E C O N O M I C S   B U L L E T I N

# Matching, Specialties and Wage Inequality

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## *Abstract*

Empirical evidence show that there is a negative relation between policies that accelerate the matching process in labor market and "within-group" wage inequality. We apply the standard "search and matching" framework to construct a labor market model with ex-ante heterogeneous workers, so as to examine and interpret this phenomenon. We show that a composition effect working through the effective rate of employment opportunities (decision pattern through which individuals accept or reject jobs in which they are less specialized) is responsible for the increase in within group inequality as matching process is accelerated.

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# 1. INTRODUCTION

The literature behind within group wage inequality[1] in labor market and its increase in 80's and 90's is huge. Several explanations have been given for the interpretation of this phenomenon; existence of ability indicators -other than years of schooling- such as aptitude test scores, ability to work with other people etc. and their increasing role in the level of wages, increasing returns to skills (due to skill biased technical change)[2], plant specific wage differentials (workers in large plants earn more than those in small ones), the 'bundling' phenomenon (related with increasing industry specific wage differentials) which characterizes the 'productive abilities' possessed by an individual etc. This article focuses on the last of these interpretations, that of 'bundling'. According to this theory, the separate productive abilities possessed by a worker, (the wage is the sum of their prices) cannot be 'unbundled' and sold to the highest bidder (Mandelbrot 1962). Based on this conclusion, Heckman *et al* (Heckman and Scheinkman 1987; Heckman and Sedlacek 1985), developed theoretical and econometric models proving the hypothesis that the unit price of a productive attribute varies across sectors (see also the paradigm of Levy and Murnane (1992)). It seems that the theory of 'bundling' offers a good explanation for the inter-industry wage differentials that constitute together with 'plants differentials' the main source of within group wage inequality. Studies (see Katz & Author (1999)) show that the trend of the overall within group wage inequality in United States is increasing between 1960-95[3].

Simultaneously, during almost the same period of time several developments in the way that economic agents (workers/firms) meet each other in the labor market environment took place. Specifically, according to recent OECD studies (see Martin (2000)), technological advances in matching process[4] between employers and employees took place in several industrial countries.

Such improvements in the matching process contribute to the decrease of search frictions in labor market. The decrease of such frictions enables workers to reject jobs in which they are less productive. In other words, as the matching process is accelerated, we go from a 'random assignment' model (individuals accept any kind of job) of labor market to a Roy's model of self-selection (individuals accept only jobs in which they are specialized in). Heckman and Sedlacek (1985), show that self-selection reduces intra and intersectoral wage inequality. However, their model ignores the crucial role of search frictions to the transition from the random assignment case to the self-selection case. Therefore, they fail to answer why within group inequality seems to be unaffected from such developments in matching process.

In our work by using a search and matching framework, we show that a composite effect working through match acceptance probabilities is responsible for the increase of within group inequality as matching process is accelerated.

The framework used for the construction of the model is based on Albrecht and Vroman (2002)[5]. In their model, as in ours, they assume two types of workers

[1] Wage inequality between workers with the same socioeconomic characteristics such as education, age, race etc.

[2] For an overall review of theories and research on skill biased technical change look at Acemoglu (2001).

[3] For wage inequality in UK see Machin (1996).

[4] Technological advances in matching process include reforms such as the computerization of employment offices, job advertising on internet, job-search assistance policies, governmental subsidies into policies helping matching process etc.

[5] Another related paper is that of Acemoglu (1999).

the fraction of whom is exogenously given and two types of jobs. Furthermore, 'mismatches' are less productive than 'proper' matches and free entry for firms is assumed. However, they seem to classify workers and jobs according to the educational level possessed (in case of workers) or required (in case of jobs). Therefore, they assume that low-educated workers can not perform 'high-educated' jobs. Thus, they focus on across groups wage inequality. In our model, workers can perform both kind of jobs, since they differ in the job they are specialized for and not in their educational level. Another source of heterogeneity in our paper is the difference in the level of observed and unobserved skills (other than education) possessed by individuals. Moreover, Albrecht and Vroman examine the impact of skill-biased technical change on income distribution and unemployment. The model presented in this article, focuses on the effect of technological advances in matching on wage inequality.

The paper is organized as follows. In the next section, the basic model is presented. Section 3, examines the nature of equilibrium and the results from the comparative statics analysis. Section 4, simulates the model and measures the level of wage inequality. Finally, section 5, concludes.

## 2. THE MODEL

### 2.1. Assumptions

A continuous-time model with risk neutral and infinite lived workers is considered. The case, where workers have similar socioeconomic characteristics (education, age and race) is examined. Workers differ in the occupation in which they are specialized and in the level of observed and unobserved skills[6], they have. The distribution of specialties[7] across workers is assumed exogenous. Specifically, workers are of two types; *1* and *2*. The fraction of type-*1* and type-*2* individuals in the total population is $p$ and $1-p$, respectively. The population of workers is normalized to unity.

Workers are either employed or unemployed, and jobs are either filled or vacant. Jobs are also of two types (*1* and *2*). Filled jobs 'die' at an exogenous rate $\delta$. We assume a 'free' entry regime for vacancies, i.e. vacancies are created, whenever it is profitable to do so (the long-run nature of the model allows the assumption of the free entrance). Each firm offer only one job. Firms and workers, discount the future at the same rate $r$. The production technology is the following:

$$x_{ij} = \begin{cases} y + a, if\ i = j \text{ where } i, j = 1, 2. \\ y + ka, k \in [0, 1), if\ i \neq j \text{ where } i, j = 1, 2. \end{cases} \quad (1)$$

where $i$, denote the type of worker and $j$ denote the type of job. The variable $a$, is assumed to be a random variable uniformly distributed between zero and one, realized by workers before entering the labor market. So $a$, is considered as a measure of skills. When $i \neq j$, $a$, is multiplied by $k$, which indicates the 'price' of

[6] By the word skills, we mean all productive abilities possessed by a worker except from education and experience. Observed are those skills which can be measured eg. IQ tests and unobserved those which can not be observed by the econometrician eg. ability to talk in a pleasing and persuasive way.

[7] Individuals can perform jobs that are not specialized for (or alternatively they can perform 'intersectoral' jobs), like the fishermen and hunters in Roy's model (1951).

skills in case of a mismatch[8]. In case of a 'proper' match the value of $k$ is equal to one. For the rest of the model analysis, we will set $k = 1/2$. The variable $y$, is the same for all workers and it can be considered as the return of the socioeconomic characteristics (education, age and race). The cost of the filled job is $w_{ij} + c$, i.e. the wage paid plus a fixed cost which is paid by the firm, even if the job is vacant. Unemployed workers receive unemployment insurance benefit, $b$.

Workers and vacancies meet each other randomly, according to a Pissarides matching function, $m(u, v)$, where $u$, is the unemployment rate (since the population of workers is normalized to one) and $v$, is the measure of vacancies. Moreover, we assume that the matching function exhibits constant returns to scale[9]. Hence, the arrival rate for workers is $m(\theta)$, where $\theta = v/u$, is the measure of labor market tightness. The usual properties hold for $m(\theta)$, i.e. $m'(\theta) > 0$ and $\lim_{\theta \to 0} m(\theta) = 0$ . The arrival rate for jobs is $m(\theta)/\theta$ with $[m(\theta)/\theta]' < 0$, and $\lim_{\theta \to 0} [m(\theta)/\theta] = \infty$.

Let $\varphi$, denote the fraction of type-*1* vacancies and $\gamma$, the fraction of type-*1* unemployed. There will be a reservation value for $a$, for each type of workers, above which the employment in a job of different type will not be worthwhile. Thus, across workers of the same type the effective arrival rate of employment opportunities will differ. Specifically, workers having $a$, above the reservation value, will match only with vacancies of the same type and therefore the effective arrival rate of employment will be $\varphi m(\theta)$ (for type-*1*) or $(1 - \varphi)m(\theta)$ (for type-*2*). On the other hand, for those having $a$, under the reservation value, the effective arrival rate is $m(\theta)$.

## 2.2. Match Formation and Wages

As stated above, the workers within each specialty will be of two categories; those matching with both type of jobs ($a$, below reservation value), and those matching only with the same type of jobs ($a$, above reservation value). Let $\tilde{a}_1$, and $\tilde{a}_2$, denote the reservation values for each type of workers. For the purpose of our analysis, we will use the following notation; $U_i^n(a_i^n|\ a_i^n \leq \tilde{a}_i) = U_i^n(a_i^{n-})$, is the value of unemployment for the $n$-th worker of type $i$, having $a$, less or equal to the reservation value of his type. Similarly, the value of unemployment for workers with $a > \tilde{a}_i$, will be $U_i^n(a_i^n|\ a_i^n > \tilde{a}_i) = U_i^n(a_i^{n+})$. $W_{ij}(a_i^n)$, and $W_{(ij|\ i=j)}(a_i^n) = W_i(a_i^n)$, are the values to employment for $n$-th worker of type $i$, to $j$-th job when $a \leq \tilde{a}_i$, and the value to employment for $n$-th worker of type $i$, to $j$-th job when $a > \tilde{a}_i$, respectively. $J_{ij}(a_i^n)$, and $J_{(ij|\ i=j)}(a_i^n) = J_i(a_i^n)$, are the values of a filled job and their definition is similar to that of $W$'s. Finally, $V_j$, is the value of a vacancy of type $j$.

The wage paid to each worker is determined through a Nash bargaining process with equal bargaining power between the two sides ($\beta = 1/2$). The Nash

[8] Like in Heckman and Scheinkman (1987), the hypothesis of uniform pricing of skills across sectors is rejected. In our case, the highest 'bidder' for the specific skills possessed by type-*1* worker is type-*1* firm.

[9] Most empirical studies, such as Anderson and Burgess (2000) and Burda (1993) find that a log-linear approximation to matching function with constant returns to scale fits the data quite well.

bargaining conditions will be:

$$W_{ij}(a_i^n) - U_i^n(a_i^{n-}) = \frac{1}{2}[W_{ij}(a_i^n) + J_{ij}(a_i^n) - U_i^n(a_i^{n-}) - V_j] \qquad (2).$$
$$W_i(a_i^n) - U_i^n(a_i^{n+}) = \frac{1}{2}[W_i(a_i^n) + J_i(a_i^n) - U_i^n(a_i^{n+}) - V_j] \qquad (3).$$

In the following paragraph, we develop expressions for the value functions of the model.

The value functions for workers are the following:

*I)Unemployed*

$$rU_1^n(a_1^{n-}) = b + m(\theta)\varphi[W_{11}(a_1^n) - U_1^n(a_1^{n-})] + m(\theta)(1-\varphi)[W_{12}(a_1^n) - U_1^n(a_1^{n-})] (4a).$$
$$rU_2^n(a_2^{n-}) = b + m(\theta)(1-\varphi)[W_{22}(a_2^n) - U_2^n(a_2^{n-})] + m(\theta)\varphi[W_{21}(a_2^n) - U_2^n(a_2^{n-})] (4b).$$
$$rU_1^n(a_1^{n+}) = b + m(\theta)\varphi[W_1(a_1^n) - U_1^n(a_1^{n+})] (5a).$$
$$rU_2^n(a_2^{n+}) = b + m(\theta)(1-\varphi)[W_2(a_2^n) - U_2^n(a_2^{n+})] (5b).$$

Equations *(4a)-(5b)*, imply that the flow value of unemployment is equal to the sum of the opportunity cost of work plus the expected capital gain from changing status (from unemployment to employment). The relative equations for the marginal worker (the one who is indifferent between working or not in a job of different type) will be:

$$rU_1^n(\tilde{a}_1) = b + m(\theta)\varphi[W_{11}(\tilde{a}_1) - U_1^n(\tilde{a}_1)] \qquad (6a).$$
$$rU_2^n(\tilde{a}_2) = b + m(\theta)(1-\varphi)[W_{22}(\tilde{a}_2) - U_2^n(\tilde{a}_2)] \qquad (6b).$$

Equations *(6a)* and *(6b)* are the conditional version of *(4a)* and *(4b)*, with $W_{12}(a_1^n) - U_1^n(a_1^{n-}) = W_{21}(a_2^n) - U_2^n(a_2^{n-}) = 0$.

*II) Employed*

The expressions for the flow value of employment are:

$$rW_{ij}(a_i^n) = w_{ij}(a_i^n) + \delta[U_i^n(a_i^{n-}) - W_{ij}(a_i^n)] \qquad (7a).$$
$$rW_i(a_i^n) = w_{(ij|\ i=j)}(a_i^n) + \delta[U_i^n(a_i^{n+}) - W_i(a_i^n)] \qquad (7b).$$

where $w_{ij}(a_i^n)$, is the wage paid to a worker of type $i$, with $a \leq \tilde{a}_i$, employed in a job of type $j$ and $w_{(ij|\ i=j)}(a_i^n) = w_i(a_i^n)$, is the wage paid to a worker of type $i$, with $a > \tilde{a}_i$. The flow value of employment for each type of worker is equal to the flow return of employment, $w$, plus the expected capital loss as result of job destruction.

The corresponding value functions for the jobs are the following:

*III) Filled*

$$(r+\delta)J_{ij}(a_i^n) = y + ka - w_{ij}(a_i^n) - c + \delta V_j, \qquad (8a).$$
$$\text{where } k = 1 \text{ for } i = j, \text{and } k = 1/2, \text{ for } i \neq j.$$
$$(r+\delta)J_i(a_i^n) = y + a - w_i(a_i^n) - c + \delta V_j \qquad (8b).$$

*IV) Vacant*

$$rV_1 = -c + \frac{m(\theta)}{\theta} \left\{ \begin{array}{c} \gamma \left( \begin{array}{c} F(\tilde{a}_1) \int_0^{\tilde{a}_1} [J_{11} - V_1] f(a_1) da_1 + \\ (1 - F(\tilde{a}_1)) \int_{\tilde{a}_1}^1 [J_1 - V_1] f(a_1) da_1 \end{array} \right) \\ +(1-\gamma) F(\tilde{a}_2) \int_0^{\tilde{a}_2} [J_{21} - V_1] f(a_2) da_2 \end{array} \right\} \qquad (9a).$$

$$rV_2 = -c + \frac{m(\theta)}{\theta} \left\{ \begin{array}{c} (1-\gamma) \left( \begin{array}{c} F(\tilde{a}_2) \int_0^{\tilde{a}_2} [J_{22} - V_2] f(a_2) da_2 + \\ (1 - F(\tilde{a}_2)) \int_{\tilde{a}_2}^1 [J_2 - V_2] f(a_2) da_2 \end{array} \right) \\ +\gamma F(\tilde{a}_1) \int_0^{\tilde{a}_1} [J_{12} - V_2] f(a_1) da_1 \end{array} \right\} \qquad (9b).$$

where, $F(a_i)$,denote the cumulative distribution function (*cdf*), and $f(a_i)$, the probability density function (*pdf*) of $a_i$.The flow value of a vacant job is equal to the cost of holding the vacancy plus the expected capital gain when the job is filled. When a vacancy matches with a worker of the same type, two are the possible scenarios; either the worker has $a \leq \tilde{a}_i$, or has $a > \tilde{a}_i$. The probability of meeting a type-*1*(*2*) worker with $a \leq \tilde{a}_i$ is $[m(\theta)/\theta]\gamma F(\tilde{a}_1)$ $([m(\theta)/\theta](1-\gamma)F(\tilde{a}_2))$, while vacancies meet type-*1*(*2*) workers with $a > \tilde{a}_i$, at rate $[m(\theta)/\theta]\gamma(1 - F(\tilde{a}_1))$ $([m(\theta)/\theta](1-\gamma)(1 - F(\tilde{a}_2)))$. On the other hand, when the worker is of different type, the match is worthwhile only if $a \leq \tilde{a}_i$. Since, firms know only the distribution of $a$'s, they form expectations about their potential capital gain in each case. The 'free' entry assumption implies that the value of type-*1* and type-*2* vacancies must be zero, that is:

$$V_1 = V_2 = 0 \qquad (10).$$

From equations , (*10*), (*7a*), (*7b*), (*8a*), (*8b*) and (*2*), (*3*), we get that the wage paid to each type of worker in each case is:

$$\begin{aligned} w_{ij}(a_i^n) &= \frac{1}{2}(y + ka - c) + \frac{1}{2} r U_i^n(a_i^{n-}), \qquad (11a). \\ \text{where } k &= 1 \text{ for } i = j, \text{and } k = 1/2, \text{ for } i \neq j. \\ w_i(a_i^n) &= \frac{1}{2}(y + a - c) + \frac{1}{2} r U_i^n(a_i^{n+}) \qquad (11b). \end{aligned}$$

The wage paid is a weighted average of the net output of the match and the flow value of unemployment.

By substituting, the 'free' entry condition (equation (*10*)), into equations *(8a), (8b),* we get:

$$\begin{aligned} J_{ij}(a_i^n) &= \frac{\frac{1}{2}[y + ka - c - rU_i^n(a_i^{n-})]}{r + \delta}, \qquad (12a). \\ \text{where } k &= 1 \text{ for } i = j, \text{and } k = 1/2, \text{ for } i \neq j. \\ J_i(a_i^n) &= \frac{\frac{1}{2}[y + a - c - rU_i^n(a_i^{n+})]}{r + \delta} \qquad (12b). \end{aligned}$$

Since $\beta = 1/2$ and by combining equations (*10*), (*11a*), (*11b*), *(7a)* and *(7b)*, we get that; $J_{ij}(a_i^n) - V_j = W_{ij}(a_i^n) - U_i^n(a_i^{n-})$, and $J_i(a_i^n) - V_j = W_i(a_i^n) - U_i^n(a_i^{n+})$, i.e. as soon as it is profitable for the worker to match with a specific job, then it is profitable for the firm as well and vice versa.

## 2.3. The steady state

Only steady state equilibria are examined in this model. In the steady state flow of workers out of unemployment should be equal to the flow of workers back to unemployment, that is:

$$
\begin{aligned}
m(\theta)F(\tilde{a}_1)\gamma u + m(\theta)(1-F(\tilde{a}_1))\varphi\gamma u &= \delta(p-\gamma u) && (13a).\\
m(\theta)F(\tilde{a}_2)(1-\gamma)u + m(\theta)(1-F(\tilde{a}_2))(1-\varphi)(1-\gamma)u &= \delta[1-p-(1-\gamma)u] && (13b).
\end{aligned}
$$

Equation (*13a*), is the steady state condition for type-*1* workers, where equation (*13b*), is the steady state condition for type-*2* workers.

## 3. EQUILIBRIUM

DEFINITION 1. A steady-state equilibrium is a six tuple $\{\tilde{a}_1,\, \tilde{a}_2,\, \gamma,\, \varphi,\, \theta,\, u\}$, that satisfy the following conditions: i) Workers' and firms' choices constitute a Nash equilibrium, ii) The creation of vacancies satisfies the 'free' entry conditions, and iii) The flow of workers out of unemployment should be equal to the flow of workers back to unemployment.

From equations (*6a*) - (*7a*) and (*11a*), (*11b*), we get that the reservation values of workers are (for the derivation see appendix):

$$
\begin{aligned}
rU_1^n(\tilde{a}_1) &= y + \frac{\tilde{a}_1}{2} - c = \frac{2(r+\delta)b + m(\theta)\varphi[\tilde{a}_1/2]}{2(r+\delta)} && (14a).\\
rU_2^n(\tilde{a}_2) &= y + \frac{\tilde{a}_2}{2} - c = \frac{2(r+\delta)b + m(\theta)(1-\varphi)[\tilde{a}_2/2]}{2(r+\delta)} && (14b).
\end{aligned}
$$

From equations (*4a*) - (*5b*), (*7a*), (*7b*), (*11a*), (*11b*), (*12a*), (*12b*), (*9a*) and (*9b*) and after tedious algebra, the zero value conditions (equation (*10*)), can be written as (for the derivation see appendix):

$$
c = \frac{m(\theta)}{\theta}\left\{ \begin{array}{c} \gamma \left( \begin{array}{c} (1-F(\tilde{a}_1))\frac{(1-\tilde{a}_1)(y-c-b)+(1/2)-(\tilde{a}_1^2/2)}{2(r+\delta)+m(\theta)\varphi} + \\ F(\tilde{a}_1)\frac{2(r+\delta)[y+(\tilde{a}_1/2)-c-b]\tilde{a}_1+m(\theta)[(3/2)+\varphi](\tilde{a}_1^2/2)}{[2(r+\delta)+m(\theta)][2(r+\delta)]} \end{array} \right) + \\ (1-\gamma)F(\tilde{a}_2)\frac{2(r+\delta)[y+(\tilde{a}_2/2)-c-b]\tilde{a}_2+m(\theta)(\varphi-1)(\tilde{a}_2^2/2)}{[2(r+\delta)+m(\theta)][2(r+\delta)]} \end{array} \right\} \quad (15a).
$$

$$
c = \frac{m(\theta)}{\theta}\left\{ \begin{array}{c} \gamma F(\tilde{a}_1)\frac{2(r+\delta)[y+(\tilde{a}_1/2)-c-b]\tilde{a}_1+m(\theta)(1-\varphi)(\tilde{a}_1^2/2)}{[2(r+\delta)m(\theta)][2(r+\delta)]} + \\ (1-\gamma)\left( \begin{array}{c} (1-F(\tilde{a}_2))\frac{[1-\tilde{a}_2][y-c-b]+(1/2)-(\tilde{a}_2^2/2)}{2(r+\delta)+m(\theta)(1-\varphi)} + \\ F(\tilde{a}_2)\frac{2(r+\delta)[y+(\tilde{a}_2/2)-c-b]\tilde{a}_2+m(\theta)[(\tilde{a}_2^2\varphi)/4]}{2(r+\delta)[2(r+\delta)+m(\theta)]} \end{array} \right) \end{array} \right\} \quad (15b).
$$

In order to find the equilibrium values for $\{\tilde{a}_1,\, \tilde{a}_2,\, \gamma,\, \varphi,\, \theta,\, u\}$, we have to solve the system of the following equations (6x6 non-linear system); (*15a*), (*15b*), (*14a*), (*14b*), (*13a*) and (*13b*).

### 3.1. Comparative statics

The subject of interest in this paper is the shifts in the matching function due to technological advances in matching and their impact on wage inequality. We assume a meeting function of the form, $m(v,\, u) = A\sqrt{vu}$, so $m(\theta) = A\sqrt{\theta}$, where $A$, is the parameter capturing the technological advances in matching. In the baseline case, we assume that, $p = 1/3$, $b = 0.1$, $y = 1$, $c = 0.3$, $\delta = 0.2$, $r = 0.05$, and $A = 1.8$. *Table I*, presents the results for the comparative statics analysis for $A$. As $A$, increases, the probability of matching with a job of the same type increases, therefore workers become pickier and as a result the reservation value of $a$ decreases. The fact that $\tilde{a}_1$, is always greater than $\tilde{a}_2$, incorporates the assumption of 'asymmetry' in the population, i. e. $p < 1 - p$. In the baseline case, the fraction of type-*1* unemployed is quite small. However, as the constant of matching increases, $\gamma$ increases as well. The basic reason for this is the following: given that most workers are type-*2* ($p = 2/3$), it is extremely difficult for type-*1* workers to find a job of the same type.

**TABLE I**: Comparative Statics for A-Solution with $m(\theta)$=$A\sqrt{\theta}$

| | $A = 1.8$ | $A = 2$ | $A = 2.2$ | $A = 2.4$ | $A = 2.6$ | $A = 3.4$ | $A = 100$ | $A = 100000$ |
|---|---|---|---|---|---|---|---|---|
| $\tilde{a}_1$ | 0.8021 | 0.7653 | 0.7519 | 0.7186 | 0.7020 | 0.5803 | 0.0077 | 0 |
| $\tilde{a}_2$ | 0.4193 | 0.3141 | 0.2415 | 0.2088 | 0.1788 | 0.1334 | 0.0054 | 0 |
| $\gamma$ | 0.2808 | 0.3040 | 0.3114 | 0.3244 | 0.3277 | 0.3559 | 0.4138 | 0.4109 |
| $\varphi$ | 0.3933 | 0.3478 | 0.3035 | 0.2837 | 0.2601 | 0.2350 | 0.4125 | 0.4098 |
| $\sqrt{\theta}$ | 1.7595 | 1.8450 | 1.9431 | 1.9613 | 2.0031 | 1.9202 | 1.9005 | 1.9150 |
| $u$ | 0.0716 | 0.0641 | 0.0558 | 0.0514 | 0.0465 | 0.0387 | 0.0020 | 0 |
| $Y$ | 1.1612 | 1.1988 | 1.2331 | 1.2494 | 1.2675 | 1.2988 | 1.4939 | 1.5 |

As a result of this, the gain from search for type-*1* individuals is so small that even the high skilled ($\tilde{a}_1 = 0.8021$), accept a type-*2* job. This implies that the effective rate of employment opportunities for the most of type-*1* workers is $m(\theta)$ and not $\varphi m(\theta)$. Moreover, the fact that $\varphi$, is low (due to the overrepresentation of type-*2* individuals in the population), further implies that a large number of type-*1* workers are employed by type-*2* jobs. On the other hand type-*2* workers are pickier when they choose their job ($\tilde{a}_2 = 0.4193$), i. e. for the most of them the effective rate of employment opportunities is $(1 - \varphi)m(\theta)$. Hence, the 'indifference' of type-*1* workers regarding their employment future resulting to their employment by type-*2* jobs, justifies the low value of $\gamma$. As an increase in matching opportunities takes place (increase in $A$), we move from a mixed-type pattern of equilibrium to a more ex-post segmented pattern. Both types of workers become pickier, leading to a increase in $\gamma$ and a decrease in $\varphi$ (as $A$ increases, it is more profitable for the firms to open type-*2* jobs). The reason why $\varphi$, increases after a certain value of $A$, is because the negative effect of being more profitable to open type-*2* jobs is dominated by the positive effect of being more difficult for type-*1* workers to find jobs of the same type.

Overall unemployment rate is decreasing in $A$, since an increase in $A$, reduces the impact of search frictions, and enables individuals to match at a faster rate. The average duration of unemployment in the baseline case for type-*1* and type-*2* workers with $a \geq \tilde{a}_i$, is (number of months in a year $*$ $(1/(m(\theta) * \varphi))$ $= 9.6338$), $9.6338$ and (number of months in a year $*$ $(1/(m(\theta) * (1 - \varphi)))$ $=$ $6.2452$), $6.2452$ months respectively. The corresponding duration of unemployment for those with $a < \tilde{a}_i$, is $3.789$ months. The average duration of a vacancy is (number of months in a year $*$ $(\theta/m(\theta))$ $= 11.73$)$11.73$ months. When $A = 3.4$, there is a large decrease in the average duration of the 'unproductive' state (unemployed worker/vacant job); $1.838$ months for workers (with $a < \tilde{a}_i$), $7.8215(2.4027)$ months for type-*1(2)* workers with $a \geq \tilde{a}_i$ and, $6.7772$ months for vacancies.

The fifth row of *Table I*, shows the effects of the increase in the technological parameter of matching faction on the measure of labor market tightness. The increase in $A$ is implicitly equivalent with an increase in productivity (the number of low productive 'mismatches' decreases), leading to a rotation of the job creation line anticlockwise and a shift of Beveridge curve towards the origin. Hence labor market tightness (slope of job creation line) increases. However, there is a negative influence on labor market tightness due to the reduction of job acceptance by workers. This negative effect seems to be dominated by the implicit increase in productivity for the first five values of $A$. On the contrary this does not hold for the last three values.

Finally, the last row of *Table I*, shows the impact of $A$ on aggregate output ($Y$) (for the formula see appendix).

## 4. WAGE INEQUALITY

The computation of wage inequality, presupposes the knowledge of the distribution of wages. By using the formulas for the wage paid in each case (different types of workers - different skills)[10], we apply 'Monte Carlo' simulation to get the distribution of wages. *Table II*, presents the impact of fluctuations in 'matching technology' on four inequality measures; the coefficient of variation, the Gini coefficient, the Theil Entropy Index and the variance of the natural logarithm of earnings. As we note, all inequality measures register an increase as $A$ goes from 1.8 to 3.4, and decrease for the extreme values of $A$ (100, 100000). The last row of *Table II*, shows the % change in inequality for each measure as $A$ increases from 1.8 to 3.4. The greatest increase is registered by the variance of natural logarithm of wages, where the lowest by the coefficient of variation.

**TABLE II**: Income Inequality

| A | C.V. | G.C. | T.I. | Var. |
|---|---|---|---|---|
| 1.8 | 0.239558 | 0.137692 | 0.02876 | 0.060334 |
| 2 | 0.241444 | 0.138843 | 0.02927 | 0.061703 |
| 2.2 | 0.242667 | 0.139556 | 0.029575 | 0.062425 |
| 2.4 | 0.24338 | 0.139999 | 0.029779 | 0.062996 |
| 2.6 | 0.244149 | 0.140436 | 0.029972 | 0.063438 |
| 3.4 | 0.2448 | 0.140938 | 0.030256 | 0.064587 |
| 100 | 0.240596 | 0.13873 | 0.029482 | 0.064105 |
| 100000 | 0.240517 | 0.138688 | 0.029464 | 0.064065 |
| % change between A=1.8 & A=3.4 | 2.188 | 2.357 | 5.2 | 7.04 |

As $A$ increases, we go from a 'random assignment' model to a model of self-selection. However, we note that inequality decreases only for the last two values of $A$. The reason for this is that the 'mismatched' workers (who are from the low skill ranks), bridge the gap between proper matched high and low skill workers. This can be clearly illustrated with the following example: Let assume that we have two workers; a low and a high skilled. The wage earned by high-skilled is 2 and the wage paid to low-skilled is 0.5. Suppose now a third worker with wage 1.5 in case of a 'proper' match and 0.75 in case of a 'mismatch'. The coefficient of variation for the first two workers is 0.85. If we add the third mismatched worker we get 0.74. As search frictions disappear due to the large increase in $A$ (100, 100000), the negative impact of self-selection on inequality dominates the positive aforementioned effect.

[10] For the workers who match with both type of vacancies ($a_i \leq \tilde{a}_i$), the formula used for the wage is the average of the wage earned in each case weighted with the fraction of the corresponding type of vacancies, $\varphi$.

## 5. CONCLUSION

We have constructed a model which highlights the influence of technological advances in matching process between employers and employees on within-group wage inequality. According to our analysis, a composition effect related with the matching process seems to be the cause for the increase in within group inequality. The model is consistent with the empirical evidence from US economy showing a positive relation between wage inequality and developments in the way that economic agents meet each other within a labor market environment. However, for high values of the technological parameter of the matching function ($A$) -frictionless case-, the inequality decreases, a fact which is consistent with the theory of 'self-selection'.

## 6. APPENDIX

**Derivation of** *aggregate output (Y)*

Aggregate output is computed as expected productivity of type-*1* workers times type-*1* employment plus expected productivity of type-*2* workers times type-*2* employment, specifically:

$$\begin{aligned} Y &= (p-\gamma u)\left\{ \begin{array}{c} F(\tilde{a}_1)\left[ \begin{array}{c} \varphi(y+\int_0^{\tilde{a}_1} a_1 f(a_1)da_1)+ \\ (1-\varphi)(y+\int_0^{\tilde{a}_1} \frac{a_1}{2} f(a_1)da_1) \end{array} \right] \\ +(1-F(\tilde{a}_1))(y+\int_{\tilde{a}_1}^1 a_1 f(a_1)da_1) \end{array} \right\} + \\ & \quad (1-p-(1-\gamma)u)\left\{ \begin{array}{c} F(\tilde{a}_2)\left[ \begin{array}{c} (1-\varphi)(y+\int_0^{\tilde{a}_2} a_2 f(a_2)da_2)+ \\ \varphi(y+\int_0^{\tilde{a}_2} \frac{a_2}{2} f(a_2)da_2) \end{array} \right] \\ +(1-F(\tilde{a}_2))(y+\int_{\tilde{a}_2}^1 a_2 f(a_2)da_2) \end{array} \right\} \end{aligned}$$

**Derivation of** *the flow value of unemployment for each type of worker*:

By substituting (*11a*) into (*7a*), we get:

$$W_{11}(a_1^n) = \frac{\frac{1}{2}(y+a-c)+\frac{1}{2}rU_1^n(a_1^{n-})+\delta U_1^n(a_1^{n-})}{r+\delta} \qquad (A1).$$

$$W_{12}(a_1^n) = \frac{\frac{1}{2}(y+\frac{1}{2}a-c)+\frac{1}{2}rU_1^n(a_1^{n-})+\delta U_1^n(a_1^{n-})}{r+\delta} \qquad (A2).$$

By substituting (*10*) into (*2*) and then the result into (*4a*), we get:

$$\begin{aligned} rU_1^n(a_1^{n-}) &= b+\frac{m(\theta)\varphi}{2}[W_{11}(a_1^n)+J_{11}(a_1^n)-U_1^n(a_1^{n-})] \\ & \quad +\frac{m(\theta)(1-\varphi)}{2}[W_{12}(a_1^n)+J_{12}(a_1^n)-U_1^n(a_1^{n-})] \qquad (A3). \end{aligned}$$

Finally, by substituting (*A1*), (*A2*) and (*12a*) into the above equation we get

$$rU_1^n(a_1^{n-}) = \frac{2(r+\delta)b+m(\theta)[y-c+(1+\varphi)(a/2)]}{2(r+\delta)+m(\theta)} \qquad (A4)$$

By following a similar procedure, we get

$$rU_2^n(a_2^{n-}) = \frac{2(r+\delta)b + m(\theta)[y - c + (1-(\varphi/2))a]}{2(r+\delta) + m(\theta)} \qquad (A5).$$

$$rU_1^n(a_1^{n+}) = \frac{2(r+\delta)b + m(\theta)\varphi[y + a - c]}{2(r+\delta) + m(\theta)\varphi} \qquad (A6).$$

$$rU_2^n(a_2^{n+}) = \frac{2(r+\delta)b + m(\theta)(1-\varphi)[y + a - c]}{2(r+\delta) + m(\theta)(1-\varphi)} \qquad (A7).$$

**Derivation of** (*14a*):

Since the marginal worker is indifferent between the two type of jobs, $W_{12}(a_1^n) = U_1^n(a_1^{n-})$. Hence, from (*A2*) we get:

$$\frac{\frac{1}{2}(y + \frac{1}{2}\tilde{a}_1 - c) + \frac{1}{2}rU_1^n(\tilde{a}_1) + \delta U_1^n(\tilde{a}_1)}{r+\delta} = U_1^n(\tilde{a}_1) \Rightarrow rU_1^n(\tilde{a}_1) = y + \frac{\tilde{a}_1}{2} - c \qquad (A8).$$

By setting, $W_{12}(a_1^n) + J_{12}(a_1^n) - U_1^n(a_1^{n-}) = 0$, and by substituting (*A1*), (*A2*) and (*12a*) into (*A3*), we get:

$$rU_1^n(\tilde{a}_1) = \frac{2(r+\delta)b + m(\theta)\varphi[\tilde{a}_1/2]}{2(r+\delta)}$$

Finally by equating the above equation with (*A8*), we get (*14a*).

Equation (*14b*) is derived by following a similar procedure.

**Derivation of** (*15a*):

By substituting equations, (*10*), (*A4*), (*A5*) and (*A6*) into (*9a*), we get:

$$c = \frac{m(\theta)}{\theta}\left\{ \begin{array}{c} \gamma\left( \begin{array}{c} F(\tilde{a}_1)\int_0^{\tilde{a}_1} \frac{2(r+\delta)[y+a_1-c-b]+m(\theta)[(3/2)+\varphi]a_1}{[2(r+\delta)+m(\theta)][2(r+\delta)]}da_1 + \\ (1-F(\tilde{a}_1))\int_{\tilde{a}_1}^1 \frac{2(r+\delta)[y+a_1-c]-2(r+\delta)b}{2(r+\delta)+m(\theta)\varphi}da_1 \end{array} \right) + \\ (1-\gamma)F(\tilde{a}_2)\int_0^{\tilde{a}_2} \frac{2(r+\delta)[y+(a_2/2)-c-b]+m(\theta)[\varphi(a_2/2)-(a_2/2)]}{[2(r+\delta)+m(\theta)][2(r+\delta)]}da_2 \end{array} \right\}$$

by calculating the integrals we finally get (*15a*).

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