E C O N O M I C S   B U L L E T I N

Cournot competition and location choice with wage bargaining

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Abstract

Equilibrium locations are analyzed in location-quantity games, in which firms acquire labor inputs through bilateral monopoly relations with independent labor union. We find that the pattern of locations varies as the transport rate increase in a linear city. In a circular city, firms locate equidistant form each other.

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1. Introduction

Spatial models with Cournot competition have being increasing popular in recent years. There are two standard models of location-quantity choice that are considered in the literature: Hotelling's(1929) linear city model and Salop's(1979) circular city model.

Anderson and Neven(1991) and Hamilton et al.(1989) analyze the two stage problem of location and quantity choices in linear city model. Both papers show that the equilibrium outcome is central agglomeration. To extend their result, various other work such as Gupta et al.(1997), Mayer(2000), Chamorro Rivas(2000), Shimizu(2002), and Benassi et al.(2007) consider the same problem with different assumptions. Pal(1998) investigates a circular city duopoly model and shows that firms locate equidistant from each other. However, Matsushima(2001) demonstrates that firms agglomerate at two points in circular city model. Recently, Pal and Sarkar(2002) and Gupta et al.(2004) provide a quite characterization of the equilibrium outcomes in the location-quantity game.

In the standard models, production costs are exogenous. However, Brekke and Straume(2004) establish a location-price model in which firms' production costs are affected by their locations. The production costs in fact are endogenous in their model. This paper analyze the problem of location-quantity choice with endogenous production costs in both linear and circular city model. We do so by establishing a duopoly model in which firms employ workers through bilateral relations with independent labor unions. Workers' wages are determined in simultaneous bargaining between each firm and its labor union.

Compared with the standard models with exogenous production costs, our paper shows that the patterns of equilibrium under the linear city model depend on the value of transport rate. However, we also show that the result in circular city model is not affected by the introduction of endogenous production costs.

The paper is organized as follows. Section 2 describes the basic model. Section 3 discusses the linear city model. Section 4 discusses the circular city model. Section 5 concludes the paper.

2. The basic model

There are infinitely many consumers located uniformly at a unit length market. There are two firms who produce and sell a homogenous output to consumers. We suppose that workers are supplied to firms by the labor unions which have the objective of recent maximization. Furthermore, suppose workers' wages are determined in simultaneous bargaining. The unions' competitive or reservation wage level is assumed to be equal, and are, without loss of generality, normalized to zero. The two firms have identical technology. Firm i has to pay a linear transport cost $t|x-x_i|$ when ship a unit of good from its location x_i to a consumer at point x. And arbitrage among consumers is assumed to be infeasible.

We consider a three-stage game: first, firms simultaneously choose their locations x_1 and x_2; second, wages w_1 and w_2 are determined in simultaneous and independent bargaining; and third, each firm sets its output q_1 and q_2. As usual, the model is solved by backward induction.

Assuming that the inverse demand function at any point $x \in [0,1]$ is $p(x;x_1,x_2) = 1 - Q(x;x_1,x_2)$, where $Q(x;x_1,x_2) = q_1(x;x_1,x_2) + q_2(x;x_1,x_2)$ denotes the aggregate quantity supplied at x. In order to ensure all firms serve the entire market, we assume that $t < 4/11$[1].

Under the above assumptions, for both linear and circular city mode, firm i's $(i=1,2)$ profit function at x is

$$\pi_i(x;x_1,x_2) = (1 - Q(x;x_1,x_2) - w_i(x_1,x_2) - t|x-x_i|)q_i(x;x_1,x_2) \tag{1}$$

In the third stage, standard calculations yield the following equilibrium quantity at each x

$$q_i(x;x_1,x_2) = \frac{1 - 2w_i(x_1,x_2) + w_j(x_1,x_2) - 2t|x-x_i| + t|x-x_j|}{3} \quad (i,j=1,2; i \neq j) \tag{2}$$

Substituting (2) into (1), the profit function for firm i can be rewritten as

$$\pi_i(x;x_1,x_2) = (q_i(x;x_1,x_2))^2 \tag{3}$$

Therefore, firm i's total profits and total quantities are expressed respectively as

$$\Pi_i(x_1,x_2) = \int_0^1 \pi_i(x;x_1,x_2)\,dx \tag{4}$$

$$q_i(x_1,x_2) = \int_0^1 q_i(x;x_1,x_2)\,dx \tag{5}$$

Then, in the second stage, wages are determined by the following form: [2]

$$w_i(x_1,x_2) = arg\max_{w_i}(w_i(x_1,x_2)q_i(x_1,x_2)) \tag{6}$$

3. Linear city model

In this section, we discuss the firm's location choice in linear city markets. Let the firms locate at x_1 and x_2, with $x_1 \leq x_2$, w.l.o.g. First, from (2) and (5), the total quantities sold by each firm can be expressed as:

$$q_i(x_1,x_2) = \frac{1 - 2w_i(x_1,x_2) + w_j(x_1,x_2)}{3} + \frac{t(2x_j^2 - 4x_i^2 + 4x_i - 2x_j - 1)}{6} \tag{7}$$

[1]This can be obtained by $q_i > 0 (i=1,2)$.

[2]See, e.g., Naylor(1998).

Then, using the equilibrium quantity, (7), we can solve (6) to find the equilibrium wages as follows

$$w_i(x_1,x_2) = \frac{1}{3} + \frac{t(4x_j^2 - 14x_i^2 + 14x_i - 4x_j - 5)}{30} \tag{8}$$

At last, in stage 1, firm i chooses x_i to maximize its total profits, and let

$$\Delta_i = 1 - 2w_i(x_1,x_2) + w_j(x_1,x_2) = \frac{2}{3} + \frac{t}{30}(32x_i^2 - 22x_j^2 + 22x_j - 32x_i + 5) \tag{9}$$

Substituting (3),(7)-(9) into (4), then the total profits of each firm are given as

$$\begin{aligned} 9\Pi_1(x_1,x_2) &= \int_0^{x_1} [\Delta_1 - 2t(x_1 - x) + t(x_2 - x)]^2\, dx \\ &+ \int_{x_1}^{x_2} [\Delta_1 - 2t(x - x_1) + t(x_2 - x)]^2\, dx \\ &+ \int_{x_2}^{1} [\Delta_1 - 2t(x - x_1) + t(x - x_2)]^2\, dx \end{aligned} \tag{10}$$

$$\begin{aligned} 9\Pi_2(x_1,x_2) &= \int_0^{x_1} [\Delta_2 - 2t(x_2 - x) + t(x_1 - x)]^2\, dx \\ &+ \int_{x_1}^{x_2} [\Delta_2 - 2t(x_2 - x) + t(x - x_1)]^2\, dx \\ &+ \int_{x_2}^{1} [\Delta_2 - 2t(x - x_2) + t(x - x_1)]^2\, dx \end{aligned} \tag{11}$$

Totally differentiating Eqs. (10) and (11) with respect to x_i, respectively, the equilibrium locations can be summarized by the following proposition.

Proposition 1 *Suppose that* $t \in (0, 4/11)$. *Then*

(1) There is a unique equilibrium location, $x_1^* = x_2^* = \frac{1}{2}$, *when* $0 < t \le \frac{56}{239}$.

(2) There is only one separate equilibrium locations, $x_1^* = \frac{121t - \sqrt{15509t^2 - 1736t}}{62t} = g(t), x_2^* = 1 - x_1^*$ *when* $\frac{56}{239} < t < \frac{4}{11}$.

Since production costs are endogenous, firms' choices of location affect both market shares and production costs. Therefore, proposition 1 implies that agglomerate at the center of markets is not the unique equilibrium, dispersion is also an equilibrium outcome when transport rate t satisfies a certain range.

4. Circular city model

Similar to Section 3, let the firms locate at x_1 and x_2, and suppose $0 = x_1 \le x_2 \le \frac{1}{2}$ w.l.o.g. First, we obtain the total quantities of each firm from (5)

$$q_i(0,x_2) = \frac{1}{3}(1 - 2w_i(0,x_2) + w_j(0,x_2) - \frac{t}{4}) \tag{12}$$

Second, substituting (12) into (6), equilibrium wages are expressed as

$$w_i(0, x_2) = \frac{1}{3}(1 - \frac{t}{4}) \tag{13}$$

At last, substituting (3),(12)-(13) into (4), the total profits of firm 2 can be rewritten as

$$\begin{aligned} 9\Pi_2(0, x_2) &= \int_0^{x_2} (\frac{2}{3} + \frac{t}{12} - 2t(x_2 - x) + tx)^2 \, dx \\ &+ \int_{x_2}^{1/2} (\frac{2}{3} + \frac{t}{12} - 2t(x - x_2) + tx)^2 \, dx \\ &+ \int_{1/2}^{x_2+1/2} (\frac{2}{3} + \frac{t}{12} - 2t(x - x_2) + t(1 - x))^2 \, dx \\ &+ \int_{x_2+1/2}^{1} (\frac{2}{3} + \frac{t}{12} - 2t(1 - x + x_2) + t(1 - x))^2 \, dx \end{aligned} \tag{14}$$

The first and second order derivatives of the total profits function for firm 2 are expressed respectively as

$$\frac{d\Pi_2(0, x_2)}{dx_2} = \frac{4t^2 x_2(-2x_2 + 1)}{9}, \qquad \frac{d^2\Pi_2(0, x_2)}{dx_2^2} = \frac{4t^2(-4x_2 + 1)}{9}$$

Note that the first derivative is equal to zero at the points of $x_2 = 0$ and $\frac{1}{2}$ and is increasing in the range $[0, \frac{1}{2}]$. However, the second derivative is positive when $0 < x_2 < \frac{1}{4}$ and is negative when $\frac{1}{4} < x_2 < \frac{1}{2}$. Therefore, the total profits function of firm 2 reaches its global maximum at $x_2 = \frac{1}{2}$. Thus, if firm 1 locates at 0, $\frac{1}{2}$ is the unique location for firm 2. By symmetry, if firm 2 locates at $\frac{1}{2}$, we can conclude that firm 1 choose its optimal location at 0 in order to reach the profit maximization. Thus, we have the result stated in proposition 2.

Proposition 2 *In the location-quantity game with endogenous production costs, there is a unique equilibrium, where the two firms locate equidistant from each other on the circular.*

Our result is the same to Pal's(1998). However, we derive it by introducing endogenous production costs while the latter's production costs are exogenous. The intuitive explanation is that the wages paid by each firm are not affected by their location choices in the circular city model.

5. Conclusions

In this paper, we examine the patterns of location equilibrium in both linear city model and circular city model with endogenous production costs. Compared with the literature(for example, Anderson and Neven(1991), Hamilton et al.(1989) and Pal(1998)) which production costs are exogenous, we find that, in linear city model, agglomerate at the central markets is no longer the unique equilibrium for firms, disperse at different points is another equilibrium when transport rate satisfies some conditions. However, in circular city model, the equilibrium outcome of Pal's(1998) is robust.

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