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Fossil fuels supplied by oligopolies: On optimal taxation and rent capture

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Abstract

This article investigates the optimal taxation of a polluting exhaustible resource supplied by an oligopoly in a partial equilibrium model. A single tax/subsidy scheme is sufficient to correct both distortions arising from market power and pollution externality. Moreover, there exists an infinite family of such optimal taxation instruments. Then, I study how this set is affected by the degree of concentration of the resource suppliers. In particular, the more concentrated the extraction sector, the less falling (or the more rising) over time the optimal tax rate. Finally, although concentration tends to increase the total rent of the extraction sector, it reduces the potential tax revenues to be earned by the regulator while inducing efficiency.

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1. Introduction

Fossil fuels are exhaustible resources. Their burning in the production process generates flows of pollutants accumulating in a stock of atmospheric pollution. Moreover, the extractors of these resources are concentrated so that they are often considered to enjoy some market power. These two latter features generally imply that the pace of depletion of fossil fuels won't be optimal as soon as the decisions of suppliers or consumers are decentralized. The two sources of distortions that are resulting from a pollution externality and market power are two arguments for the regulation of markets for these resources. The economics literature has studied in particular how this can be implemented by the taxation of fossil fuels.

Nevertheless, the contributions to this issue have mostly focused on the causes of market failure separately. As a consequence, the polluting nature and the concentration in the supply side of a non-renewable resource have not been considered together. The purpose of this paper is to study the optimal taxation of a fossil fuel with its main features, all together: exhaustibility, polluting use and market power in supply side. Our contribution is thus at the crossing point of different strands of the literature on fossil fuels taxation.

First, Bergstrom (1982) proposes an analysis of the effects of a tax on a non-renewable resource. He shows that it typically transfers a part of the mining rent from the producers to the fiscal authorities.

A second strand of the literature focuses on the optimal taxation of a polluting exhaustible resource. Sinclair (1992 and 1994) and Ulph and Ulph (1994) model pollution as a stock filled by flows of carbon rejected in the atmosphere by the use of the resource. They show how the optimal tax rate should evolve over time. Daubanes and Grimaud (2007) find moreover that there is a family of such optimal tax schemes, each of them capturing a greater or lower part of the mining rent.

Another strand studies the optimal regulation through a taxation policy of a depletable resource when there is some market power in the extraction sector. The related articles concentrate on the polar case of a monopolist extractor. Bergstrom, Cross and Porter (1981), in the case of a regulator able to precommit, and Karp and Livernois (1992), without the precommitment assumption, show the existence of a family of efficiency-inducing tax/subsidy schemes. More recently, Daubanes (2007) shows that it may be possible for the regulator to raise tax revenues while taxing optimally.

Finally, the optimal taxation of a polluting good supplied by an imperfectly competitive sector has been studied in models where the resource is not explicitly non-renewable. In this context, Benchekroun and Long (1998) show that a single taxation instrument is sufficient to correct both distortions from pollution externality and market power. In the case of a monopoly, Benchekroun and Long (2002) insist on the multiplicity of optimal tax/subsidy paths. However, none of these papers considers the exhaustibility of the resource so that the firms are standard producers and pollution comes from by-product emissions.

Modeling the market for a polluting exhaustible resource supplied by an imperfectly competitive sector offers a framework to deal with the interactions between pollution and market power and their incidences on the properties of the optimal taxation policies. In particular, it allows to show that a single taxation instrument is sufficient to regulate both sources of distortions and that there is a family of such taxes bounded by the need to preserve the participation of the extractors. One can also study how the degree of concentration of the producers affects the evolution over time of the optimal taxation schemes. Finally, one can focus on the tax/subsidy profile under which the regulator collects the largest tax revenues and study how the largest tax revenues are affected by concentration.

To tackle these issues, I use a partial equilibrium model of a polluting resource depletion when it is supplied by an oligopoly. I assume standard functional forms: isoelasticity of the demand function, Cournot competition in resource supply, constancy of unit extraction cost, constancy of the pollution rate of decay and quadratic damage function. In order to get analytical and exploitable results, I assume the precommitment ability of the regulator.

In this framework, my main results are the following: an infinity of environmental time-dependant tax/subsidy profiles, adjusted by the market structure, implements the optimal allocation of the resource. The more concentrated the extraction sector, the more rising (or less falling) the optimal tax rate. A particular optimal taxation policy allows the regulator to collect a greater or lower part of the mining rent. Concentration increases this rent but decreases the potential tax revenues of the regulator.

The paper is organized as follows. Section 2 introduces the model and the optimal extraction path. In section 3, I solve the optimal taxation problem under market power and pollution externality and examine the set of optimal tax/subsidy schemes. In particular, I address the question of how concentration modifies the tax profiles, how it affects the total mining rent and the tax revenues that can be captured through an optimal policy. Section 4 concludes.

2. A partial equilibrium model

2.1 Basics

At each time $t \geq 0$, the flow of extraction in units of resource is $R(t) \geq 0$. Let $S(t)$ be the size of the reserves remaining at date t. Then,

$$S(t) = S(0) - \int_0^t R(s)\, ds,\ S(t) \geq 0,\ S(0) = S_0 \text{ given.} \tag{1}$$

The use of the extracted flow of resource generates a flow of emissions filling a stock of atmospheric pollution, $Z(t)$. The dynamics of this stock obeys[1]:

$$\dot{Z}(t) = R(t) - \delta Z(t),\ Z(0) = Z_0 \geq 0 \text{ given,} \tag{2}$$

where $\delta \geq 0$ is the rate of decay of the stock of pollution.

[1] The derivative with respect to time of any variable X is denoted by $\dot{X}$. Its rate of growth is denoted by $g_X = \dot{X}/X$.

The unit cost of extraction is constant and denoted by $c \geq 0$. Hence the total cost function is $C(R) = cR$.

There is a unitary mass of identical households whose inverse demand function for the resource is stationary[2] and iso-elastic: $P(R) = R^{-1/\alpha}$, $\alpha > 0$. Let me take their instantaneous utility from resource consumption to be the area under this demand curve: $U(R) = \frac{\alpha}{\alpha-1} R^{\frac{\alpha-1}{\alpha}}$. Moreover, the harm caused by the stock of pollution is represented by the quadratic damage function $H(Z) = \frac{\gamma}{2} Z^2$.

Hence, the net instantaneous aggregate surplus is $W = U(R) - H(Z) - C(R)$ and the social welfare is the discounted stream of these surpluses:

$$V = \int_0^{+\infty} \big(\frac{\alpha}{\alpha - 1} R(t)^{\frac{\alpha-1}{\alpha}} - \frac{\gamma}{2} Z(t)^2 - cR(t)\big) e^{-rt}\, dt, \tag{3}$$

where $r \geq 0$ is the rate of discount.

2.2 Social planner's ideal

If a utilitarian "social planner" were to control the extraction sector, her objective would be to choose the pace of extraction[3] $\{R^*(t)\}_{t\geq 0}$ which maximizes (3) under the law of motion of the reserves resulting from (1), $\dot{S}(t) = -R(t)$, the dynamics of pollution (2) and the initial conditions $S(0) = S_0$ and $Z(0) = Z_0$. This is a standard optimal control problem with an infinite horizon. Let $\lambda^* \geq 0$ and $\mu^* \leq 0$ denote the present-value costate variables associated respectively to the reserves and the atmospheric pollution.

Since the resource is necessary in the sense that $\lim_{R\mapsto 0} W = +\infty$, the extraction flow is always positive in optimum: $R^*(t) > 0$, for all $t \geq 0$. Hence, the first-order conditions of the social planner's program are:

$$\begin{aligned} \big(R^*(t)^{-1/\alpha} - c\big) &= \big(\lambda^*(t) - \mu^*(t)\big) e^{rt}, && (4)\\ \dot{\lambda}^*(t) &= 0, && (5)\\ \dot{\mu}^*(t) &= \gamma Z(t) e^{-rt} + \delta \mu^*(t), && (6) \end{aligned}$$

and the transversality conditions are:

$$\lim_{t\to+\infty} \lambda^*(t) S(t) = 0, \tag{7}$$

$$\lim_{t\to+\infty} \mu^*(t) Z(t) = 0. \tag{8}$$

Since $R^*(t) > 0$ for all t, the constant (from condition (5)) costate variable associated to the resource needs to be strictly positive: $\lambda^* > 0$. Hence, from (7), the resource is asymptotically exhausted:

$$\int_0^{+\infty} R^*(t)\, dt = S_0. \tag{9}$$

[2]The results can be easily extended to the case of a regular technical improvement in the use of the resource.

[3]Let me use superscript * to mean optimum.

Integrating equations (2) and (6), one gets:

$$\mu^*(t) = e^{\delta t}\Big[\mu^*(0) + \int_0^t \gamma Z(s) e^{-(r+\delta)s}\, ds\Big], \tag{10}$$

$$Z(t) = e^{-\delta t}\Big[Z_0 + \int_0^t R(s) e^{\delta s}\, ds\Big]. \tag{11}$$

Substituting these expressions into (8) gives the initial value of the shadow cost of pollution $\mu^*(0) = -\int_0^{+\infty} \gamma Z(t) e^{-(r+\delta)t}\, dt$. Then:

$$\mu^*(t) = -e^{\delta t} \int_t^{+\infty} \gamma Z(s) e^{-(r+\delta)s}\, ds < 0. \tag{12}$$

Differentiating (4) gives the optimal Hotelling rule. After rearranging, this results in the differential equation:

$$g_R^*(t) = -\alpha r + \alpha r c R^*(t)^{1/\alpha} + \alpha \dot{\mu}^*(t) R^*(t)^{1/\alpha} e^{rt}, \tag{13}$$

where $\dot{\mu}^*(t)$ is found from (12). Equation (9) leads to the implicit boundary condition $R^*(0) = R_0^*(S_0)$. This condition and differential equation (13) determines uniquely the optimal extraction path.

3. Correcting pollution externality and market power

Let me consider a market for the resource. Let $p(t)$ denote the associated consumer price. Pollution is a public bad. There is no uncertainty and all agents perfectly foresee the future.

There are $n \in \mathbb{N}^*$ identical extractors indexed by $i = 1, ..., n$. They are respectively endowed with a stock $S_{i0} = S_0/n$ and they respectively extract R_i, $i = 1, ..., n$. The aggregate extraction flow is $R = \sum_{1 \leq i \leq n} R_i$. These extractors play a Cournot game. Hence, the oligopoly captures the polar cases of monopoly ($n = 1$) and perfect competition ($n \mapsto \infty$).

I define the tax/subsidy scheme the regulator sets to correct the distortions that may arise due to market power and pollution externality as follows. Let $\{\theta(t)\}_{t\geq 0}$ be an *ad valorem* producer tax so that the producer price is $p(t)\tau(t) = p(t)\big(1 - \theta(t)\big)$. Assume that $\theta(t) < 1$ so that $\tau(t) > 0$ and let me restrict attention to tax profiles differentiable with respect to time[4,5]. Suppose that the regulator is able to announce credibly $\{\theta(t)\}_{t\geq 0}$ from date 0 on.

Each oligopolist maximizes the discounted stream of her spot profits subject to her exhaustibility constraint, $\dot{S}_i(t) = -R_i(t)$. Strategically, she internalizes the effect of

[4] This assumption is made for simplicity. One can show that all the optimal tax profiles are indeed differentiable with respect to time.

[5] For the sake of notational simplicity, I use the multiplicative tax denoted by τ instead of the *ad valorem* tax denoted by θ. In the following, I shall interpret the results in terms of the *ad valorem* tax. Hence, remind that τ and θ and their respective derivatives evolve in opposite directions.

its decision on the market price. The Hamiltonian of extractor i is $H_i(S_i, R_i, \lambda_i, t) = \big(\tau(t)(R_i + \sum_{j\neq i} R_j)^{-1/\alpha} R_i - cR_i\big)e^{-rt} - \lambda_i R_i$, where $\lambda_i \geq 0$ is her present-value costate variable.

The previous problem has a solution only if the gross revenue is concave. That is why I assume $\alpha > 1$. The same way as in section 2, due to the necessity of the resource, the aggregate extraction flow will always be strictly positive, whatever is the pace of taxes/subsidies: $R(t) > 0$, $\forall t \geq 0$. Hence, in symmetric equilibrium, $R_i(t) = R(t)/n > 0$, $\forall t \geq 0$, thus requiring $\lambda_i(t) > 0$, $\forall t \geq 0$, what implies asymptotic exhaustion of all stocks and then:

$$\int_0^{+\infty} R(t)\, dt = S_0. \tag{14}$$

The equilibrium extraction path under oligopoly then satisfies:

$$\big(\tau(t)\frac{\alpha n - 1}{\alpha n} R(t)^{-1/\alpha} - c\big)e^{-rt} = \lambda, \tag{15}$$

where λ is a strictly positive constant.

In this equation, λ is the discounted marginal profit of the oligopoly, independent of time at each oligopolist's optimum. For a given tax scheme $\{\tau(t)\}_{t\geq 0}$, it is decreasing in the degree of concentration. It is moreover determined by the level of the tax.

The differentiation of the latter condition leads to a modified equilibrium Hotelling rule which is a differential equation. Given a certain tax profile, $\{\tau(t)\}_{t\geq 0}$, the solution to this differential equation under the boundary condition (14) gives uniquely the extraction path under oligopoly. The resulting dynamics of extraction obeys:

$$g_R(t) = -\alpha r + \alpha g_\tau(t) + \alpha r c \frac{\alpha n}{\alpha n - 1} R(t)^{1/\alpha} \tau(t)^{-1}. \tag{16}$$

This equation tells how the time-profile of the tax affects the dynamics of extraction.

The next subsection will show that a family of tax/subsidy schemes $\{\tau^*(t)\}_{t\geq 0}$ correct all distortions, thus implementing the optimal allocation of the resource. Each of these optimal taxation policies determines a unique λ, and their set is thus bounded by the condition that λ must be positive.

3.1 Optimal tax policies

The objective is to design a tax/subsidy scheme $\{\tau^*(t)\}_{t\geq 0}$ which induces the oligopolistic extraction sector to reproduce the optimal extraction path of section 2. I am thus looking for all tax profiles such that the solution to (15), for any positive λ and under (14), is $\{R^*(t)\}_{t\geq 0}$, *i.e.* all positive functions $\tau^*(t)$ that satisfy:

$$\big(\tau^*(t)\frac{\alpha n - 1}{\alpha n} R^*(t)^{-1/\alpha} - c\big)e^{-rt} = \lambda, \tag{17}$$

where λ is any strictly positive constant[6].

[6]The strict positivity of λ is a sort of participation constraint. Indeed, this variable is the marginal net profit of the oligopoly. If the regulator wants the latter to choose the optimal extraction path, he has to ensure that, under his tax policy, this marginal profit is positive.

These functions are:

$$\tau^*(t) = R^*(t)^{1/\alpha}\big(\tau^*(0)R_0^*(S_0)^{-1/\alpha}e^{rt} + c\frac{\alpha n}{\alpha n - 1}(1 - e^{rt})\big),\ t \geq 0, \tag{18}$$

where:

$$\tau^*(0) > \underline{\tau} \equiv c\frac{\alpha n}{\alpha n - 1}R_0^*(S_0)^{1/\alpha}. \tag{19}$$

Let Θ^* denote the set of these tax functions.

Proposition 1 *There exists an infinite family of efficiency-inducing tax/subsidy paths:* $\Theta^* = \big\{\{\tau^*(t)\}_{t\geq 0} :\ (18)\ \textit{and}\ \tau^*(0) > \underline{\tau}\big\}$.

Proof of proposition 1 *See the appendix.*

Correcting several distortions generally requires the use of as many tax instruments. Proposition 1 illustrates that regulating market power in the extraction sector and correcting the environmental distortion can be done by a single instrument: *a market structure-adjusted environmental tax*. The reasons why it is so are that (1) both imperfections affect the same extraction path and only this path, and that (2) a time-dependent tax can induce almost any extraction path.

Moreover, proposition 1 tells that there exists an infinity of such instruments. This is due in particular to the exhaustibility of the resource. This constraint that the extractors face implies that the extraction problem is an exhaustion problem: the choice of each extractor is not how much to supply but when to supply. Hence, the relevant instrument to induce a certain behavior is the difference between the tax rates at different dates rather than the level of the tax rates[7].

From now, one can study how the environmental tax should adjust to the market structure, that is what the effects of n on the optimal tax profiles are. Log-differentiating equation (17) and rearranging, one finds that the motion of any optimal tax obeys:

$$g_\tau^*(t) = \frac{1}{\alpha}g_R^*(t) + r - rc\frac{\alpha n}{\alpha n - 1}R^*(t)^{1/\alpha}\tau^*(t)^{-1}, \tag{20}$$

where this growth rate appears to be increasing in the number of extractors, n. It follows that an optimal *ad valorem* tax under a more concentrated industry is less falling over time (or more rising) than the initially identical optimal tax under a less concentrated structure.

The reason for that is the following. First, a monopoly ($n = 1$) tends to be more conservative than a competitive sector ($n \mapsto +\infty$) (Stiglitz, 1976). Consistently, an oligopoly has an intermediate behavior and the more concentrated the sector, the more conservative it is. For instance, the growth rate g_R of equation (16) can be shown to be decreasing in n. Second, the more falling an *ad valorem* taxation profile, the stronger the incentives it provides the society with to postpone the extraction of the resource.

[7] For another illustration, see Daubanes (2007).

Internalizing the effect of market power on the rate of depletion thus implies that the optimal *ad valorem* tax should be less falling (or more rising)[8].

3.2 On the rent of the oligopolistic extractor

The question addressed in this subsection is how the concentration of the extractors affects the total profit of the sector. Hence, let me define all variables as functions of the number of extractors, $n \in \mathbb{N}^*$.

In what follows, let the taxation policy be any given tax/subsidy scheme $\{\tau(t)\}_{t\geq 0}$.

An oligopolistic extractor globally earns two sorts of rents: a standard scarcity rent and a market rent. Both are closely related since the former depends on the way the mine is exploited and thus on the way market power is exercised. Hence, they are not analytically identifiable. However, one can see how the profit of an oligopoly (n finite) differs from the profit of a competitive sector ($n \mapsto +\infty$). Indeed, using (15), the total profit of the oligopoly, $\pi = \int_0^{+\infty} R(t)\big(\tau(t)P(R(t)) - c\big)e^{-rt}\,dt$, can be written:

$$\pi(n) = \lambda(+\infty)S_0 + \big(\lambda(n) - \lambda(+\infty)\big)S_0 + \frac{1}{\alpha n}\int_0^{+\infty} R(n,t)^{(\alpha-1)/\alpha}e^{-rt}\,dt. \qquad (21)$$

In this expression, $\lambda(+\infty)S_0$ is the value of the mine when exploited by a competitive sector[9]. The rent of the oligopoly differs from this value in two ways: first, the mine is not exploited efficiently and, second, market power is exercised. The analysis of expression (21) then reveals two opposite effects on $\pi(n)$: $\lambda(n)$ is decreasing and the term on the far right is increasing in the degree of concentration.

Proposition 2 *Under any taxation policy, the greater the number of oligopolists, the lower the total profit of the whole extraction sector,* i.e. $\forall\{\tau(t)\}_{t\geq 0} : \tau(t) > 0, \forall t \geq 0,$ $\pi(n)$ *is decreasing in* n.

Proof of proposition 2 *See the appendix.*

This also yields that the total profit of the oligopoly increases as the number of extractors decreases. It may have some implications about the cost of regulating the oligopoly to correct market power and pollution externality. Since the taxation of a non-renewable resource typically transfers rents between the resource holders and the regulator, it may be that concentration of the formers, and thus a larger total profit, is good news for the latter. Indeed, it might be easier to collect tax revenues from a rich sector.

3.3 Cost of regulation

The regulator may not only care about efficiency. In particular, he may not be indifferent to the distribution of the social surplus resulting from his taxation policy. When choosing one optimal tax/subsidy path in the family Θ^*, the regulator may prefer not to subsidize

[8] Sinclair (1994) proposes informally a similar argument.

[9] It can be shown from (21) that the second and third terms of right-hand side disappear as n tends to the infinity, so that $\pi(+\infty) = \lambda(+\infty)S_0$

the oligopolists to a large extent and would even prefer to raise tax revenues from the extraction industry. Let me refer to Benchekroun and Long (2004) for the reasons why economies in regulation through taxes/subsidies may be desirable.

The question addressed in this subsection is how the market structure affects the maximum tax revenue the regulator can extract from (or the minimum subsidy transfer he has to give up to) the mine industry while inducing efficiency.

Among the family of optimal tax/subsidy schemes of proposition 1, one can show (see the proof of the next proposition) that the cheapest one binds the initial restriction $\tau^*(0) > \underline{\tau}$. Let $\{\underline{\tau}^*(t)\}_{t\geq 0}$ denote this particular tax/subsidy scheme. From (19), the threshold $\underline{\tau}$ is decreasing in n. This suggests that the more concentrated the industry is, the harder it is to raise funds from it while optimally regulating.

However, from above, remind that an optimal *ad valorem* tax under a more concentrated industry is less falling over time (or more rising) than the initially identical optimal *ad valorem* tax under a less concentrated structure.

Overall, the more concentrated the industry is, the larger τ must initially be whereas the more decreasing τ should be over time. Let K denote the maximum tax revenue (or, if negative, the minimum subsidy transfer) discounted at date 0 the regulator can raise from the extraction sector under an optimal tax/subsidy policy: $K = \int_0^{+\infty}(1 - \underline{\tau}^*(t))P(R^*(t))R^*(t)e^{-rt}\,dt$. The following proposition assesses the effect of the number of extractors on K, and thus on the cost of regulation.

Proposition 3 *The more concentrated the extraction sector is, the more expensive it is to regulate it,* i.e. *K is increasing in n.*

Proof of proposition 3 *See the appendix.*

The total rent of the extraction sector is generally all the larger as this sector is concentrated. However, it is then more difficult for the regulator to capture these potential tax revenues if he wants to induce efficiency. In spite of the flexibility of time-dependent taxation policies and the fact that the regulator can exploit the exhaustibility constraint, the standard result that inducing an imperfectly competitive sector to be efficient is all the more expensive as it is concentrated still holds.

4. Conclusion

In a standard partial equilibrium model, I have solved and studied the optimal taxation of a fossil fuel (polluting exhaustible resource) under oligopolistic extraction.

A single taxation instrument is sufficient to correct the distortion of the extraction path due to the external effect of pollution and market power: *a market structure-adjusted environmental tax*. The family of such optimal taxation policies is infinite. However, the regulator is limited by the need to leave a positive marginal instantaneous profit to the extractors. The examination of how this family is affected by the degree of concentration of the extraction sector reveals that: (1) the stronger the concentration, the less falling (or the more rising) the optimal tax rate, and (2) concentration increases the total rent of the sector while it reduces the potential tax revenues to be earned by the regulator through an efficiency-inducing taxation policy.

Further research in this field naturally includes the determination and analysis of the subgame perfect optimal taxation schemes. However, their explicit characterization is technically extremely difficult. Another interesting extension is to consider the entry decision of the oligopolists to deal with the effects of environmental policies on the market structure of the extraction sector. I am currently working on both of these projects.

Appendix

Proof of proposition 1 □ *Log-differentiating equation (17) and rearranging gives the differential equation:* $g_\tau(t) = \frac{1}{\alpha} g_R^*(t) + r - rc\frac{\alpha n}{\alpha n - 1} R^*(t)^{1/\alpha} \tau(t)^{-1}$. *The solutions of this equation are given by (18).*

*Among these functions, let me now eliminate those which are not strictly positive (*i.e. *such that* $\exists t \geq 0 : \tau(t) \leq 0$*) and those which don't ensure the participation constraint* $\lambda > 0$.

From (18), $\tau^*(t) > 0, \forall t \geq 0$ *is equivalent to* $\tau^*(0) R_0^*(S_0)^{-1/\alpha} + (e^{-rt} - 1)\alpha nc/(\alpha n - 1) > 0, \forall t \geq 0$, *which is equivalent to* $\tau^*(0) R_0^*(S_0)^{-1/\alpha} - \alpha nc/(\alpha n - 1) > 0$, i.e. $\tau^*(0) > \underline{\tau}$.

λ *is constant. Hence, from (17), it is equal to* $\lambda = \tau^*(0) R_0^*(S_0)^{-1/\alpha}(\alpha n - 1)/(\alpha n) - c$. *Then,* $\lambda > 0$ *is equivalent to* $\tau^*(0) > \underline{\tau}$. ■

Proof of proposition 2 □ *Let all the critical variables write as functions of parameter* n.

To examine the effects of n*, one can consider that* n *is a continuous variable,* i.e. $n \in \mathbb{R}, n \geq 1$. *Indeed, since the parameter* n*, defined as a continuous variable, affects continuously* $\lambda(n)$*,* $R(n,t), \forall t \geq 0$ *and* $\pi(n)$*, if any of these variables is monotonous in* n*, then, it is also monotonous in* n*, defined as a discrete variable.*

Let me first prove expression (21). On the one hand, from (15), $\lambda(n) = \big(\tau(t) R(n,t)^{-1/\alpha} - c - (1/(\alpha n))\tau(t) R(n,t)^{-1/\alpha}\big) e^{-rt}$. *Rearranging, one obtains:* $\big(\tau(t) R(n,t)^{-1/\alpha} - c\big) e^{-rt} = \lambda(n) + (1/(\alpha n))\tau(t) R(n,t)^{-1/\alpha} e^{-rt}$. *On the other hand,* $\pi(n) = \int_0^{+\infty} R(n,t)\big(\tau(t) P(R(n,t)) - c\big) e^{-rt}\, dt = \int_0^{+\infty} R(n,t)\big(\tau(t) R(n,t)^{-1/\alpha} - c\big) e^{-rt}\, dt$. *Using the former equation and substituting in the latter, one finds:* $\pi(n) = \int_0^{+\infty} R(n,t)\big(\lambda(n) + (1/(\alpha n))\tau(t) R(n,t)^{-1/\alpha} e^{-rt}\big)\, dt$. *Since* $\lambda(n)$ *is constant over time, the profit rewrites:* $\pi(n) = \lambda(n)\int_0^{+\infty} R(n,t)\, dt + (1/(\alpha n))\int_0^{+\infty} \tau(t) R(n,t)^{(\alpha-1)/\alpha} e^{-rt}$. *Using eventually (14) and adding and subtracting* $\lambda(+\infty) S_0$*, one gets expression (21).*

Let me now study the effect of an increase in n *on* $\lambda(n)$. *Rearranging (15), one gets:* $R(n,t) = \tau(t)^\alpha (\alpha n/(\alpha n - 1))^{-\alpha} \big(\lambda(n) e^{rt} + c\big)^{-\alpha}$. *Integrating both sides, one finds:* $S_0 = \int_0^{+\infty} \tau(t)^\alpha (\alpha n/(\alpha n - 1))^{-\alpha} \big(\lambda(n) e^{rt} + c\big)^{-\alpha}\, dt$. *Since* S_0 *is given exogenously, independently of* n*, and* $\alpha n/(\alpha n - 1)$ *is decreasing in* n*, this implies that* $\partial\lambda(n)/\partial n > 0$.

Next, let me examine how n *affects* $\pi(n)$. *From above,* $\pi(n) = \lambda(n) S_0 + (1/(\alpha n))\int_0^{+\infty} \tau(t) R(n,t)^{(\alpha-1)/\alpha} e^{-rt}\, dt$. *Hence,* $\partial\pi(n)/\partial n = \big(\partial\lambda(n)/\partial n\big) S_0 - (\alpha/(\alpha n)^2)\int_0^{+\infty} \tau(t) R(n,t)^{(\alpha-1)/\alpha} e^{-rt}\, dt + ((\alpha-1)/(\alpha^2 n))\int_0^{+\infty} \tau(t) R(n,t)^{-1/\alpha}\big(\partial R(n,t)/\partial n\big) e^{-rt}\, dt$. *Note, from (15), that* $\partial\lambda(n)/\partial n$

$= (\alpha/(\alpha n)^2)\tau(t)R(n,t)^{-1/\alpha}e^{-rt} - ((\alpha n - 1)/(\alpha^2 n))\tau(t)R(n,t)^{-1-1/\alpha}\big(\partial R(n,t)/\partial n\big)e^{-rt}$. *Thus,* $\big(\partial\lambda(n)/\partial n\big)S_0 = \int_0^{+\infty} R(n,t)\big(\partial\lambda(n)/\partial n\big)\,dt =$ $(\alpha/(\alpha n)^2)\int_0^{+\infty}\tau(t)R(n,t)^{(\alpha-1)/\alpha}e^{-rt}\,dt$ $-((\alpha n-1)/(\alpha^2 n))\int_0^{+\infty}\tau(t)R(n,t)^{-1/\alpha}\big(\partial R(n,t)/\partial n\big)e^{-rt}\,dt$. *Substituting the latter expression in the above expression of* $\partial\pi(n)/\partial n$*, one gets:* $\partial\pi(n)/\partial n = ((1-n)/(\alpha n)\int_0^{+\infty}\tau(t)R(n,t)^{-1/\alpha}\big(\partial R(n,t)/\partial n\big)e^{-rt}\,dt$.

In the latter equation, let me show that the integral is positive. The binding exhaustibility constraint, $\int_0^{+\infty} R(n,t)\,dt = S_0$*, implies:* $\int_0^{+\infty}\big(\partial R(n,t)\partial n\big)\,dt = 0$. *Recalling* $\partial g_R(n,t)/\partial n < 0, \forall t \geq 0$*, it also implies:* $\exists T > 0 : \partial R(n,t)/\partial n \geq (\leq) 0, \forall t \leq (\geq) T$ *and* $\int_0^T \big(\partial R(n,t)/\partial n\big)\,dt = -\int_T^{+\infty}\big(\partial R(n,t)/\partial n\big)\,dt > 0$. *Moreover, from (16),* $g_\tau - (1/\alpha)g_R - r < 0$*, thus implying that* $\tau(t)R(n,t)^{-1/\alpha}e^{-rt}$ *is decreasing in* t*, and then:* $\forall (t,t') \in [0,T[\times]T,+\infty[, \tau(t)R(n,t)^{-1/\alpha}e^{-rt} > \tau(t')R(n,t')^{-1/\alpha}e^{-rt'}$. *Hence, from above,* $\int_0^T \tau(t)R(n,t)^{-1/\alpha}\big((\partial R(n,t)/\partial n\big)e^{-rt}\,dt > -\int_T^{+\infty}\tau(t)R(n,t)^{-1/\alpha}\big(\partial R(n,t)/\partial n\big)e^{-rt}\,dt$, *what implies eventually that* $\int_0^{+\infty}\tau(t)R(n,t)^{-1/\alpha}\big(\partial R(n,t)/\partial n\big)e^{-rt}\,dt > 0$ *and, then, since* $1-n \leq 0$, $\partial\pi(n)/\partial n < 0$.

Using $\partial\lambda(n)/\partial n > 0$ *and* $\partial\pi(n)/\partial n < 0$*, one deduces from (21) that the term on the far right of equation (21) is decreasing in* n. ■

Proof of proposition 3 □ *From (18), one can easily show that two different taxes of* Θ^* *cannot cross,* i.e. $\forall\big(\{\tau^*(t)\}_{t\geq 0}, \{\tau^{*'}(t)\}_{t\geq 0}\big) \in \Theta^{*2}, \tau^*(0) \neq \tau^{*'}(0), \nexists T > 0 : \tau^*(T) = \tau^{*'}(T)$. *Hence,* $\{\underline{\tau}^*(t)\}_{t\geq 0} \in \Theta^*, \underline{\tau}^*(0) = \underline{\tau}$*, is such that* $\forall\{\tau^*(t)\}_{t\geq 0} \in \Theta^*, \underline{\tau}^*(t) \leq \tau^*(t), \forall t \geq 0$. *Note that, by definition, each element of* Θ^* *results in the same outcome* $\{R^*(t)\}_{t\geq 0}$ *and thus* $\{P(R^*(t))\}_{t\geq 0}$. *The tax/subsidy scheme* $\{\underline{\tau}^*(t)\}_{t\geq 0}$ *thus maximizes on* Θ^* *the discounted tax revenues (or minimizes the discounted subsidy transfers) of the regulator:* $\int_0^{+\infty}(1-\tau^*(t))P(R^*(t))R^*(t)e^{-rt}\,dt$. *From (18),* $\underline{\tau}^*(t) = R^*(t)^{1/\alpha}c\alpha n/(\alpha n - 1)$. *Hence,* $K = \int_0^{+\infty} R^*(t)e^{-rt}\big(R^*(t)^{-1/\alpha} - c\alpha n/(\alpha n-1)\big)\,dt$*, which appears to be increasing in* n. ■

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