E C O N O M I C S B U L L E T I N

Coopetition in a Mixed Duopoly Market

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Abstract

This study aims to investigate the impact of privatization on the degree of cooperation and competition in a mixed duopoly market. In this market, one semipublic firm and one private firm determine the level of two types of effort: the cooperative effort made to enlarge the total market size and the competitive effort made to increase market share. In a contest framework, our results show that the competitive effort level of the semipublic firm is smaller than that of the private firm. The more the semipublic firm is concerned for social welfare, the less it competes. On the basis of average costs, we then identify the conditions in which only the semipublic firm undertakes cooperative effort while the private firm behaves as a free rider. Besides, contrarily to common belief, our results highlight that a bad level of privatization may favor the most the free rider. Furthermore, we find that the semipublic firm always expends more cooperative effort than does the private firm.

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1 Introduction

In the business world, many firms choose to undertake not only competitive activities but also cooperative activities with each other. Studies such as Brandenburger and Nalebuff (1996), Dagnino and Padula (2002) describe situations that contain both cooperative and competitive activities in terms of coopetition. Literatures on coopetition have developed rapidly, particularly in recent years, and the concept has been used to explain many economic and social phenomena in various industries and in different countries. However, to the best of our knowledge, previous studies on coopetition have focused either on the activities of private firms or those of public firms. In other words, they have not addressed the market where there are both private and public firms. Seminal works such as Merrill and Schneider (1966), Harris and Wiens (1980), Bös (1986, 1991), Vickers and Yarrow (1988) and De Fraja and Delbono (1990) describe this market as a "mixed oligopoly market".

With privatization and deregulation waves in both developed and developing countries, phenomena of coopetition in mixed oligopoly markets have become common. For instance, the mobile phone French market is composed of three operators: Orange, SFR and Bouygues with Orange being a semipublic firm because 18.17% of its holding company, France Telecom, is controlled by the French government. Besides competing to enlarge their market shares, these operators created together in October 2003 a Wireless Link Association in an effort to offer, in the shortest period of time and at cheaper costs, the best homogenous public WiFi service covering the whole country.

Another example is the Japanese life insurance market in which Japan post, originally a pure public entity, was reformed in 2003 to be a semipublic organization to improve its efficiency. In October 2007 Japan Post was admitted to the Life Insurance Association of Japan which coordinates services for policyholders, evaluates moral hazard and so on. Japan Post and private life insurers are expected to adopt cooperative strategies to develop their common interests. However, they also compete on premiums and quantities to expand their market shares.

The purpose of this study is to investigate the coopetition in a market where there are both semipublic and private firms. For a given set of market conditions, for example, the number of firms and the timing of decisions, the activities chosen by semipublic and private firms may differ because of differences in their respective objective functions. Specifically, if one firm is semipublic rather than private, how does this affect market equilibrium?

To answer this question, we consider the simplest duopoly market that contains one semipublic firm and one private firm. Following Bös (1991), Matsumura (1998), Barcena-Ruiz and Garzon (2003) and Matsumura and Kanda (2005), we assume that the semipublic firm is only a partial social welfare maximizer with pure social welfare maximizer as a special case[1]. This assumption is not irrelevant because privatized firms with mixed ownership must also respect the interests of private shareholders. As a result of this, they cannot behave as pure welfare maximizers.

Our research is also inspired by Chung (1996), Krishnamurthy (1999) and Dearden and Lilien (2001), who model coopetition in a contest framework. The merit of using a contest

[1]See, for example, De Fraja and Delbono (1990), White (1996), Fjell and Pal (1996), Mujumdar and Pal (1998) and Pal (1998) for the case where public firms' objective is pure social welfare maximization.

framework to model coopetition is that it allows taking into account agents' competitive efforts, which are neglected in the traditional Cournot and Bertrand models. Ngo (2006) argues that many economic and social phenomena can be viewed as coopetition contests in which agents spend resources in order to win one or more prizes. Many examples of coopetition contests can be found in real life: employees compete with each other for promotion in organizational hierarchies but also work collectively to develop their firms; domestic firms compete for market share but also join together against foreign firms; athletes compete for prizes but are mutually responsible for attracting the spectators.

Regarding players' efforts, Chung (1996) analyzes only the competitive effort levels expended by players to increase their probability of winning the prize which expands with aggregate competitive efforts. Unlike Chung (1996) and following Krishnamurthy (1999) and Dearden and Lilien (2001), we assume that each firm chooses two types of effort level: cooperative effort and competitive effort. The latter aims to increase their respective market shares while the former is expended to enlarge the common market size. In other words, we develop a coopetition model of effort levels in a mixed duopoly market.

Our work contributes to this literature in two ways. First, while previous studies assume that payoff functions of players are similar, we consider the general case in which players pursue different objectives and thus have distinct payoff functions. Second, following Ngo (2006) we employ a two-stage model. As pointed out by Dumez and Jeunemaître (2006), there are two types of coopetition. In a one-stage game, cooperation and competition occur simultaneously in a multi-dimension framework while in a two-stage or multi-stage game, cooperation and competition take place sequentially.

The remainder of this study is organized as follows. In Section 2, we develop a model of cooperative and competitive effort in a mixed oligopoly market that contains both private and semipublic firms. The model is used to derive several interesting results. Concluding remarks are given in Section 3.

2 The model

For simplicity, we consider a model of two firms. Firm A is semipublic, while firm B is private. These firms are supposed to play a two-stage game.

In the first stage, both firms simultaneously choose their cooperative effort levels. The cooperative effort level is denoted by y_i for $i \in \{A, B\}$, which increases the total market. The overall market demand function is:

$$y = a + y_A + y_B$$

where a represents initial demand without any cooperative effort.

In the second stage, both firms simultaneously choose their competitive effort levels, which are denoted by x_i. Competitive efforts can enhance individual firms' competitive power and market shares. That is, if there is no competitive effort, the market is equally divided to two firms. Otherwise, the pie share of firm i is determined by the ratio $x_i/(x_A + x_B)$.

Let the inverse demand function be $p(\cdot)$. Assume that $p'(\cdot) < 0$. Average cost denoted by c is assumed to be constant and the same for both firms.

Then, the objective function of firm B is[2]:

$$U_B = \Pi_B = (p-c)\, y \frac{x_B}{x_A + x_B} - k_x x_B - k_y y_B^2 \tag{1}$$

where $p \equiv p(y)$, $k_x x_B$ and $k_y y_B^2$ represent the costs of expending competitive and cooperative efforts respectively[3].

Because A is a semipublic firm, its objective function is:

$$U_A = \alpha W + (1-\alpha)\, \Pi_A \tag{2}$$

where W, which represents the social surplus, is the sum of the producer's profit and the consumer's surplus; the parameter $\alpha \in [0,1]$ can be interpreted at two levels. At one level, it represents the weight of the government's participation in the firm A. At the other level, it can be regarded as the importance level attributed to the government's objective, i.e. the social welfare, in contrast with the profit objective. $\alpha = 0$ signifies that the firm A is solely concerned about its profit. $\alpha = 1$ means that the firm A aims to maximize the social welfare regardless of its profit.

It follows that:

$$\Pi_A = (p-c)\, y \frac{x_A}{x_A + x_B} - k_x x_A - k_y y_A^2 \tag{3}$$

and the social surplus can be written as:

$$W = \int_0^y p(q)\, dq - cy - k_x (x_A + x_B) - k_y \left(y_A^2 + y_B^2\right) \tag{4}$$

To derive the extensive form game, we solve the game by backward induction. That is, the equilibrium in the second stage is derived on the basis of the first stage before the first stage has been played. Once the equilibrium in the second stage is determined, the equilibrium in the first stage is derived by using the results from the second stage.

The second stage is described below.

The first-order conditions with respect to x_i are:

$$\frac{\partial U_A}{\partial x_A} = \frac{(1-\alpha)(p-c)\, x_B y - k_x (x_A + x_B)^2}{(x_A + x_B)^2} = 0$$
$$\frac{\partial U_B}{\partial x_B} = \frac{(p-c)\, x_A y - k_x (x_A + x_B)^2}{(x_A + x_B)^2} = 0$$

Given $\alpha \in [0,1)$, the equilibrium competitive effort levels are:

$$x_A^* = \frac{(1-\alpha)^2 (p-c)\, y}{(2-\alpha)^2 k_x} \; ; \; x_B^* = \frac{(1-\alpha)(p-c)\, y}{(2-\alpha)^2 k_x} \tag{5}$$

[2]Here we consider a deterministic outcome competition in which each party receives a share of what is under dispute. The equivalent results under a probabilistic competition, i.e. in a winner-take-all contest, can be derived under the assumption of risk neutrality.

[3]The ratio $x_i/(x_A + x_B)$ is concave in x_i while the overall market demand function is linear in y_i. Thus, in order to guarantee the existence of the optimal x_i and y_i, we assume that the cost functions of x_i and of y_i are, respectively, linear and quadratic.

and the market shares of public and private firm are:

$$s_A^* = \frac{1-\alpha}{2-\alpha} \; ; \; s_B^* = \frac{1}{2-\alpha}$$

When $\alpha \to 1$, it is reasonable to assume that $x_A^* \to 0, x_B^* = \varepsilon$ (where ε represents a very small positive number) and the market shares are $s_A^* \to 0, s_B^* \to 1$.

These results are used to state the following lemma.

Lemma 1 (relationship between competitive effort levels)**.**
Both competitive effort levels satisfy: $x_A^* = (1-\alpha)\, x_B^*$. *Furthermore, the relationship between market shares of firm A and B:* $s_A^* = (1-\alpha)\, s_B^*$.

This result implies that the competitive effort level of the semipublic firm is below that of the private firm. The more concerned is the semipublic firm for social welfare (the closer is α to 1), the less it competes. As a result of this, the market share of the public firm is never bigger than that of the private firm.

Several comments on the equilibrium cooperative effort levels shown in (5) are warranted. In this context, consider the case in which only one variable changes.

First, $\partial x_i^*/\partial k_x < 0$ and $\partial x_j^*/\partial k_x < 0$ have the simple and intuitive implication that the higher is the cost level, the lower is the competitive effort.

Second, consider the relationship between competitive and cooperative effort levels. From (5), the following derivatives are obtained:

$$\frac{\partial x_A^*}{\partial y_A} = \frac{\partial x_A^*}{\partial y_B} = \frac{(1-\alpha)^2 \{p'y + p - c\}}{(2-\alpha)^2 k_x} = \frac{(1-\alpha)^2 \left\{p\left(1-\frac{1}{e}\right) - c\right\}}{(2-\alpha)^2 k_x}$$
$$\frac{\partial x_B^*}{\partial y_B} = \frac{\partial x_B^*}{\partial y_A} = \frac{(1-\alpha) \{p'y + p - c\}}{(2-\alpha)^2 k_x} = \frac{(1-\alpha) \left\{p\left(1-\frac{1}{e}\right) - c\right\}}{(2-\alpha)^2 k_x}$$

where $e \equiv -\frac{p}{p'y}$ is the price elasticity of demand.

Thus, if the demand function is sufficiently price elastic, i.e. $e > p/(p-c)$ then we have the following results: (i) $\partial x_i^*/\partial y_i > 0$ implying that both types of effort spent by a firm are complements; (ii) $\partial x_i^*/\partial y_j > 0$ implying that both types of effort spent by two different firms are also complements. By contrast, if the demand is sufficiently price inelastic, i.e. $e < p/(p-c)$ then we have the following results: (i) $\partial x_i^*/\partial y_i < 0$ implying that both types of effort spent by a firm are substitutes; (ii) $\partial x_i^*/\partial y_j < 0$ implying that both types of effort spent by two different firms are also substitutes.

In general, competition and cooperation are considered as two polar opposites, that is, a higher level of cooperation naturally leads to a lower level of competition and *vice versa*. On the contrary, in a coopetitive game, the relation between competition and cooperation can be positive or negative depending on the price elasticity level of the demand. The following lemma summarizes these results.

Lemma 2 (relationship between competitive and cooperative efforts)**.**
If e is sufficiently large ($e > p/(p-c)$)*, then both types of effort are complements. In contrast, if e is sufficiently small* ($e < p/(p-c)$)*, then both types of effort are substitutes.*

At present, we analyze the first stage of the game. Substituting x_A^*, x_B^* into equations (1) and (2) yields:

$$U_A = \frac{1}{(2-\alpha)^2}(p-c)\,y - \alpha p y - k_y\left(y_A^2 + \alpha y_B^2\right) + \alpha \int_0^y p\,(q)\,dq$$

$$U_B = \frac{1}{(2-\alpha)^2}(p-c)\,y - k_y y_B^2$$

The first-order conditions with respect to y_A and y_B are:

$$\frac{\partial U_A}{\partial y_A} = \frac{1}{(2-\alpha)^2}p'y + \frac{1}{(2-\alpha)^2}(p-c) - \alpha p'y - 2k_y y_A = 0 \tag{6}$$

$$\frac{\partial U_B}{\partial y_B} = \frac{1}{(2-\alpha)^2}p'y + \frac{1}{(2-\alpha)^2}(p-c) - 2k_y y_B = 0 \tag{7}$$

To obtain interior solutions from equations (6) and (7), one can remark that the following condition must be satisfied: $p'y + p - c = p\left(1 - \frac{1}{e}\right) - c \geq 0$

From equations (6) and (7), the following proposition can be derived.

Proposition 1 (degree of cooperation)**.**
Define that $\bar{c} \equiv p\left[1 - \frac{1}{e}\left(1 - \alpha\,(2-\alpha)^2\right)\right]$ *and* $\underline{c} \equiv p\left(1 - \frac{1}{e}\right)$*. Then, there are three outcomes of cooperative effort levels corresponding to three different average cost levels.*

1. *If average cost is high, i.e.* $c > \bar{c}$ *, both firms expend no cooperative effort. Moreover, equilibrium competitive effort levels are:* $(x_A^*, x_B^*) = \left(\frac{(1-\alpha)^2(p(a)-c)a}{(2-\alpha)^2 k_x}, \frac{(1-\alpha)(p(a)-c)a}{(2-\alpha)^2 k_x}\right)$.

2. *If average cost is moderate, i.e.* $\underline{c} < c < \bar{c}$ *, only the semipublic firm expends cooperative effort. The private firm free rides.*

3. *If average cost is low, i.e.* $c < \underline{c}$ *then, both firms expend cooperative effort.*

Proof.
From equation (6), the following condition is necessary for $x_A^* > 0$: $c < \bar{c}$.
From equation (7), the following condition is necessary for $x_B^* > 0$: $c < \underline{c}$.
It is easy to verify the following inequality since $\alpha\,(2-\alpha)^2 \geq 0$: $\underline{c} < \bar{c}$.

From these above inequalities, all three cases in relation to average costs can be derived. If the inequality $c < \bar{c}$ is not satisfied, then the best strategy for both firms is to produce no output ($y_A = 0$ and $y_B = 0$). The equilibrium competitive effort levels can be derived by substituting $y_A = 0$ and $y_B = 0$ into equations (6) and (7). □

Proposition 1 has interesting implications. When α rises, while the semipublic firm behaves more like a public firm, it has ambiguous effect on private firm's behavior as a free rider. In fact, when $\alpha < 2/3$, the case 2 is more likely to arise because $\partial\bar{c}/\partial\alpha = p/e.(2-\alpha)(2-3\alpha) > 0$ and $\partial\underline{c}/\partial\alpha = 0$. This property implies that the semipublic firm behaves more like a public firm and the private firm is more likely to free ride. However, when $\alpha > 2/3$, $\partial\bar{c}/\partial\alpha < 0$, surprisingly, rising α will lead the private firm less likely to free ride. The nearer α is to 0 (semi public cares less and less about welfare) or to

1 (semi public cares more and more about welfare), the free ride seems to be the less probable. In the context of a quantity-setting oligopoly, De Fraja and Delbono (1990) show that welfare may be higher when a public firm maximizes profits rather than welfare. Their results suggest that, in some cases, a public firm should be privatized. Our results obtained from a coopetitive framework indicate that in order to mitigate the free rider problem, it is not enough to privatize public firm; the privatization level also needs to be taken into consideration. More precisely, the free ride is less likely to happen when the semi public firm acts either much like public firm or much like private firm. Contrarily to common belief, a mixture around the middle of the two forms may favor free ride behavior of the private rival firm.

Regarding the price elasticity of demand, the smaller is e, the more likely is the case 2 to arise because $\frac{\partial \bar{c}}{\partial e} = \frac{p\left(1-\alpha(2-\alpha)^2\right)}{e^2} < \frac{p}{e^2} = \frac{\partial \underline{c}}{\partial e}$. This implies that the lower is the price elasticity of demand, the more likely is the private firm to free ride.

Next, we try to make a general comparison of cooperative effort levels which yields the following proposition.

Proposition 2 (cooperative effort levels)**.**
The private firm has no strategic incentive to spend more cooperative effort than the semipublic firm. That is, $y_B^* \leq y_A^*$ with strict equality when $c > \bar{c}$ or $\alpha = 0$.

Proof.
In case 1, i.e. $c > \bar{c}$, both firms do not spend cooperative effort ($y_A^* = y_B^* = 0$).
In case 2 and 3, just by subscribing (7) from (6), we get

$$\begin{aligned} \alpha p' y &= 2k_y \left(y_B^* - y_A^*\right) \\ \Rightarrow y_B^* - y_A^* &= \frac{\alpha p' y}{2k_y} \leq 0, \end{aligned}$$

equality holds if and only if $\alpha = 0$. □

3 Concluding remarks

In this study, we developed a coopetition model of a mixed duopoly market. We built the model to describe a situation in which both public and private firms determine their levels of competitive effort to expand their market shares after having chosen their cooperative effort levels to maximize the total market size.

Our results showed that the competitive effort level of the semipublic firm is below that of the private firm. The more concerned is the semipublic firm about social welfare, the less it competes. On the basis of average cost, we then identified the conditions under which only the semipublic firm expends cooperative effort while the private firm behaves as a free rider. Besides, there exists a critical level of privatization at which the free ride is the most likely to occur. Furthermore, we found that the semipublic firm always expends more cooperative effort than does the private firm. Hence, our analysis generates many insights of interest to the government, public and private firms. In fact, the government can use the privatization level as a mean to regulate the competitiveness of public firms and to mitigate free-rider problem. However, it must keep in mind that a

bad mixture of public and private forms may favor free ride behavior of private rival firms. Our model also offers an explanation for why public firms almost always contribute the most in cooperative associations with private firms but the former often have difficulties in competing with the latter.

Despite these contributions, our research is incomplete in several respects. Following two out of such several aspects are the most interesting and important. First, we assumed that both firms choose their effort levels simultaneously. However, in reality, semipublic firms may choose their cooperative effort levels before private firms do. Thus, a Stackelberg model may be more appropriate than a Nash one. Second, the extent to which the semipublic firm cares about social welfare is implicitly assumed to be common knowledge: the private firm knows how much importance the semipublic firm attaches to the social surplus. If there exists some asymmetric information about that and the private firm is risk averse, cooperative and competitive effort levels at equilibrium may change. The extension of our model following these directions remains for future research.

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