Economics Bulletin

The examination of the validity of the Divisia price index for the almost ideal demand system model: Some Monte Carlo results

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Abstract

In this paper, we have investigated about the validity of an exact price index suggested by Feenstra and Reindorf (2000) in the almost ideal demand (AID) system model. This index can express by the use of the Divisia index with the weighted shares, and it has been evaluated using data on the expenditure shares and prices at two data points. Our Monte Carlo experiments show that the Divisia index does not perform so suitable estimates in any design. And then we find that the Divisia index would yield the poor estimates against the AID price index.

Acknowledgements The author is grateful to Professors Kazuhiro Ohtani and Hisashi Tanizaki for their guidance and helpful comments
Citation: Ogura, Manami, (2008) "The examination of the validity of the Divisia price index for the almost ideal demand system model: Some Monte Carlo results." *Economics Bulletin,* Vol. 3, No. 14 pp. 1-10
Submitted: November 15, 2007. **Accepted:** March 6, 2008.
URL: http://economicsbulletin.vanderbilt.edu/2008/volume3/EB-07C50005A.pdf

1. Introduction

Feenstra and Reinsdorf (2000) suggested an exact price index for the almost ideal demand (AID) model by using data on budget shares and prices at two comparison points, and geometric mean. They then expressed it as a convenient and exact measure by using the Divisia price index. In many empirical researches of demand systems, the linearized AID (LAID) model with the linearly approximated price index has often been employed. While the use of the linearly approximated price index brings some problems in estimates, for example, the bias in estimates, the inconsistency of estimator, etc. (See also Buse 1994, 1998, and Pashardes 1993), the linearized model must be still efficient in the aspect of recent time series estimation (e.g., Attfield 1997, and Duffy 2002, 2003).[1]

There were several previous studies reminded some problems for the LAID model. Green and Alston (1990), and Buse (1994) suggested the appropriate formula of elasticity for the LA/AID model, pointing out the biases and inconsistency of estimators. Buse (1994) also concluded the use of the Stone price index should be avoided in the AID model. In addition to them, there were the previous studies noticed for biases of estimates by Pashardes (1993), and Alston, et al. (1994). Not only the improvement of formula in elasticity but also the examination of the valid price index to replace the Stone index was conducted by Moschini (1995)[2], Asche and Wessells (1997), and Buse and Chan (2000). Our expectations obtained from their results are that the performance of invariant Laspeyres and Tornqvist indices are superior to the Stone index, and the LAID model is equivalent to the AID model if the systems are evaluated at the point of price normalization to unity.

In this paper, we investigate the validity of the Divisia price index as the alternative index to replace the existing Stone price index. The Divisia index is an exact index for the AID model evaluated using data. We expect that, if the Divisia index is more efficient in the AID model, the performance of it must be superior to the Stone index in biases of estimates.

2. The model

2.1 LA/AID model

The AID system model provided by Deaton and Muellbauer (1980) is given by

$$w_{it} = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + \beta_i \ln(x_t / P_t) + u_{it}, \qquad i, j = 1, \ldots, n, \quad t = 1, \ldots, T. \tag{1}$$

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[1] When price variables are nonstationary, particularly, in the industrialized economies such as Japan, U.K, and U.S, the cointegrated demand systems are expressed by the linear relationships. In footnote 7 of Buse and Chan (2000), they touched upon long run equilibrium relationships between shares, prices, and expenditure variables.

[2] Moschini (1995) conducted a Monte Carlo experiment using three price indices of Tornqvist, Paasche, and Laspeyres, which are invariant to the change in the units of measurement, and the Stone index, which is not invariant to the change in units of measurement; he has shown that the Stone index results in a less accurate approximation of the true price index as compared to the other three price indices are used.

where w_{it} is the budget share of ith commodity in period t, p_{jt} is the price of jth commodity, $\ln(x_t / P_t)$ is the log real income with an aggregate price index (deflator) $\ln P_t$, and u_{it} is the error terms with *i.i.d* (0, **Σ**). The true AID price index is a non-linear form as follows:

$$\ln P_t = \alpha_0 + \sum_{k=1}^{n} \alpha_k \ln p_{kt} + 1/2 \sum_{j=1}^{n} \sum_{k=1}^{n} \gamma_{jk} \ln p_{jt} \ln p_{kt} . \tag{2}$$

The AID model should satisfy the adding-up, homogeneity, and symmetry conditions; the adding-up restriction is given by $\Sigma_i \alpha_i = 1$ and $\Sigma_i \gamma_{ij} = \Sigma_i \beta_i = 0$. This restriction is automatically satisfied, and the n -1 equations are used in the estimation. The homogeneity restriction for price parameters is $\Sigma_j \beta_{ij} = 0$, and the symmetry restriction is $\beta_{ij} = \beta_{ji}$.[3] (See also Buse (1998) about the rejection bias of homogeneity in the LA/AID model)

The AID system model of (1) is classified into the translog family with the translog indirect utility function. Generally, the form of price index must follow to the form of utility function, so the AID model has the translog price index. On the other hand, Feenstra and Reinsdorf (2000) have devised an exact index for the AID model from the view of data point.

The Divisia price index P^D suggested by Feenstra and Reinsdorf (2000) will be defined as a share-weighted integral of price changes, between an initial and final data point. Let w_t equal $w(\mathbf{p}_t, x_t)$, which is a function of budget shares and $\dot{\mathbf{p}}_t$ denote the vector of log price changes $\ln(\mathbf{p}_1/\mathbf{p}_0)$. The Divisia price index is generally expressed as:

$$\ln P^D = \int_0^1 w_t \cdot \dot{p}_t dt . \tag{3}$$

They showed that the integral for P^D can be evaluated by a two-thirds weight on the shares implied by the geometric mean of the initial and final values for prices and income. Therefore, equation (3) is redefined by

$$\ln P_t^D = \left[\frac{1}{6} w(p_0, x_0) + \frac{2}{3} w(p_{0.5}, x_{0.5}) + \frac{1}{6} w(p_1, x_1) \right] \cdot \ln(p_1 / p_0) , \tag{4}$$

where $\ln(\mathbf{p}_1/\mathbf{p}_0)$ denotes a vector with elements of the $\ln(p_{i1} / p_{i0})$. Thus, this index can express as the linear form by the data. It is an advantage of the use of the Divisia price index in empirical

[3] Although these conditions are derived from the true price index, whether they are also satisfied in the linearized price index cannot be guaranteed. Chen (1993) has shown that the linearized AID model with the Stone index satisfies the adding-up and homogeneity conditions. However, with regard to symmetry, it is not guaranteed whether or not the condition is satisfied. This unsatisfaction will be applied to Divisia price index.

research as well as the Stone index. The first term in the right-hand side of (4) can be approximately equal to w_{it}.

The Stone price index P^S, which is popularly used in the empirical estimation of demand system, is expressed in

$$\ln P_t^S = \sum_{i=1}^{n} w_{it} \ln p_{it}. \tag{5}$$

In many empirical studies, nonlinear index (2) can be often replaced to the Stone index of (5). From several previous studies (See also Buse 1994, and Pashardes 1993), we found the estimators have biases and are inconsistent by using the linearized approximately index instead of true AID price index (2). In light of these problems, we investigate the validity of the Divisia price index by Feenstra and Reinsdorf (2000) in order to improve the extent of the bias in linear price index. We supposed if the Divisia price index is an exact index evaluated using the data, the biases of estimates must be so small that we can ignore or mitigated it.

2.2 Price elasticities

Two alternative formulas for uncompensated price elasticities using the LA/AID estimates are represented in this section. The formulas about uncompensated price elasticity ε_{ij} from the AID and LA/AID models are well summarized by Alston et al. (1994, pp.352-353). We use the same expressions as them, so that we omitted the particulars of definition about these formulas.
In the AID model of equation (1) with (2), the uncompensated price elasticity is derived from

$$\varepsilon_{ij}\,(AI) = -\delta_{ij} + \gamma_{ij} / w_i - \beta_i / w_i\,(\alpha_j + \sum_{k=1}^{n} \gamma_{kj} \ln p_k). \tag{6}$$

where δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$ for $i = j$ or $\delta_{ij} = 0$ for $i \neq j$). Because this definition requires estimates of α_i, price elasticities calculated by the LA/AID model must have large biases derived from the constant term α_i^*. Treating the shares in the Stone index as the constant[4], the associated elasticity becomes

$$\varepsilon_{ij}\,(LA) = -\delta_{ij} + \gamma_{ij} / w_i - (\beta_i w_j / w_i). \tag{7}$$

Equation (7) must be the appropriate formula for the Divisia price index because the first term in the right-hand side of (4) can be approximately equal to w_{it}. Two formulas from (6) and (7) are

[4] Under the unit price, α_0 is set to expenditure in the base period such that α_i equals to the predicted budget share. Expenditure equals to one, and then α_0 will be zero.

compared in the Monte Carlo experiments of section 3. And we investigate the validity of the Divisia price index from the data.

3. Monte Carlo experiments

3.1 The Monte Carlo design

We generate the data of three commodities allowing two settings such as (i) multi-collinearity (four cases) and (ii) price variance (two cases). The parametric structure is a constant because their change does not have an effect on experimental results (See Buse 1994), and they were chosen to satisfy the adding-up, homogeneity and symmetry conditions. The vector of budget share is defined as $\mathbf{w}$ = (0.4, 0.3, 0.3), and the chosen vectors are $\boldsymbol{\beta}$ = (0.35, -0.2, -0.15) and $\boldsymbol{\alpha}$ = (0.05, 0.5, 0.45).[5] Therefore the vector of income elasticities is set at $\boldsymbol{\eta}$ = (2.0, 0.33, 0.50).[6] And the matrix of price parameters $\boldsymbol{\gamma}$ is defined by satisfying the adding-up, homogeneity and symmetry restrictions. Log prices are drawn from a multivariate normal distribution with a covariance matrix of high, medium, low or mixed collinearity, and small or large price variance. For high collinearity case, the correlation structure of log prices is: ρ_{12} = 0.99, $\rho_{13} = \rho_{23}$ = 0.98. For the mixed collinearity case, they are: ρ_{12} = 0.75, ρ_{13} = -0.5, ρ_{23} = 0.5. Log prices are normalized at one data point in order to be consistent with the AID model as Asche and Wessells (1997). As they showed, the Stone index will be equal to the AID price index at data point of unit price, and so do the Divisia index. Log real income is generated from the AID cost function (Deaton and Muellbauer, 1980), $\ln(x_t / P_t) = U\beta_0 \exp(\Sigma\beta_k \ln p_{kt})$ with the utility index $U = 1$ and $\beta_0 = 1$. The new observed shares w_{it}^* are generated by $w_{it}^* = w_{it} + u_{it}$ with the disturbance drawn from a multivariate normal distribution with the variance-covariance matrix of a positive definite matrix. All shares should satisfy with the (0, 1) interval. At each of the 8 design points, 1000 iterations were performed in the LAID model with the generated budget shares. The observations were used sample sizes of 35 and 100.

Let each calculated price elasticity in the *p-th* iteration be $\hat{\varepsilon}_{ij}^{(p)}$ and the true values of the uncompensated price elasticity be ε_{ij}^{0}.[7] Then, the Mean Square Error (MSE) for each value of price

[5] If the share equations are evaluated at unit prices, the constant terms are determined by $w_i = \alpha_i + \beta_i$.

[6] The income elasticity is defined as $\eta_i = 1 + \beta_i / w_i$.

[7] From Deaton and Muellbauer (1980)'s suggestion, the true uncompensated price elasticities are determined by $\varepsilon_{ij} = (s_{ij}^* / w_i) - w_j\eta_i$, where $s_{ij}^* = \gamma_{ij} + \beta_i\beta_j \ln(x_t / P_t) - w_i\delta_{ij} + w_i w_j$. The negativity condition of the Slutsky matrix,

elasticities can be expressed as:

$$\text{MSE} = 1/M\sum_{p=1}^{M}(\hat{\varepsilon}_{ij}^{(p)} - \varepsilon_{ij}^{0})^2 , \qquad i, j = 1,\ldots,n \tag{8}$$

where M = 1000 is the number of iterations in the Monte Carlo experiments.

3.2 Simulation results

Each calculated elasticities and their Mean Square Errors (MSE) obtained in the Monte Carlo experiments are listed in Tables 1. We present the results of one experimental design point there. At each design, the aggregate MSE of two price indices are recorded in Table2. The main results of our experiments can be summarized as follows:

(i) Table 1 shows each estimate of elasticities and MSE, having the high collinearity with a large price variance. In both indices of Table 1, the use of the ε (AI) formula derives the inaccurate estimates than the ε (LA) formula in much elasticity. For instance, the inaccuracies produced from the under-estimates of ε_{22} and ε_{23} and the over-estimate of ε_{13} are remarkable. These can be due to the following factors: first, the bias of the price parameters for the second and third equations would be large[8] because these parameters are derived from the adding-up and homogeneity condition; the biases from other parameters accumulate through their conditions. Second, the intercept in the LAID model in equation (1) may have contributed to the bias. For instance, in the Stone index, because the intercept in the LAID model is given by $\alpha_i^* = \alpha_i - \beta_i \alpha_0$, it becomes apparent that using α_i^* generates the biased estimates of elasticities. Then, we can write ε (AI*) = ε (AI) + $\beta_i \beta_j \alpha_0 / w_i$. Accordingly, we may systematically over-estimate (or under-estimate) the price elasticity ε (AI*). Thus, we anticipated that the calculated bias of ε (AI) becomes large since the use of it in the LAID model contains the extra intercept bias. On the other hand, in the comparison of two indices, some performances of the Stone index show an enough approximation to the ε (AI) formula than the Divisia index. As indicated in Table 1, the difference between them is mainly attributed to the large bias of ε_{23} emerging as a result of under-estimate, and that of ε_{13} as a result of over-estimate in the Divisia index. These biases can be related to the above first factor for the homogeneity condition. Table 1 also shows that both indices generate the unbiased estimates for the income elasticities, and are the precision to the η_x formula.

(ii) Table2 shows the aggregate MSE of elasticities calculated from two indices. We find that the results by data set with a large price variance are accurate than ones with a small variance in both price indices. In particular, the use of ε (LA) formula would bring the

whose elements $\{s_{ij}\}$ are negative semi-definite, is not satisfied at all data point, but will be satisfied at data point of unit price.

[8] The calculation results in parameters are omitted. The results for them are available from the author on request.

better unbiased estimates against the ε (*AI*) formula. As the results of Buse (1994) have also shown through his Monte Carlo results, the use of ε (*LA*) formula is more appropriate in the LAID model. Further, we find that the ε (*AI*) and ε (*LA*) formulas would bring both inaccurate estimates when the multi-collinearity among log prices is low. Contrary to this, as the degree of multi-collinearity increases, the accuracy of estimates also increases. For instance, when the log prices have the high collinearity with a large variance, the Stone index has the most favorable approximation to the AID price index of all cases. Buse and Chan (2000) also resulted that the high collinearity made the accuracy of estimates well. While this tendency is true of the Stone price index, it would not be necessarily applied to the Divisia index. In the Divisia index, having the middle or mixed collinearity with a large covariance shows a good approximation to the AID price index as well as the Stone index, but for high collinearity, the approximation is not so good. For the income elasticities, the Stone index generates the unbiased estimates in any design. In the Divisia index, the use of η_x formula would produce the suitable estimates under high collinearity; however, in the low or mixed collinearity, the use of it may yield the poor estimates. Especially, in the case of the low collinearity with a small variance, the use of η_x formula must cause a large bias unfortunately.

For the true AID price index, the Divisia index would yield the poorer approximation in many designs than the Stone index. Especially, under low or mixed collinearity with a small variance, the use of the Divisia index causes the biased estimates and the precision of approximation gets poor. Although the Divisia index shows the superior performance to the Stone index in the particular cases of high or medium collinearity with a small variance, the approximation to the AID price index is not so remarkable.

Alston et al. (1994) showed that, even if the sample size increases from 35 to 70, the above performance does not change. In our calculations, the same results are obtained when the sample size increases from 35 to 100.

4. Conclusion

In this paper, we have investigated about the validity of the Divisia price index in the AID model. Feenstra and Reindorf (2000) used the Divisia index as an exact price index in the AID model, and the index has been evaluated using data on the expenditure shares and prices at two data points. In our experiments, we used the Stone index as a benchmark of the comparison to the Divisia index. Our results show that the Divisia index yields the poorer approximation to the AID price index in many designs than the Stone index. As the case may be, the use of Divisia index would derive the largely biased estimates. Thus the Divisia index is not always preferable to the Stone index in terms of accuracy of approximation. In other words, the indiscriminate use of the

Divisia index instead of the true AID price index is not recommended in applied research.

In addition, we also obtain the expected results that demonstrate the unsuitability of ε (*AI*) formula in the comparison to the ε (*LA*) formulas. The estimates calculated from the ε (*AI*) formula would not relatively yield good results because the values estimated from it have the additional bias when applying it to the LAID model. As long as the ε (*AI*) formula is used, the bias of estimates can be serious; however, high collinearity among log prices would slightly improve the precision of elasticities for the approximation. On the contrary, the ε (*LA*) formula would produce the accurate estimates. This conclusion is consistent with Buse and Chan (2000). Thus, neither do the Stone nor Divisia indices will produce the accurate estimates if the ε (*AI*) formula is used to calculate price elasticity.

Acknowledgements

The author is grateful to Professors Kazuhiro Ohtani and Hisashi Tanizaki for their guidance and helpful comments

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Table1. Mean Square Errors of the calculated elasticities when log prices have large variance and high collinearity (T=35)

Elasticity	True Value	Divisia index $\varepsilon(AI)$	Divisia index $\varepsilon(LA)$	Stone index $\varepsilon(AI)$	Stone index $\varepsilon(LA)$
(I) Price					
ε_{11}	-0.746	-0.869 (0.016)	-1.175 (0.190)	-0.865 (0.015)	-1.160 (0.172)
ε_{12}	-0.635	-0.128 (0.260)	-0.332 (0.112)	-0.136 (0.248)	-0.335 (0.089)
ε_{13}	-0.619	-0.079 (0.541)	-0.373 (0.057)	-0.086 (0.285)	-0.380 (0.057)
ε_{21}	-0.111	0.269 (0.145)	0.499 (0.407)	0.293 (0.164)	0.520 (0.399)
ε_{22}	-0.468	-1.045 (0.332)	-0.892 (0.161)	-1.047 (0.336)	-0.895 (0.182)
ε_{23}	0.249	-0.168 (0.418)	0.052 (0.049)	-0.181 (0.186)	0.044 (0.042)
ε_{31}	-0.293	0.236 (0.285)	0.241 (0.266)	0.251 (0.296)	0.249 (0.294)
ε_{32}	0.398	-0.089 (0.237)	-0.087 (0.240)	-0.097 (0.245)	-0.098 (0.246)
ε_{33}	-0.605	-1.146 (0.294)	-1.141 (0.348)	-1.157 (0.304)	-1.158 (0.306)
(II) Income		η_x		η_x	
η_1	2.00	1.880 (0.014)		1.874 (0.016)	
η_2	0.33	0.339 (9.65e-5)		0.329 (0.75e-6)	
η_3	0.50	0.487 (0.17e-3)		0.505 (0.28e-4)	

Notes: Numbers in parenthesis indicates MSE, which is the mean squares error for each parameter. Values are average of 1,000 of estimated elasticities. The calculation of elasticities uses the mean of budget share w_i.

Table2 Aggregate MSE of the calculated elasticities (T=35)

Design Point	Divisia index			Stone index		
	$\varepsilon(AI)$	$\varepsilon(LA)$	η_x	$\varepsilon(AI)$	$\varepsilon(LA)$	η_x
(i) Small variance						
High collinearity	2.291	2.102	0.036	2.349	2.132	0.027
Medium	2.439	2.234	0.026	2.508	2.282	0.025
Low	4.413	10.913	26.520	3.039	2.806	0.020
Mixed	2.493	4.073	4.788	2.425	2.256	0.025
(i) Large variance						
High collinearity	2.528	1.837	0.015	2.082	1.791	0.016
Medium	2.131	1.832	0.015	2.126	1.833	0.025
Low	2.387	2.009	0.080	2.371	2.051	0.021
Mixed	2.121	1.838	0.356	2.089	1.832	0.024

Notes: Values indicate the aggregate mean square error of elasticities. (i) and (ii) indicate the degree of price variance. High collinearity, medium - , low - , and mixed collinearity indicate the correlation between log prices.