E C O N O M I C S B U L L E T I N

Relative Deprivation with Imperfect Information

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Abstract

The paper proposes an alternative index of relative deprivation which allows for selection of the reference group and imperfect information, two central elements of modern theories of social justice. An application to real data and a simulation on artificial data illustrate the use and some properties of the index.

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1. Relative deprivation

In an article appeared in 1979 on the Quarterly Journal of Economics, Yitzhaki (1979) proposed an index of relative deprivation based on a concept of relative deprivation developed first by Stouffer *et Al.* (1949) in a study of the US army and later formalized by Runciman (1966) in a theory of social justice. In this article we propose to extend the Yitzhaki index so as to encompass the notions of imperfect information and reference group already present in Runciman theory.

The basic idea behind the Yitzhaki index is that the sense of deprivation felt by individuals is determined by the relative position they occupy in the income scale. Relative deprivation is measured by adding the distances that separate the income of each individual from the incomes of all those people occupying an upper rank in the income scale. The average of these distances measures the individual relative deprivation and the average relative deprivation for all individuals measures relative deprivation in a society. In discrete terms the formulas for individual and collective relative deprivation are as follows:

$$RD_i = \frac{1}{n} \sum_{j>i} (y_j - y_i) \tag{1}$$

$$RD = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j>i} (y_j - y_i) \tag{2}$$

Where y = Income vector with $y = (y_1, ..., y_n)$; y_i and y_j = income vectors sorted in ascending order of income and n = sample or population size. Yitzhaki showed that the relative deprivation index for a society is equal to the absolute Gini index, a finding that generated a stream of contributions which explored further the properties and use of such index (Hey and Lambert, 1980; Berrebi and Silber, 1985; Chakravarty, 1995 and 1997; Bossert, D'Ambrosio and Peragine, 2004).

The underlying assumption of the RD index is that individuals are identical in all attributes except income. This is a very strong assumption given that the sense of deprivation that individuals may feel depends from both the *objective* situation they face - the relative position determined by relative income - and the *subjective* perception of this same relative position. Indeed, the subjective aspect of relative deprivation was a central argument in Runciman theory (1966): *"Relative Deprivation should always be understood to mean a sense of deprivation; a person who is 'relatively deprived' need not be 'objectively' deprived in the more usual sense that he is demonstrably lacking something"*. (p.10-11) Moreover, the RD index constrains the reference group by definition and *ex-ante* to the group of people with larger incomes. In Runciman theory the mechanism of group selection is also important: *"(...) The questions to ask are first, to what group is a comparison being made? second, what is the allegedly less well-placed group to which the person feels that he belongs? (...)"* (p. 14).

The RD index also excludes two other important aspects of relative deprivation. The first is the possibility that individuals may feel deprived because of imperfect information about the reference group. If I compare myself with someone whose attributes I know only in part, my sense of deprivation may arise simply because of this imperfect information. The second aspect is the possibility of negative values of relative deprivation which would depict 'satisfaction'. An individual or a society may well feel satisfied instead of deprived and exhibit negative deprivation. Yitzhaki saw the deprivation function as the

complement of the 'satisfaction' function. It seems useful instead to devise a function which allows for negative and positive values where negative values stand for 'satisfaction' and positive values stand for 'deprivation'.

2. Relative deprivation with imperfect information

We want to construct an index which allows for group selection, imperfect information and negative values.

The *reference group* is defined as the likes of the person observed. It is composed by all other individuals in a population each weighted by the degree of likeness with the person observed. The assumption is that people sense of deprivation depends on both the relative income position they occupy and on the relative personal characteristics they have. This is rather natural. If I have a lower income of someone with the same age and education as I have I will feel more deprived than if I have a lower income of someone older and better educated than I am. Allowing for such selection mechanism is simple. We build a measure that considers actual income as well as income conditioned on a set of personal characteristics. This can be done in several ways, for example with propensity score matching or linear regression. For illustrative purposes, we use a linear regression approach. We can run an income (y_i) equation on a vector of personal characteristics (X_i) and then estimate the predicted incomes (y_i^p) for all individuals as follows:

$$y_i = \alpha + \beta X_i + \delta_i + \varepsilon_i \tag{3}$$
$$y_i^p = a + bX_i \tag{4}$$
$$r_i = y_i - y_i^p \tag{5}$$

where δ_i is the effect of unobserved factors on income and ε_i is the measurement error.

Intuitively, predicted income can be interpreted as the income that an individual expects on the basis of his or her own personal characteristics relatively to those of others. With *perfect* information, individuals will be aware of all factors that determine income. In this case, predicted values of income are equal to observed values of income and the residuals and relative deprivation would be nil. With *imperfect* information relative deprivation will also depend on the fact that individuals do not observe all factors that explain income. It is irrelevant whether the unobserved factors are personal characteristics or other factors such as discrimination. What matters is that these factors are unobserved.

We can construct an alternative index of relative deprivation by combining the ideas described about the reference group and imperfect information with the more traditional concept proposed by Yitzhaki as follows:

$$RDI_i = \frac{1}{n}\sum_{j>i}(y_j - y_i) - \frac{1}{n}\sum_{j>k}\left(y_j^P - y_i^P\right) \tag{6}$$

$$RDI = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j>i}(y_j - y_i) - \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j>k}\left(y_j^P - y_i^P\right) \tag{7}$$

Where y_i and y_j are the income vectors ranked in ascending order of income and y_i^P and y_j^P are the predicted income vectors ranked in ascending order of predicted income.[1] The

[1]Note that the ranking of the same individual according to income and predicted income may be

RDI index has two components. The first is the RD Yitzhaki index already explained. The second is the same index calculated on the predicted values of income. We call this component RP. The RDI_i index is simply the difference between these two components.

As already mentioned and as shown by Yitzhaki (1979), the Yitzhaki index is equivalent to the absolute gini index (the mean times the Gini index) - μG. Given that the means of income and predicted incomes are the same by definition, our RDI index can be re-written as $RDI = \mu(G_y - G_{y^P})$ with $RD = \mu G_y$ and $RP = \mu G_{y^P}$. However, as we will see in the next section, the individual RDI_i scores and its RD_i and RP_i components are also worth estimating for graphic purposes.

The RDI index has several advantages vis-à-vis the RD index. It is closer to the Stouffer-Runciman concept of relative deprivation. It enables the researcher to define the criteria that people may use to identify the reference group by choosing the regressors of the income equation. It combines the ideas that relative deprivation may arise from relative income rank and from relative income conditional on personal characteristics. It allows for different degrees of information as the residuals change depending on the characteristics of the sample and on the regressors selected. The index can also result in positive and negative values of deprivation with negative values standing for satisfaction rather than deprivation.

3. Applications

We illustrate the use of the individual score RDI_i and index RDI using a data set of 534 observations on hourly wages extracted from the Unites States 1985 current population survey.[2] Suppose we want to compare relative wage deprivation between genders using the RDI_i and RDI formulas in (6) and (7). The objective is to understand which gender feels more deprived in terms of wages.

We can first calculate the RDI index with its two components RD and RP for the two genders and compare the two figures as we do in Table I where it is shown that relative deprivation is higher for women (1.8) than for men (1.0). The difference between the RD and RP components establishes that women are more deprived than men. Figure 1 plots the RD_i component against the RP_i component of the RDI_i index. We can appreciate whether each individual (each dot in the figure) is relatively deprived or relatively satisfied. Those relatively 'deprived' will have an RD_i score larger than the RP_i score and will be found above the 45 degrees line. Those relatively 'satisfied' will have a RP_i score larger than the RD_i score and will be found below the 45 degrees line.

We can also study stochastic dominance of the two gender distributions by plotting the Pen's parade and the Lorenz curve of the RDI_i score (Figure 2). Both the Pen's parade and the Lorenz curve for females are everywhere above those of males. Therefore, we can conclude (with a second degree confidence) that relative deprivation for women is higher than relative deprivation for men. We can also say that the proportion of satisfied individuals is much larger among men than among women as shown in both Figures 1 and 2.

One additional advantage of the RDI index over the RD index is the capacity of the former to capture cross-group variations in personal characteristics. This is like increasing

different. This is marked in the formula by the subscripts in the sum signs $j > i$ and $j > k$ where i and k stand for different rankings.

[2] These data are publicly available for download on the following web site:
http://www.economicswebinstitute.org/data/wagesmicrodata.xls

or decreasing the difference between the δ_i for males and females in equation (3) and it is shown by the Pen's parades plotted in Figure 3 and based on simulated data. In the four panels, the observed wage and the R-squared (set at 50%) are the same. Therefore the combined variance of $\delta_i$and ε_i remains fixed and the RD curve shown in the top-left panel of Figure 3 is the same for all simulations. What changes across the other three panels is the difference in predicted wages between men and women. We can see in Figure 3 that the gender bias of the residuals decreases from the top-right to the bottom-left panel. The RDI_i curves reflect changes in gender differences that the traditional RD curve would not be able to capture.

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Table I - RDI and Its Components

	Obs.	Mean	Std. Dev.
All			
RDI	533	1.4	1.6
RD	533	2.7	1.7
RP	533	1.3	1.1
Males			
RDI	288	1.0	1.5
RD	288	2.3	1.7
RP	288	1.3	1.1
Females			
RDI	245	1.8	1.5
RD	245	3.1	1.7
RP	245	1.3	1.0

Figure 1 - RDI Components

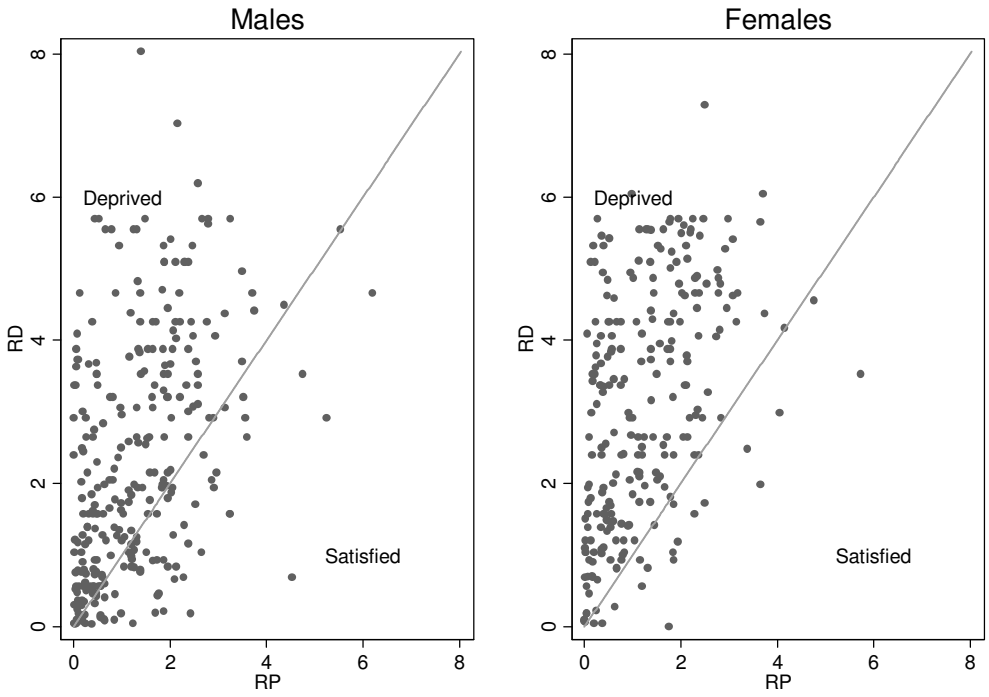

Figure 2 - Stochastic Dominance

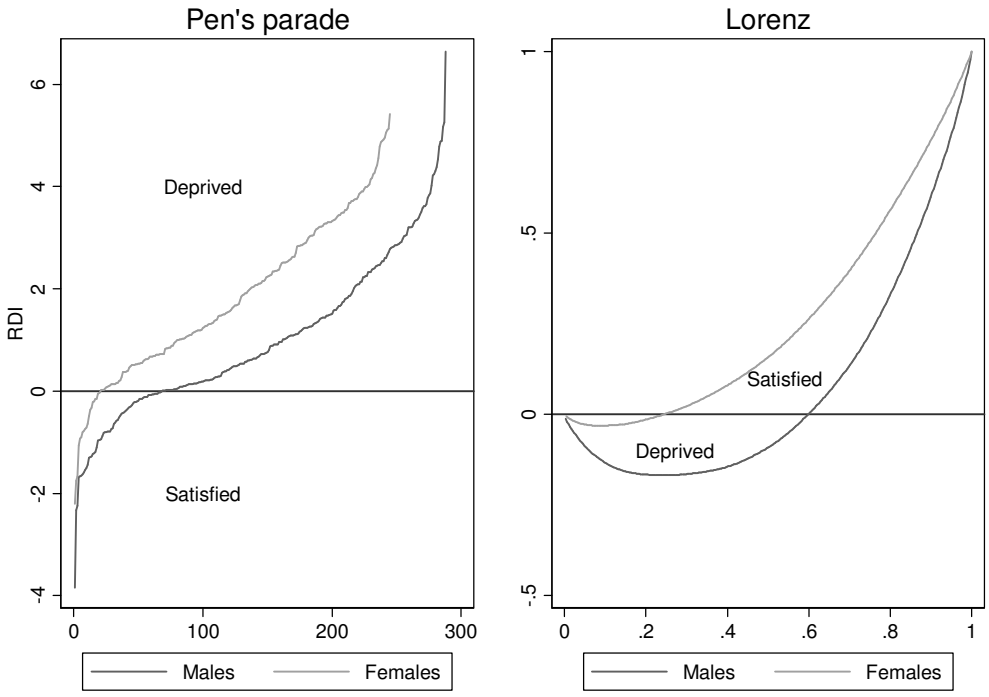

Figure 3 - Simulations