surface is tangent to the production frontier $e - e - He$ in all directions. Samuelson's rule is thus equivalent to $\sum \text{MRS}(\gamma) = 1$ for all $\gamma \in \mathbb{R}$. Proposition 1 shows that a specific version of this rule also holds in the second-best optimum (3). This finding relies on the observation that a tax financed increase in public good provision affects the agents' demand for private consumption x and leisure l in a particular way, namely in proportion to the weights γ^{oc} and $1 - \gamma^{oc}$. As long as this constraint is taken into account, the logic behind Samuelson's rule can be applied to the model with distortionary taxation as well.

Proposition 1 complements earlier analyses that have characterized the second-best optimum by means of the marginal rate of substitution MRS_{Gx} or MRS_{Gl} , i.e. by assuming that either x or l serves as the unit of account. In the present context, these cases refer to the weights $\gamma = 1$ and $\gamma = 0$ respectively. As long as $\gamma^{oc} \neq 1$ and $\gamma^{oc} \neq 0$, the corresponding marginal cost of public funds $\text{MCF} = \sum \text{MRS}(\gamma)$ thus deviate from unity. These deviations, however, do not reflect the excess burden of distortionary taxation, but the difference between a thought experiment where it is assumed that an increase in G is financed by reducing only private consumption x or leisure l , whereas the real experiment of tax financing affects x and l simultaneously.

Note that the term $\text{MRS}(\gamma)$ is decreasing in the parameter γ provided that $U_x > U_l$. Since utility maximization of each agent implies $U_x/U_l = q/w = 1 + t_x$, the inequality $U_x > U_l$ must hold in the optimum (3). As a consequence of Proposition 1, we thus have $\sum \text{MRS}(\gamma) > 1$ in second best if and only if $\gamma < \gamma^{oc}$.⁷ In other words, a further increase in public good provision increases welfare if and only if the corresponding change in private consumption takes place by a γ -percent reduction in x and a $(1 - \gamma)$ -percent reduction in l , where $\gamma < \gamma^{oc}$ (see Figure 1b). This observation immediately leads to the main result of this note, namely that a local increase in public good provision is desirable in the second-best optimum (3) provided that the additional expenditures can be financed by lump-sum taxation $T > 0$.

Proposition 2: *Consider the second-best optimum (3) and assume that additional government expenditures can be financed by means of a lump-sum tax T . Then a marginal increase in public good provision G is welfare improving as long as the distortionary tax rate $(q - 1) > 0$ or the revenue from distortionary taxation $(q - 1)x > 0$ is kept constant in equilibrium.*

Proof: Consider the second-best quantity G^S and the corresponding price q^S . Assume now that G^S is marginally increased to \tilde{G} . Since the indirect utility $V(q, w, T, G)$ is

⁷This property means that the second-best indifference surface appears to be flatter or steeper than the production frontier depending on whether one moves from S towards some direction $\gamma < \gamma^{oc}$ or $\gamma > \gamma^{oc}$ (see Figure 1b).

increasing in G and decreasing in q , we must have $dq/dG > 0$ in second best. Therefore, the expenditure level \tilde{G} requires some consumer price $\tilde{q} > q^S$ as long as the lump-sum tax $T = 0$ is not changed. Assume instead that q^S remains constant and that the difference $\tilde{G} - G^S > 0$ is financed by a lump-sum tax \tilde{T} . Then we have $x(q^S, 1, \tilde{T}, \tilde{G}) + l(q^S, 1, \tilde{T}, \tilde{G}) = x(\tilde{q}, 1, 0, \tilde{G}) + l(\tilde{q}, 1, 0, \tilde{G})$. Since $\tilde{q} > q^S$ leads to a substitution effect between the commodities x and l , the inequality $x(q^S, 1, \tilde{T}, \tilde{G}) > x(\tilde{q}, 1, 0, \tilde{G})$ must hold. Hence, the lump-sum tax \tilde{T} reduces private consumption (x, l) into some direction $\tilde{\gamma} < \gamma^{oc}$ (see Figure 1 (b)). Therefore, $\sum \text{MRS}(\tilde{\gamma}) > 1$ which in turn implies that the increase in G is welfare improving. Note that the same argument can be used if the revenue $(q - 1)x(\cdot)$ is kept constant because this assumption also implies a positive lump-sum tax $\hat{T} := (1/H)(\tilde{G} - G^S)$. In this case, the price \hat{q} is chosen which is implicitly determined by means of the condition $(\hat{q} - 1)x(\hat{q}, 1, \hat{T}, \tilde{G}) = (q^S - 1)x(q^S, 1, 0, G^S)$. Because of $\hat{T} > 0$, one obtains $\hat{q} < \tilde{q}$ which implies that the demand $l(\hat{q}, 1, \hat{T}, \tilde{G}), x(\hat{q}, 1, \hat{T}, \tilde{G})$ must also lie on the right of the line AD in Figure 1. \square

The intuition behind Proposition 2 is straightforward: Since the lump-sum tax is always more efficient than the distortionary tax at the margin,⁸ an increase in public good provision must be welfare improving in second best provided that additional expenditures can be financed without further increasing the excess burden of taxation. As mentioned in the introductory section, earlier analyses on the link between public good provision and distortionary taxation have concentrated on the question whether the properties $\sum \text{MRS}_{Gx} > 1$, $\sum \text{MRS}_{Gl} > 1$, or $G^S < G^F$ are fulfilled in second best. This literature has shown that none of these inequalities hold in general even if it is assumed that the demand functions $x(\cdot)$ and $l(\cdot)$ are independent of G . In contrast, Proposition 2 only relies on the assumption that $U(x, l, G)$ is strictly quasiconcave. Hence, the finding is based on a pure substitution effect which takes place irrespective of whether the above-mentioned inequalities are satisfied. In particular, no assumption concerning the effect of G on the demand $x(\cdot)$ of the taxed commodity is made. In this way, a robust property of the second-best optimum is established that confirms the intuitive claim that public expenditures should not be carried out as far as would be done if lump-sum taxation were available.

It should be noted that the reasoning that underlies Propositions 1 and 2 can also be applied if one allows for a vector $x = (x_1, \dots, x_n)$ of consumption commodities and a corresponding vector of excise taxes $t_i = (q_i - 1)$, $i = 1, \dots, n$. In this case, the second best problem can be analyzed in two steps. First, the optimal prices (taxes) $q(G) = (q_1(G), \dots, q_n(G))$ are derived for an exogenous level of public expenditures G . Then utility $U(x, l, G)$ is maximized subject to the requirement $x = \hat{x}(G) := x(q(G), 1, G)$, $l = \hat{l}(G) := l(q(G), 1, G)$, and the production constraint $e - \sum_{i=1}^n x_i - l - G/H \geq 0$. Note that the constraint $(x, l) = (\hat{x}(G), \hat{l}(G))$ represents the agents' (modified) offer curve along the path of optimal prices $q_i(G)$. Accordingly, Samuelson's rule is obtained in second best if private consumption is expressed by means of a commodity bundle

⁸This means that the marginal cost of public funds MCF of the lump-sum tax is below the MCF of the distortionary tax irrespective of whether the latter exceeds unity.

$\sum_{i=1}^n \gamma_i x_i + \gamma_0 l$ where the parameters $(\gamma_0, \gamma_1, \dots, \gamma_n)$ are proportional to the derivatives $(\partial \hat{l} / \partial G, \partial \hat{x}_1 / \partial G, \dots, \partial \hat{x}_n / \partial G)$, and $\sum_{i=0}^n \gamma_i = 1$. This property is analogous to Proposition 1. Using the structure of the second-best prices $q_i(G)$, Proposition 2 can then be established as well.

References

- Atkinson, A. B., and N. H. Stern (1974) "Pigou, Taxation and Public Goods" *Review of Economic Studies* **41**, 119-128.
- Atkinson, A. B. and J. E. Stiglitz (1980) *Lectures on Public Economics*, McGraw-Hill: London.
- Ballard, C. L., and D. Fullerton (1992) "Distortionary Taxes and the Provision of Public Goods" *Journal of Economic Perspectives* **6**, No 3, 117-131.
- Chang, M. C. (2000) "Rules and Levels in the Provision of Public Goods: The Role of Complementarities between the Public Good and Taxed Commodities" *International Tax and Public Finance* **7**, 83-91.
- de Bartolomé, C. A. (1998) "Is Pigou Wrong? Can Distortionary Taxation Cause Public Spending to Exceed the Efficient Level?", Discussion Paper, University of Colorado at Boulder.
- Gaube, T. (2000) "When Do Distortionary Taxes Reduce the Optimal Supply of Public Goods?" *Journal of Public Economics* **76**, 151-180.
- Gaube, T. (2005) "Financing Public Goods With Income Taxation: Provision Rules vs. Provision Level" *International Tax and Public Finance* **12**, 319-334.
- Gronberg, T., and L. Liu (2001) "The Second-Best Level of a Public Good: An Approach Based on the Marginal Excess Burden" *Public Economic Theory* **3**, 431-453.
- Håkonsen, L. (1998) "An Investigation Into Alternative Representations of the Marginal Cost of Public Funds" *International Tax and Public Finance* **5**, 329-343.
- Mayshar, J. (1990) "On Measures of Excess Burden And Their Application" *Journal of Public Economics* **43**, 263-289.
- Mirrlees, J. A. (1994) "Optimal Taxation And Government Finance" in *Modern Public Finance* by J.M. Quigley and E. Smolensky, Eds., Harvard University Press: Cambridge/Mass., 213-231.
- Myles, G. D. (1995) *Public Economics*, Cambridge University Press: Cambridge.
- Pigou, A. C. (1947) *A Study in Public Finance*, Macmillan: London.

Sandmo, A. (1998) "Redistribution And the Marginal Cost of Public Funds" *Journal of Public Economics* **70**, 365-382.

Triest, R. K. (1990) "The Relationship Between the Marginal Cost of Public Funds And Marginal Excess Burden" *American Economic Review* **80**, 557-566.

Wilson, J. D. (1991) "Optimal Public Good Provision With Limited Lump-Sum Taxation" *American Economic Review* **81**, 153-166.