E C O N O M I C S   B U L L E T I N

School and subject choices in education

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Abstract

Effects of school choice have not been satisfactorily resolved empirically and theoretically. I provide a theory of school choice. I show that a positive assortive matching between teachers and students maximizes the production of education. Also the production can be augmented by letting students choose subjects that they are good at rather than do everything.

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1. Introduction

Effects of school choice are controversial. Hoxby (2002) finds positive effects in the USA (Milwaukee, Michigan, and Arizona) and Angrist, Bettinger, and Kremer (2005) in Columbia. On the contrary, Cullen, Jacob, and Levitt (2005) find no effects in the USA (Chicago) and nor do Heish and Urquiola (2006) in Chile.

Theoretically, Beseley and Ghatak (2005) show that school choice is efficient, when principal (parent) and agent's (teacher's) missions are similarly aligned. They, however, omit students. Since education is a dialogue between teachers and students, students need to be taken into account to explain effects of school choice.

I bring back students as one of major components in education. I will show under what conditions school choice has maximal effects on education. And all the empirical and theoretical arguments above do not address students' time allocation. So I will present, once school choice has been completed, how students need to allocate their time to augment the maximal effects of school choice.

2. Illustration

Whatever forms are taken, school choice is essentially matching between students and teachers. This is analogous to matching in a marriage market. So an illustration helps get the intuition about assortive matching (Becker 1973).

Suppose that there are teachers (x) and students (y) and their uni-dimensional quality is either high h or low l such that $x_h > x_l$ and $y_h > y_l$. A production function of education f is positive and increasing in each argument. If high quality students learn more when high quality teachers teach than low quality teachers do, then $f(x_h, y_h) > f(x_l, y_h)$. Similarly $f(x_h, y_l) > f(x_l, y_l)$ for low quality students. If $f(x_h, y_h) - f(x_l, y_h) > f(x_h, y_l) - f(x_l, y_l)$, then teachers and students are complementary, because marginal product of teachers is greater when matched with high quality students than low quality students. When the terms are rearranged, the equation becomes $f(x_h, y_h) + f(x_l, y_l) > f(x_h, y_l) + f(x_l, y_h)$. The last inequality shows that when teachers and students are complementary, homogeneous matching maximizes the total product of education.

3. Model

As illustrated above, the only assumptions of f are positiveness and increasing in each argument. The assumptions are realistic in the case of education as long as education does not dull students and higher quality teachers (students) teach (learn) better. To generalize the case above, differentiability is needed. This is a technical assumption and does not change the substance of the model. So a function of assortive matching is quite general and realistic.

Among various functional forms that satisfy all the three assumptions, a modified Kremer's (1993) O-ring model is relevant in this case. At first glance, the model seems too extreme to be realistic because it is sensitive to even one input quality. It may not be the case, however. If an ignorant teacher teaches only what students know already or one disruptive student interrupts the whole class, the product of education is naught. Also the model demonstrates the importance of complementarity as extreme as it gets.

Consider expected production of education of subject $l \in \{1,2,\cdots L\}$ per unit time in a school,

$$E(y_l) = k^{\alpha}(\prod_{i=1}^{n} q_{il})(\prod_{j=1}^{m} p_{jl})(n+m)B_l$$

where E is the expectation operator, y_l is the total product of education of subject l per unit time, k is a fixed amount of school facility, α is a parameter of k, $q_{il} \in [0,1]$ is pedagogy quality (or mistake rate) of teacher i on subject l, $p_{jl} \in [0,1]$ is learning quality (or mistake rate) of student j on subject l, n and m are exogeneously fixed numbers of teachers and students, respectively, for reasons of financial constraint, legal restriction or physical capacity of the school, and B_l is output per teacher and student per time unit with a single unit of k, if they complete their task without any mistake ($q_{il}, p_{jl} = 1$). q_{il} and p_{jl} are continuously distributed exogeneously. $n+m$ teachers and students produce education simultaneously. Teachers and students teach and learn inelastically without facing labor-leisure choice.

For simplicity, assume that the principal of the school is risk neutral and can observe the quality of teachers by qualification test scores and quality of students by previous test scores. Each subject is independent of each other. Then $E(y_l)$ becomes y_l and the maximization of the sum is the same as the sum of the maximization. So I focus on the principal's problem on subject l.[1]

$$\max_{k,\{q_{il}\},\{p_{jl}\}} k^{\alpha}(\prod_{i=1}^{n} q_{il})(\prod_{j=1}^{m} p_{jl})(n+m)B_l - \sum_{i=1}^{n} w(q_{il}) - \sum_{i=1}^{m} c(p_{jl}) - rk$$

where $w(q_{il})$ is a C^2 wage function of teachers ($w'(\cdot) > 0$), $c(p_{jl})$ is a C^2 compensation function[2] of students ($c'(\cdot) > 0$), and r is a rental rate.

The first order condition with respect to q_{il} is

$$\frac{dw(q_{il})}{dq_{il}} = \frac{dy_l}{dq_{il}} = (\prod_{h \neq i} q_{hl})(\prod_{j=1}^{m} p_{jl})(n+m)B_l k^{\alpha} (>0)$$

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[1] A fixed amount of tuition can be added in the problem, but it does not affect main results.

[2] Compensations can be pecuniary, e.g. scholarship, or non-pecuniary, e.g. compliments, attention.

Similarly, the first order condition with respect to p_{jl} is

$$\frac{dc(p_{jl})}{dp_{jl}} = \frac{dy_l}{dp_{jl}} = (\prod_{h \neq j} p_{hl})(\prod_{i=1}^{n} q_{il})(n+m)B_l k^{\alpha} (>0)$$

The conditions explain "bad apples" story. For example, when even one teacher's (student's) quality drops by half, any other teacher's (student's) marginal product drops by half. If this happens, other teachers (students) are discouraged from teaching (learning).

Now get cross derivatives:

$$\frac{d^2 y_l}{dq_{il} d(\prod_{h \neq i} q_{hl})} = (\prod_{j=1}^{m} p_{jl})(n+m)B_l k^{\alpha} (>0)$$

$$\frac{d^2 y_l}{dq_{il} d(\prod_{j=1}^{m} p_{jl})} = (\prod_{h \neq i} q_{hl})(n+m)B_l k^{\alpha} (>0)$$

$$\frac{d^2 y_l}{dp_{jl} d(\prod_{h \neq j} p_{hl})} = (\prod_{i=1}^{n} q_{il})(n+m)B_l k^{\alpha} (>0)$$

$$\frac{d^2 y_l}{dp_{jl} d(\prod_{i=1}^{n} q_{il})} = (\prod_{h \neq j} p_{hl})(n+m)B_l k^{\alpha} (>0)$$

The positive cross derivatives indicate that teachers and students' own and cross qualities are complementary. In this case, when teachers and students of the same quality are matchedm total production is maximized. Furthermore, the maximization condition is optimal for *each* teacher and student (Becker, 1973). Thus, $q_{il} = q_l$ and $p_{jl} = p_l$ at the optimum. For simplicity, assume that the distribution of q_l and p_l are same. Then $q_{il} = p_{jl} = q_l$. From now on, the optimality is satisfied.

The first order condition with respect to *k* is

$$\alpha k^{\alpha-1} q_l^{n+m} (n+m) B_l = r$$

or

$$k = \left(\frac{\alpha q_l^{n+m} (n+m) B_l}{r} \right)^{\frac{1}{1-\alpha}}$$

k is increasing in q_l. So, at the optimum, more facilities are needed as better teachers teach better students.

The analysis generalizes the illustration above: high (low) quality teachers should match with high (low) quality students to maximize the total product of education. And this is good individually as well as collectively. Suppose a Nobel laureate teaches economics. But it is meaningless if students are so ignorant that they do not understand what the teacher says. The teacher and students become discouraged from teaching and learning. On the other hand, suppose that an ignorant, yet motivated teacher puts all the efforts. Again, it is meaningless if the teacher teaches what students already know. Teachers and students need to match according to their quality to increase efficiency in education.

Now I turn to time allocation of students. I simplify the model to its bare-bone to focus on time allocation only: consider *m* number of students, no teachers (*n*=0); and one unit of capital (*k*=1)[3]. Each student has T units of time, study *L* subjects, and allocate time equally to each subject. Then total profit is

$$\sum_{l=1}^{L}\left[\left(\frac{mT}{L}\right)q_l^m mB_l - mc(p_l) - r\right] \tag{1}$$

If students focus only one subject that they are best at, then the total product is

$$\sum_{l=1}^{L}\left[(m_l T)q_l^{m_l} m_l B_l - mc(p_l) - r\right] \tag{2}$$

where m_l is a number of students studying subject *l* such that $m = \sum_{l=1}^{L} m_l$. Compare (1) and (2) by (2)-(1),

$$T\sum_{l=1}^{L} B_l q_l^{m_l}\left(m_l^2 - q_l^{m-m_l}\frac{m^2}{L}\right)$$

This shows that if there are too many subjects to study, or more likely, the quality of students attending subject *l* is too low, it is better to let students do only what they do best. It may be argued that students go to school to do better what they are not good at. But, as assumed in the model, it could be wasting time for students to (be forced to) learn what they are bad at *inherently*. There is no point to make genetically obese students learn sprinting or genetically tone-deaf students learn music. From the efficiency point of view, it is more efficient for them to choose cooking or mathematics instead, if they are better at them.

Moreover unless all the qualities of students on *L* subjects are perfectly correlated, subject choice leads to more equal distribution of final outcome of education for each student. Intuitively, the argument is similar to the one of comparative advantage in international economics. Efficiency increases and the inequality gap can decrease.

[3] Since teachers and students' quality are homogeneous, it does not matter whether teachers are added or not.

5. Conclusion

If teachers and students are complementary, if they know the complementarity, if they know each other's quality, and if they can move freely, then they will positively assortively match. It is, however, more likely that movement incurs costs (transaction costs) and they do not know each other's quality (information costs). In that case someone who controls the information (principals or the Department of Education) can reduce transaction costs by eliminating legal obstructions (e.g. teachers union) or red tapes, and information costs by announcing the information. Or if there are no coordination costs, the information controller can coordinate matching directly. So school choice has a potential to lead to a higher level of education, when transaction costs and information costs are reduced.

And yet, school choice is not the end of the story. Even when teachers and students are optimally matched, it is not efficient if students (are forced to) learn what they are bad at. Instead, it is more efficient to allocate time to subjects that students are good at (subject choice). In addition, the combination of school and subject choice has potential to increase not only efficiency but also equity in education.

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