

## Tighter Price Cap Regulation on Consumer Surplus and Utility

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### *Abstract*

Kang, Weisman, and Zhang (2000) demonstrated that, under a tighter price cap, consumer welfare increases with the independence of demands. Conversely, the tighter price cap may negatively impact consumer welfare in situations involving interdependent demands. This paper identifies an overlooked property of Kang et al. (2000), namely, that the cross-price effects should be symmetrical without an income effect and demand functions of a Hicksian type. This paper reveals the invariant conclusion that consumers always benefit from a tighter price cap. Besides, it further derives the effects of the tighter price cap on consumer utility, and concludes that the net price elasticity of demand is the key to the effect of the average revenue constraint.

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## 1. Introduction

Kang, Weisman, and Zhang (2000) adopted a unique price regulation to demonstrate that consumer welfare, in terms of consumer surplus, cannot be increased by tighter price caps given interdependent demands. Conversely, it was also argued that, under a tighter price cap, consumer welfare increases with the independence of demands.<sup>1</sup> However, their model contains a hidden property. When this hidden property is revealed, their model invariably concludes that consumers always benefit from such a tighter price cap policy. Furthermore, unlike Kang *et al.* (2000) this paper considers how a tighter price cap affects consumer utility given different revenue constraint settings.

## 2. Theoretical Framework

Kang *et al.* (2000) considered a price cap on a monopoly of two related products,  $Q_1$  and  $Q_2$ . The consumer demand for each product depends on the price of that product ( $P_i$ ) and the price of the other product ( $P_j$ ) in a linear function:

$$Q_i = \alpha_i - a_i P_i + b_i P_j, \text{ for } i, j = 1, 2 \text{ and } i \neq j. \quad (1)$$

The coefficients  $\alpha_i$  and  $a_i$  must be positive, but  $b_i$  can have any value provided that  $a_i > |b_i| > 0$ , meaning that the own-price effect must exceed the cross-price effect in terms of absolute value. The price cap  $\bar{P}$  is defined as the weighted average of the two prices:

$$\bar{P} = \sum_{i=1}^2 w_i P_i, \quad (2)$$

where  $w_i$  is the weight governed by  $w_i \in (0, 1)$  and  $\sum_{i=1}^2 w_i = 1$ .

Based on this set up, through profit maximization subject to the price cap of Eq. (2) for the optimal prices, the consumer surplus (CS) may be calculated from the demand functions for Eq. (1) using the optimal prices as follows:

$$CS = \int_{P_1}^{\hat{P}_1} Q_1(z_1, P_2) dz_1 + \int_{P_2}^{\hat{P}_2} Q_2(P_1, z_2) dz_2, \quad (3)$$

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<sup>1</sup> Law (1995) demonstrated that, given independent demand, a tightening average revenue constraint can reduce the consumer surplus in situations where marginal costs differ between products.

where  $\hat{P}_1$  and  $\hat{P}_2$  are reservation prices. Kang *et al.* (2000) identified the effects of a change in the price cap on consumer welfare:<sup>2</sup>

$$\frac{dCS}{d\bar{P}} = \sum_{i \neq j}^{i=1} Q_i \frac{b_i[2a_i w_j + (b_i + b_j)w_i] - a_i[2a_j w_i + (b_i + b_j)w_j]}{2a_i[(b_i + b_j)w_i w_j + a_j w_i^2 + a_i w_j^2]}. \quad (4)$$

Since the denominator of Eq. (4) is always positive based on the second-order condition of profit maximization, the welfare effect depends entirely on the sign of the numerator. Subsequently, three scenarios are sorted from this equation depending on whether  $b_i = 0$ ,  $b_i \neq 0$  or  $b_i < 0$  (Kang *et al.*, 2000, Proposition 2). Among these scenarios, only the first one delivers an unambiguously negative derivative. Meanwhile, the other two scenarios are inconclusive, and Kang *et al.* (2000) concludes that consumers may not benefit from a tighter price cap regulation.

### 3. The Revision and Proof of the Kang *et al.* (2000) Model

The areas in which the method used in the handling of the model of Kang *et al.* (2000) are lacking are presented as follows: “*The model should have symmetric cross-price effects without income effects*”. When exhibiting voluntary behavior, consumers make a utility maximization decision. “*If a consumer does not exhibit voluntary and rational behavior, the consumer surplus has no meaning.*” In other words, based on the meaning of the consumer surplus, a consumer must be able to make voluntary choices (i.e., consumer utility maximization). In particular, when each consumer maximizes utility in a complete market, and each consumer faces the same prices, this model can be described using a representative consumer.<sup>3</sup> Let  $H_i$  denote the Hicksian demand for goods  $i$ . The expenditure function is  $E(P)$ , where  $P = (P_1, P_2)$ . The revision is explained as follows:

$$\frac{\partial Q_1}{\partial P_2} = \frac{\partial H_1}{\partial P_2} = \frac{\partial^2 E(P)}{\partial P_2 \partial P_1} = \frac{\partial^2 E(P)}{\partial P_1 \partial P_2} = \frac{\partial H_2}{\partial P_1} = \frac{\partial Q_2}{\partial P_1}. \quad (5)$$

The first and fifth equalities follow from the lack of income effects. Meanwhile, the second and fourth equalities follow from the property of the expenditure function. Moreover, the third equality follows from  $\partial E(P)/\partial P_i$  and  $\partial^2 E(P)/\partial P_j \partial P_i$  being continuous at points  $P_i$  and  $P_j$  in the linear demand

<sup>2</sup> The variable  $Q_i$  is lacking in the model of Kang *et al.* (2000), and can thus be omitted; however, this omission does not affect either their or our conclusion.

<sup>3</sup> Details of the proof of the inference can be found in Appendix A.

functions.

To sum up, as the model abstracts itself from the income effects (Kang *et al.*, 2000), then the postulated demand functions are of the Hicksian type (Varian, 1992). Accordingly, the cross-price effects must be equal, namely:

$$b_1 = b_2 = b. \quad (6)$$

Subsequently, Eq. (4) can be reduced to:

$$\begin{aligned} \frac{dCS}{d\bar{P}} &= \sum_{\substack{i=1 \\ i \neq j}}^2 Q_i \frac{b[2a_i w_j + 2b w_i] - a_i[2a_j w_i + 2b w_j]}{2a_i[2b w_i w_j + a_j w_i^2 + a_i w_j^2]} \\ &= \sum_{\substack{i=1 \\ i \neq j}}^2 Q_i \frac{2a_i b w_j + 2b^2 w_i - 2a_i a_j w_i - 2a_i b w_j}{2a_i[2b w_i w_j + a_j w_i^2 + a_i w_j^2]} \\ &= \sum_{\substack{i=1 \\ i \neq j}}^2 Q_i \frac{2b^2 w_i - 2a_i a_j w_i}{2a_i[2b w_i w_j + a_j w_i^2 + a_i w_j^2]} < 0. \end{aligned} \quad (7)$$

Based on the assumption that provides a basis for Eq. (1)  $a_i > |b| > 0$ , this derivative of Eq. (7) is always negative, and a tighter price cap always increases the consumer surplus. This conclusion holds even when both products are complementary, that is, if  $b < 0$  (*cf.* Kang *et al.*, 2000, Proposition 2 [iii]). Consequently, the change in consumer welfare owing to a tighter price cap depends less on whether the demands are independent or interdependent than on the nature of the interdependence. Since the model ignores the income effect, the interdependence is symmetric. Thus, the model has the invariant conclusion as expressed in Eq. (7).

#### 4. The Effects of a Tighter Price Cap on Consumer Utility

Evaluating the consumer's utility is more straightforward than evaluating the consumer surplus in multi-product circumstances. The next question to be asked concerns the effect of a tighter price cap on utility. Let  $V(P, M)$  denote the indirect utility function, where  $P$  is an  $n \times 1$  vector of price and  $M$  is the fixed amount of money available to a representative consumer.

First of all, in a lagged revenue (non-contemporaneous) weight constraint setting, this paper pays attention to cases where the price cap is binding, i.e.,

$\bar{P} = \sum_{i=1}^n w_i P_i$ , for  $w_i \in (0,1)$  and  $\sum_{i=1}^n w_i = 1$ , where  $\bar{P}$  denotes the price cap. The sign of the derivative of the utility with respect to the price cap is given

by:

$$\begin{aligned}\frac{dV(P, M)}{d\bar{P}} &= \sum_{i=1}^n \left[ \frac{\partial V(P, M)}{\partial P_i} \right] \left[ \frac{\partial P_i}{\partial \bar{P}} \right] = \sum_{i=1}^n \left[ -Q_i \frac{\partial V(P, M)}{\partial M} \right] \left[ \frac{1}{\partial \bar{P} / \partial P_i} \right] \\ &= -\frac{\partial V(P, M)}{\partial M} \sum_{i=1}^n \frac{Q_i}{w_i} \leq 0.\end{aligned}\quad (8)$$

This analysis demonstrates that the indirect utility is non-decreasing with a tighter price cap, regardless of the cross-price effects.

Next, in an average revenue constraint (contemporaneous weight) setting, this paper considers cases in which the price cap is binding, i.e.,  $\bar{P} = \sum_{i=1}^n (Q_i / \sum_{j=1}^n Q_j) P_i$ . In the case involving independent demand, the effect of the derivative of the utility with respect to the price cap is illustrated as follows:

$$\begin{aligned}\frac{dV(P, M)}{d\bar{P}} &= \sum_{i=1}^n \left[ \frac{\partial V(P, M)}{\partial P_i} \right] \left[ \frac{\partial P_i}{\partial \bar{P}} \right] = \sum_{i=1}^n \left[ -Q_i \frac{\partial V(P, M)}{\partial M} \right] \left[ \frac{1}{\partial \bar{P} / \partial P_i} \right] \\ &= -\frac{\partial V(P, M)}{\partial M} \sum_{i=1}^n Q_i \left[ \frac{Q_i}{\sum_{i=1}^n Q_i} + \sum_{j=1}^n \frac{P_j (\sum_{i=1}^n Q_i) \frac{\partial Q_j}{\partial P_i} - P_j Q_j \frac{\partial \sum_{i=1}^n Q_i}{\partial P_i}}{(\sum_{i=1}^n Q_i)^2} \right] \\ &= -\frac{\partial V(P, M)}{\partial M} \sum_{i=1}^n Q_i \left[ \frac{Q_i}{\sum_{i=1}^n Q_i} + \sum_{j=1}^n \frac{(P_j - \bar{P}) \frac{\partial Q_j}{\partial P_i}}{\sum_{i=1}^n Q_i} \right].\end{aligned}\quad (9)$$

This analysis demonstrates that the sign of the derivative of the utility with respect to this price cap is ambiguous. We further consider that the cross-price effects of the demand function are zero; in other words, the consumers' demands are all independent. The effect of the derivative of the utility with respect to the price cap is rewritten as follows:

$$\frac{dV(P, M)}{d\bar{P}} = -\frac{\partial V(P, M)}{\partial M} \sum_{i=1}^n Q_i \left[ \frac{Q_i + (P_i - \bar{P}) \frac{\partial Q_i}{\partial P_i}}{\sum_{i=1}^n Q_i} \right]$$

$$\begin{aligned}
&= -\frac{\partial V(P, M)}{\partial M} \sum_{i=1}^n \left\{ \left[ \sum_{i=1}^n Q_i \right] \left[ 1 + \frac{P_i - \bar{P}}{Q_i} \frac{\partial Q_i}{\partial (P_i - \bar{P})} \right] \right\} \\
&= -\frac{\partial V(P, M)}{\partial M} \left[ \sum_{i=1}^n Q_i \right] \left[ \sum_{i=1}^n \frac{1}{1 - \varepsilon_i} \right], \tag{10}
\end{aligned}$$

where  $\varepsilon_i = -\frac{P_i - \bar{P}}{Q_i} \frac{\partial Q_i}{\partial (P_i - \bar{P})}$  denotes the net price (the individual price minus the price cap) elasticity of demand. The indirect marginal utility of income is non-decreasing in  $M$ . Three possibilities exist: (1) If all values of 1 minus the net price elasticity of demand share the same sign and  $1 - \varepsilon_i > 0 \quad \forall i$ , then the consumer utility is non-increasing in  $\bar{P}$ . (2) If all values of 1 minus the net price elasticity of demand share the same sign and  $1 - \varepsilon_i < 0 \quad \forall i$ , then the consumer utility is non-decreasing in  $\bar{P}$ . (3) If not all values of 1 minus the net price elasticity of demand share the same sign, then the sign of the derivative of the utility with respect to the price cap is ambiguous. This implies that a tighter price cap may reduce indirect utility, and that the net price elasticity of demand is the key to the effect of the average revenue constraint.

## 5. Conclusion

Kang *et al.* (2000) showed that, with a tighter price cap, consumer welfare increases if demands are independent. Conversely, a tighter price cap may reduce consumer welfare in situations involving interdependent demands. However, the cross-price effects should be symmetrical without income effects and Hicksian type demand functions. This paper draws the invariant conclusion that consumers always benefit from a tighter price cap. In addition, this paper further derives the effects of the tighter price cap on consumer utility. First, in a lagged revenue (non-contemporaneous) weight constraint setting, this paper demonstrates that indirect utility is non-decreasing with a tighter price cap regardless of the cross-price effects. Finally, in an average revenue constraint (contemporaneous weight) setting, the paper finds that a tighter price cap may reduce indirect utility, implying that the net price elasticity of demand is the key to the effect of the average revenue constraint.

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## Appendix A

When each consumer maximizes utility and is subject to budget constraints:

$$\begin{aligned} \max_{X_1^i, X_2^i} U^i(X_1^i, X_2^i) \\ \text{s.t. } P_1 X_1^i + P_2 X_2^i = m^i, \text{ for } i=1, 2, \dots, N. \end{aligned} \quad (11)$$

Here  $U^i(X_1^i, X_2^i)$  expresses the utility function of consumer  $i$ ,  $X_1^i$  represents the demand for the first goods of consumer  $i$ ,  $X_2^i$  is the demand for the second goods of consumer  $i$ ,  $m^i$  is the income of consumer  $i$ , and  $N$  represents the number of consumers. Then, a representative consumer exists with the maximization of utility and a budget constraint:

$$\begin{aligned} \max_{Q_1, Q_2} U(Q_1, Q_2) = \sum_{i=1}^N U^i(X_1^i, X_2^i) = U^1 + U^2 + \dots + U^N \\ \text{s.t. } P_1 Q_1 + P_2 Q_2 = \sum_{i=1}^N m^i, \text{ for } i=1, 2, \dots, N. \end{aligned} \quad (12)$$

Here,  $Q_1 = \sum_{i=1}^N X_1^i$  and  $Q_2 = \sum_{i=1}^N X_2^i$ . As market demand is the summation of each consumer's demand, this statement can be confirmed by proving that the optimal solution for a representative consumer is equal to the sum of the optimal solution for each consumer.

### Proof:

Denote the Lagrangian function of Eq. (11) by:

$$L^i = U^i(X_1^i, X_2^i) + \lambda^i (m^i - P_1 X_1^i - P_2 X_2^i), \text{ for } i = 1, 2, \dots, N. \quad (13)$$

By partially differentiating  $L^i$  with respect to  $X_1^i, X_2^i$ , the first-order conditions are as follows:

$$\frac{\partial L^1}{\partial X_1^1} = \frac{\partial U^1}{\partial X_1^1} - \lambda^1 P_1 = 0, \quad \frac{\partial L^2}{\partial X_1^2} = \frac{\partial U^2}{\partial X_1^2} - \lambda^2 P_1 = 0, \dots, \quad \frac{\partial L^N}{\partial X_1^N} = \frac{\partial U^N}{\partial X_1^N} - \lambda^N P_1 = 0. \quad (14)$$

$$\frac{\partial L^1}{\partial X_2^1} = \frac{\partial U^1}{\partial X_2^1} - \lambda^1 P_2 = 0, \quad \frac{\partial L^2}{\partial X_2^2} = \frac{\partial U^2}{\partial X_2^2} - \lambda^2 P_2 = 0, \dots, \quad \frac{\partial L^N}{\partial X_2^N} = \frac{\partial U^N}{\partial X_2^N} - \lambda^N P_2 = 0. \quad (15)$$

Then, the summations for each consumer from Eq. (14) and Eq. (15) are:

$$\sum_{i=1}^N \frac{\partial U^i}{\partial X_1^i} - P_1 \sum_{i=1}^N \lambda^i = 0; \quad (16)$$

$$\sum_{i=1}^N \frac{\partial U^i}{\partial X_2^i} - P_2 \sum_{i=1}^N \lambda^i = 0. \quad (17)$$

Denoting the Lagrangian function of Eq. (12) by:

$$L = \sum_{i=1}^N U^i + \lambda (\sum_{i=1}^N m^i - P_1 \sum_{i=1}^N X_1^i - P_2 \sum_{i=1}^N X_2^i).$$

and partially differentiating L with respect to  $Q_1$ ,  $Q_2$ , the first-order conditions are as follows:

$$\begin{aligned} \frac{\partial L}{\partial Q_1} &= \sum_{i=1}^N \frac{\partial U^i}{\partial Q_1} - \lambda P_1 \\ &= \sum_{i=1}^N \left( \frac{\partial U^i}{\partial X_1^i} \frac{\partial X_1^i}{\partial Q_1} \right) - \lambda P_1 \\ &= \sum_{i=1}^N \frac{\partial U^i}{\partial X_1^i} - \lambda P_1 \\ &= 0; \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\partial L}{\partial Q_2} &= \sum_{i=1}^N \frac{\partial U^i}{\partial Q_2} - \lambda P_2 \\ &= \sum_{i=1}^N \left( \frac{\partial U^i}{\partial X_2^i} \frac{\partial X_2^i}{\partial Q_2} \right) - \lambda P_2 \\ &= \sum_{i=1}^N \frac{\partial U^i}{\partial X_2^i} - \lambda P_2 \\ &= 0, \end{aligned} \quad (19)$$

where  $\lambda = \sum_{i=1}^N \lambda^i$ . Eq. (16) is equal to Eq. (18), and Eq. (17) is equal to Eq.

(19). Hence the optimal solution for a representative consumer equals the sum of the optimal solution for each consumer.<sup>4</sup>

Q.E.D.

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<sup>4</sup> Here, the first-order conditions are utilized to confirm this inference. The second-order conditions are also derived in this paper and the analytical results are consistent with the inference. However, due to the space limitations, the derivation of the second-order conditions is not presented in this paper.