E C O N O M I C S B U L L E T I N

Public monopoly, mixed oligopoly and productive efficiency: a generalization

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Abstract

The paper considers a cost-reducing investment by the public sector. We compare the investment in the public monopoly to that in the mixed oligopoly. Without specifications of the demand and cost functions, we show that the investment in the public monopoly is higher thanthat in the mixed oligopoly. Our result is a generalization of the result of Nishimori and Ogawa (2002), which investigate the effects of deregulation on the public sector's investment by assuming the linear demand and linear cost functions.

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1 Introduction

In many countries, including those in Europe, Central Asia, South Asia, and Africa, we can observe the state-owned public enterprises or the public sector. The public sectors are still having enormous influence in some developing countries and transitional economies.

The paper considers a strategic cost-reducing investment by the public sector. The purpose of this paper is to compare the incentive of the investment by the public sector in two regimes: the public monopoly and the mixed oligopoly. We show that the investment in the public monopoly is higher than that in the mixed oligopoly. This implies that the emergence of private firms has a negative impact for the incentive of the investment by the public sector. Our result has an important implication in the policy issue for developing countries and transitional economies. An important feature of our analysis is its generality. That is, we do not specify the demand and cost functions, and use only mild assumptions. Further, we consider not only the mixed oligopoly with no entry but also the mixed oligopoly with free entry. These generalizations guarantees the robustness of our result.

Our result is a generalization of the result of Nishimori and Ogawa (2002), which investigate the effects of deregulation on the cost-reducing investment by public sector. They assume that the inverse demand function is linear, and the marginal cost of each firm is constant. We remark the limitation of assuming the linear environment. It is well known that when the marginal cost of each firm is constant, the output of each private firm is zero if the public firm is as efficient as each private firm. Then, assumption that the public firm is less efficient than each private firm is indispensable. In fact, Nishimori and Ogawa (2002) assume inefficiency of the public firm. However, there exists many empirical studies that support the equality of cost efficiency between public and private firms. For example, see Boyd (1986), Iordanoglou (2001), and Millward (1982).[1] According to our results under general formulation, if cost function of each firm is strictly convex, the investment in the public monopoly is higher than that in the mixed oligopoly when the public firm and each private firm have same technologies.[2] These facts indicate the importance of our results.

Nishimori and Ogawa (2002) compare the public monopoly and the mixed oligopoly with no entry. We compare the investment in the public monopoly to that in the mixed oligopoly with free entry in addition to this comparison. Another difference in formulation between our model and their specification is as follows: in our model, the public firm chooses the investment, while in Nishimori and Ogawa's (2002) model, the public firm chooses its marginal cost. Since marginal cost is a decreasing function of the investment, our model is essentially a generalization of Nishimori and Ogawa's (2002) model.

Now, we mention other related literature.[3] For a discussion of the strategic cost-reducing investment in standard Cournot model, see Brander and Spencer (1983). Some papers investigate the strategic R&D competition in the mixed oligopoly. By using a patent race model similar to Laury (1979), Delbono and Denicolò (1993) studies R&D competition in a mixed duopoly. In their model, each firm chooses only R&D expenditure to maximize its payoff. Assuming that imitation is easy, Poyago-Theotoky (1998) investigates the role of a public firm in the free-rider

[1] Willner (2001) briefly summarizes contributions of empirical results of efficiency under different ownership.

[2] Strictly convexity of cost function is assumed by many papers of the mixed oligopoly. Typically, the quadratic cost function is used. Foe example, see De Fraja and Delbono (1989) and Matsumura (1998).

[3] De Fraja and Delbono (1989) and Matsumura (1998) provide fundamental results in the field of the mixed oligopoly.

problem of R&D. Ishibashi and Matsumura (2006) consider a patent race model where each firm chooses not only R&D expenditure but also its innovation size. These three papers use patent race models. By using location model, Matsumura and Matsushima (2004) study cost-reducing activities of public and private firms in a mixed duopoly. While all papers which mentioned does not use Cournot fashion model, this paper considers the two stage game of Cournot fashion.

The rest of the paper is organized as follows. In Section 2, we present definitions and a model. In Section 3, we show our comparative static result and the main result. The Appendix includes proofs of lemmata.

2 Basic Model

We consider $n+1$ firms that engage in quantity competition. The zero-th firm is a state-owned public enterprise whose objective is to maximize social welfare. Firm i ($i = 1, 2, \ldots, n$) is private and seeks to maximize its profit.

The inverse demand function is given by $P(Q)$, where P is the price and Q is the total output. Let the cost function of a state-owned public firm be $C^s(q_0, I)$ and that of a private firm be $C(q_i)$. I is the cost-reducing investment by the public sector. The cost expense for the cost-reducing investment is $f(I)$.

Social welfare comprises consumer surplus and the firms' profit:

$$W = \int_0^Q P(q)dq - PQ + \sum_{i=0}^{n} \Pi_i = \int_0^Q P(q)dq - C^s(q_0, I) - \sum_{i=1}^{n} C(q_i) - f(I)$$

where Π_i is firm i's profit.

We consider the following regimes: (i) public monopoly, (ii) mixed oligopoly with no entry, and (iii) mixed oligopoly with free entry. In a public monopoly, there exists one state-owned public firm which maximizes social welfare. In a mixed oligopoly, there exists one public firm and n private firms. In (ii), the number of private firms is exogenous, while in (iii) it is endogenous. Let W^P, W^M, and W^F denote welfares in public monopoly, mixed oligopoly with no entry, and mixed oligopoly with free entry, respectively.

We present assumptions.

Assumption 1. P is twice differentiable and $P' < 0$.

Assumption 2. $C' > 0, C'' \geq 0, C^s_q > 0, C^s_{qq} \geq 0, C^s_I < 0, C^s_{II} \geq 0$ and $C^s_{qI} < 0$.

Assumption 3. $P''q + P' < 0$.

Assumption 4. $f'(I) > 0$ and $f''(I) > 0$.

All assumptions are standard, and we need not explain these assumptions.

3 Results

In this section, we present and discuss our results.

Public Monopoly First, we consider the case of the public monopoly. In this game, the state-owned public firm chooses the investment at the first stage, and it chooses the output at the second stage. We solve this game by backward induction. In the second stage, given I, the public firm decides its output to maximize social welfare, and, thus, the optimality condition is as follows:

$$P(q_s^*) - C_q^s(q_s^*, I) = 0$$

where q_s^* is the function of I.

In the first stage, the public firm decides its investment. The optimality condition in the first stage is as follows:

$$\begin{aligned} \frac{dW^P}{dI} &= \Big(P(q_s^*) - C_q^s(q_s^*, I)\Big)\frac{dq_s^*}{dI} - C_I^s(q_s^*, I) - f'(I) \qquad (1)\\ &= -C_I^s(q_s^*, I) - f'(I) = 0. \qquad (2) \end{aligned}$$

Let I^P denote the optimal investment in the public monopoly. Note that $-C_I^s(q_s^*, I^P) - f'(I^P) = 0$.

Comparison to Mixed Oligopoly with No Entry Next, we consider the case of the mixed monopoly with no entry. The game is summarized as follows. At the first stage, the state-owned public firm chooses the investment. At the second stage, given the public firm's investment, each firm chooses output simultaneously. We use the subgame perfect Nash equilibrium as the equilibrium concept.

We solve this game by backward induction. In the second stage, given I, $n+1$ firms decide their outputs. We are interested in the equilibrium in which the outcomes of private firms are symmetric, i.e., $q_1 = q_2 =, \dots, = q_n$. Let q_s^{**} denote the equilibrium output of the public firm and q_p^{**} denote the equilibrium output of each private firm. Furthermore, Q^{**} denote the equilibrium total output. The equilibrium conditions are as follows:

$$P(Q^{**}) + q_p^{**}P'(Q^{**}) - C'(q_p^{**}) = 0, \qquad (3)$$
$$P(Q^{**}) - C_q^s(q_s^{**}, I) = 0. \qquad (4)$$

We introduce an additional assumption.

Assumption 5. $q_s^{**} > 0$ and $q_p^{**} > 0$.

This assumption requires that each firm produces positive output. This assumption is usually satisfied in the models of Cournot competition.

Under the standard assumptions, we obtain the following lemma by using equation (3) and (4).

Lemma 1. Suppose that assumptions 1–5 are satisfied. Then, (i) $dq_p^{**}/dI < 0$ and (ii) $dq_s^{**}/dI > 0$.

The proof of this lemma is in the Appendix. According to this lemma, the output of each private firm is decreasing in I, and the output of the public firm is increasing in I. This result is very intuitive, so we do not explain more. The key of this result is strategic substitute in oligopolistic quantity-competition.

In the first stage, the public firm decides its investment. The optimality condition in the first stage is as follows:

$$\frac{dW^M}{dI} = n\Big(P(Q^{**}) - C'(q_p^{**})\Big)\frac{dq_p^{**}}{dI} - C_I^s(q_s^*, I) - f'(I) \tag{5}$$

$$= -nq_p^{**}P'(Q^{**})\frac{dq_p^{**}}{dI} - C_I^s(q_s^*, I) - f'(I) = 0, \tag{6}$$

where we use equation (3) and the envelope theorem. Let I^M denote the optimal investment in the mixed oligopoly. Note that $-nq_p^{**}P'(Q^{**})dq_p^{**}/dI - C_I^s(q_s^*, I^M) - f'(I^M) = 0$.

Now, we compare investments in two regimes.

Before present our main result, we state the following lemma.

Lemma 2. Suppose that assumptions 1–5 are satisfied. Then, for all I, $q_s^*(I) > q_s^{**}(I)$.

The proof of this lemma is in the Appendix. According to this lemma, the output of the public firm in the public monopoly is higher than the output of the public firm in the mixed oligopoly.

The main result in this part is as follows.

Proposition 1. *Suppose that assumptions 1–5 are satisfied. Then, $I^P > I^M$.*

According to this proposition, the investment in the public monopoly is higher than that in the mixed oligopoly.

To prove this proposition, we derive the derivative of social welfare in the mixed oligopoly at I^P (the optimal investment in the public monopoly). From equation (6), we have the following:

$$\frac{dW^M}{dI}\Big|_{I=I^P} = -nq_p^{**}P'(Q^{**})\frac{dq_p^{**}}{dI} - C_I^s(q_s^{**}, I^P) - f'(I^P), \tag{7}$$

$$= -nq_p^{**}P'(Q^{**})\frac{dq_p^{**}}{dI} + C_I^s(q_s^*, I^P) - C_I^s(q_s^{**}, I^P) - C_I^s(q_s^*, I^P) - f'(I^P), \tag{8}$$

$$= -nq_p^{**}P'(Q^{**})\frac{dq_p^{**}}{dI} + \{C_I^s(q_s^*, I^P) - C_I^s(q_s^{**}, I^P)\}, \tag{9}$$

where we use equation (2).

Since $dq_p^{**}/dI < 0$ by lemma 1, the first term of right-hand side of equation (15) is negative. Further, since $C_{qI}^s < 0$ by assumption 2, $q_s^* > q_s^{**} \Rightarrow C_I^s(q_s^*, I) < C_I^s(q_s^{**}, I)$. Thus,

$$C_I^s(q_s^*, I^P) - C_I^s(q_s^{**}, I^P) < 0.$$

These facts imply that

$$\frac{dW^M}{dI}\Big|_{I=I^P} < 0.$$

This indicates that $I^P > I^M$.[4]

[4]We can show that

$$\frac{dW^M}{dI}\Big|_{I=\bar{I}} < 0$$

for all $\bar{I} \geq I^P$. This fact guarantees correctness of our result.

Comparison to Mixed Oligopoly with Free Entry Now, we consider the case of the mixed oligopoly with free entry. The game is summarized as follows. At the first stage, the state-owned public firm chooses the investment. At the second stage, after observing I private firms choose whether to enter the market. After observing the number of entering private firms n, each firm chooses output simultaneously. We are interested in the equilibrium in which $q_1 = q_2 =, \ldots, = q_n$. Let q_s^f denote the equilibrium output of the public firm and q_p^f denote the equilibrium output of each private firm. Furthermore, n^f denote the equilibrium number of private firms and Q^f denote the equilibrium total output. The optimality conditions in the third stage are same as (3) and (4). In the free entry equilibrium, the zero-profit condition holds:

$$P(Q^f)q_p^f - C(q_p^f) = 0. \tag{10}$$

The following assumption requires positive production of firms.

Assumption 6. $q_s^f > 0$ and $q_p^f > 0$.

Lemma 3. Suppose that assumptions 1–4 and 6 are satisfied. Then, $dq_p^f/dI = 0$.

The proof of this lemma is in the Appendix. This lemma says that the output of each private firm is independent of the investment by the public firm.

The optimality condition of the public firm in the first stage is as follows:

$$\begin{aligned} \frac{dW^F}{dI} &= = n\Big(P(Q^f) - C'(q_p^f)\Big)\frac{dq_p^f}{dI} - C_I^s(q_s^f, I) - f'(I) + \Big(P(Q^f)q_p - C(q_p^f)\Big)\frac{dn^f}{dI} && (11)\\ &= -C_I^s(q_s^f, I) - f'(I) = 0, && (12) \end{aligned}$$

where we use Lemma 3 and equation (10). Let I^F denote the optimal investment. Note that $-\partial C^s(q_s^f, I^F)/\partial I - f'(I^F) = 0$.

We state the result similar to Lemma 2.

Lemma 4. Suppose that assumptions 1–4 and 6 are satisfied. Then, for all I, $q_s^*(I) > q_s^f(I)$.

The proof of this lemma is same as that of Lemma 2, and so we omit.

The main result in this part is as follows.

Proposition 2. *Suppose that assumptions 1–4 and 6 are satisfied. Then, $I^P > I^F$.*

We can prove this proposition along similar steps to Proposition 1. From equation (12), we have the following:

$$\begin{aligned} \frac{dW^F}{dI}\Big|_{I=I^P} &= -C_I^s(q_s^f, I^P) - f'(I^P), && (13)\\ &= C_I^s(q_s^*, I^P) - C_I^s(q_s^f, I^P) - C_I^s(q_s^*, I^P) - f'(I^P), && (14)\\ &= C_I^s(q_s^*, I^P) - C_I^s(q_s^f, I^P), && (15) \end{aligned}$$

where we use equation (2). Since $C_{qI}^s < 0$ by assumption 2, $q_s^* > q_s^f \Rightarrow C_I^s(q_s^*, I) < C_I^s(q_s^f, I) \Rightarrow C_I^s(q_s^*, I^P) - C_I^s(q_s^f, I^P) < 0$. Therefore, we have

$$\frac{dW^F}{dI}\Big|_{I=I^P} < 0,$$

and so $I^P > I^F$.

Appendix: Proofs of Lemmata

Proof of Lemma 1: By using the implicit function theorem, we obtain the following equation:

$$D\begin{pmatrix} dq_p^*/dI \\ dq_s^*/dI \end{pmatrix} = \begin{pmatrix} 0 \\ C^s_{qI} \end{pmatrix}$$

where

$$D = \begin{pmatrix} nP''q_p + (n+1)P' - C_{qq} & P''q_p + P' \\ nP' & P' - C^s_{qq} \end{pmatrix}.$$

We can check $|D| > 0$.
(i)

$$\frac{dq_p^{**}}{dI} = -\frac{(P''q_p + P')C^s_{qI}}{|D|} < 0.$$

(ii)

$$\frac{dq_s^{**}}{dI} = \frac{\{nP''q_p + (n+1)P' - C_{qq}\}C^s_{qI}}{|D|} > 0.$$

■
Proof of Lemma 2: Take any I. In the public monopoly, the optimality condition of the public firm given I is the following:

$$P(q_s^*) = C^s_q(q_s^*, I).$$

In the mixed oligopoly, the optimality condition of the public firm given I is the following:

$$P(q_s^{**} + nq_p^{**}) = C^s_q(q_s^{**}, I).$$

Since $nq_p^{**} > 0$ by assumption 1, we have $q_s^*(I) > q_s^{**}(I)$. ■
Proof of Lemma 3: By using the implicit function theorem, we obtain the following equation:

$$G\begin{pmatrix} dq_p^f/dI \\ dq_s^f/dI \\ dn^f/dI \end{pmatrix} = \begin{pmatrix} 0 \\ C^s_{qI} \\ 0 \end{pmatrix}$$

where

$$G = \begin{pmatrix} nP''q_p + (n+1)P' - C'' & P''q_p + P' & P''q_p^2 + P'q_p \\ nP' & P' - C^s_{qq} & P'q_p \\ nP'q_p + P - C' & P'q_p & P'q_p^2 \end{pmatrix}.$$

We can check $|G| = q_pC^s_{qq}\{(P' + q_pP'')(P - C') - q_pP'(P' - C'')\} < 0$.

$$\begin{aligned} \frac{dq_p^f}{ds} &= -\frac{C^s_{qI}}{|G|}\Big((P''q_p + P')P'q_p^2 - P'q_p(P''q_p^2 + P'q_p)\Big) && (16) \\ &= 0 && (17) \end{aligned}$$

■

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